AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

THE MATHEMATICAL THEORIES OF LOGIC AND PROBABILITIES.

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思维规律的研究

的

思维规律，

论其所依据者

逻辑的数学理论以及
概率。

由

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TO

JOHN RYALL, LL.D.

VICE-PRESIDENT AND PROFESSOR OF GREEK

IN QUEEN'S COLLEGE, CORK,

THIS WORK IS INSCRIBED

IN TESTIMONY OF FRIENDSHIP AND ESTEEM

到

约翰·赖尔，法学博士

副校长兼希腊语教授

在科克女王学院，

谨以此书献给

以证友谊与敬意

PREFACE.

The following work is not a republication of a former treatise by the Author, entitled, "The Mathematical Analysis of Logic." Its earlier portion is indeed devoted to the same object, and it begins by establishing the same system of fundamental laws, but its methods are more general, and its range of applications far wider. It exhibits the results, matured by some years of study and reflection, of a principle of investigation relating to the intellectual operations, the previous exposition of which was written within a few weeks after its idea had been conceived.

That portion of this work which relates to Logic presupposes in its reader a knowledge of the most important terms of the science, as usually treated, and of its general object. On these points there is no better guide than Archbishop Whately's "Elements of Logic," or Mr. Thomson's "Outlines of the Laws of Thought." To the former of these treatises, the present revival of attention to this class of studies seems in a great measure due. Some acquaintance with the principles of Algebra is also requisite, but it is not necessary that this application should have been carried beyond the solution of simple equations. For the study of those chapters which relate to the theory of probabilities, a somewhat larger knowledge of Algebra is required, and especially of the doctrine of Elimination, and of the solution of Equations containing more than one unknown quantity. Preliminary information upon the subject-matter will be found in the special treatises on Probabilities in "Lardner's Cabinet Cyclopædia," and the "Library of Useful Knowledge," the former of these by Professor De Morgan, the latter by Sir John Lubbock; and in an interesting series of Letters translated from the French of M. Quetelet. Other references will be given in the work. On a first perusal the reader may omit at his discretion, Chapters X., XIV., and XIX., together with any of the applications which he may deem uninviting or irrelevant.

In different parts of the work, and especially in the notes to the concluding chapter, will be found references to various writers, ancient and modern, chiefly designed to illustrate a certain view of the history of philosophy. With respect to these, the Author thinks it proper to add, that he has in no instance given

前言。

本书并非作者先前一部题为《逻辑的数学分析》的论著的再版。其较早的部分确实致力于同一目标，并且以建立同一套基本规律体系开篇，但其方法更为一般，应用范围也远为广阔。它呈现的是经过数年研究与思考而成熟起来的、关于理智操作的一种研究原则的成果，而该原则此前的阐述，是在其构想形成后数周之内写成的。

本书中与逻辑相关的部分，预设读者具备这门科学中最重要的术语的知识(按通常的处理方式)，以及对其一般对象的了解。在这些方面，没有比沃特利大主教的《逻辑要义》或汤姆森先生的《思维规律纲要》更好的指南了。对这类研究的兴趣在当下的复兴，在很大程度上似乎应归功于前者。此外还需要对代数原理有所了解，但这一应用不必超出解简单方程的范围。对于研究那些与概率论相关的章节，则需要稍多一些的代数知识，尤其是消元学说以及含有一个以上未知量的方程的解法。关于题材的预备知识，可在《拉德纳万有文库》和《实用知识文库》中关于概率的专门论著中找到，前者出自德摩根教授之手，后者出自约翰·卢伯克爵士之手；还可在一系列由凯特勒先生的法文著作译出的有趣书信中找到。本书中还将给出其他参考文献。初次阅读时，读者可自行决定略去第x、xiv和xix章，以及任何他认为不吸引人或无关的应用部分。

在本书的不同部分，尤其是在最后一章的注释中，将会发现对古今多位作者的引用，这些引用主要是为了阐明关于哲学史的某种观点。关于这些引用，作者认为有必要补充说明，他从未在任何情况下给出

a citation which he has not believed upon careful examination to be supported either by parallel authorities, or by the general tenor of the work from which it was taken. While he would gladly have avoided the introduction of anything which might by possibility be construed into the parade of learning, he felt it to be due both to his subject and to the truth, that the statements in the text should be accompanied by the means of verification. And if now, in bringing to its close a labour, of the extent of which few persons will be able to judge from its apparent fruits, he may be permitted to speak for a single moment of the feelings with which he has pursued, and with which he now lays aside, his task, he would say, that he never doubted that it was worthy of his best efforts; that he felt that whatever of truth it might bring to light was not a private or arbitrary thing, not dependent, as to its essence, upon any human opinion. He was fully aware that learned and able men maintained opinions upon the subject of Logic directly opposed to the views upon which the entire argument and procedure of his work rested. While he believed those opinions to be erroneous, he was conscious that his own views might insensibly be warped by an influence of another kind. He felt in an especial manner the danger of that intellectual bias which long attention to a particular aspect of truth tends to produce. But he trusts that out of this conflict of opinions the same truth will but emerge the more free from any personal admixture; that its different parts will be seen in their just proportion; and that none of them will eventually be too highly valued or too lightly regarded because of the prejudices which may attach to the mere form of its exposition.

To his valued friend, the Rev. George Stephens Dickson, of Lincoln, the Author desires to record his obligations for much kind assistance in the revision of this work, and for some important suggestions.

5, Grenville-place, Cork,
Nov. 30*th.* 1853.

对于他经仔细审查后认为既无平行权威支持、亦不符合其所出著作之总体主旨的引文，他未曾采信。尽管他本乐于避免引入任何可能被解释为炫耀学问的内容，但他感到，无论对他的主题还是对真理而言，正文中的陈述都应当附有可供验证的手段。而如今，在结束一项其规模鲜有人能凭其表面成果加以判断的工作之际，倘若允许他片刻谈及自己从事并如今搁置此项任务时的心情，他愿说，他从未怀疑它值得他竭尽全力；他感到，它所能揭示的任何真理都不是私人的或任意的东西，其本质不依赖于任何人的意见。他完全清楚，有学识、有能力的学者在逻辑问题上所持的意见，与他的整部论证与程序所依据的观点直接相反。尽管他相信那些意见是错误的，他也意识到自己的观点可能不知不觉地受到另一种影响而扭曲。他尤其感到那种理智偏见的危险，这种偏见是长期关注真理的某一特定方面所容易产生的。但他相信，从这种意见的冲突中，同一真理只会更加摆脱任何个人掺杂而浮现出来；其各个部分将以其恰当的比例被看待；并且，由于可能附着于其阐述的单纯形式之上的种种偏见，其中任何部分最终都不会被评价过高或看待过轻。

作者希望向他尊敬的朋友、林肯的乔治·斯蒂芬斯·迪克森牧师记录他的感激之情，感谢他在修订本书过程中给予的诸多善意帮助，以及提出的一些重要建议。

5, Grenville-place, Cork, 1853年
11月30日。

CONTENTS.

CHAPTER I.

Nature and Design of this Work, ... 1

CHAPTER II.

Signs and their Laws, ... 17

CHAPTER III.

Derivation of the Laws, ... 28

CHAPTER IV.

Division of Propositions, ... 37

CHAPTER V.

Principles of Symbolic Reasoning, ... 48

CHAPTER VI.

Of Interpretation, ... 59

CHAPTER VII.

Of Elimination, ... 74

CHAPTER VIII.

Of Reduction, ... 87

目录。

章 i

本书的性质与设计， 1

章 ii

符号及其规律， 17

章 iii

规律的推导， 28

章 iv

命题的分类， 37

章 v

符号推理原理， 48

章 vi

论解释， 59

章 vii

论消去法， 74

章 viii

论还原， 87

CHAPTER IX.

Methods of Abbreviation, ... 100

CHAPTER X.

Conditions of a Perfect Method, ... 117

CHAPTER XI.

Of Secondary Propositions, ... 124

CHAPTER XII.

Methods in Secondary Propositions, ... 137

CHAPTER XIII.

Clarke and Spinoza, ... 143

CHAPTER XIV.

Example of Analysis, ... 169

CHAPTER XV.

Of the Aristotelian Logic, ... 174

CHAPTER XVI.

Of the Theory of Probabilities, ... 187

CHAPTER XVII.

General Method in Probabilities, ... 194

CHAPTER XVIII.

Elementary Illustrations, ... 211

CHAPTER XIX.

Of Statistical Conditions, ... 227

CHAPTER XX.

Problems on Causes, ... 247

CHAPTER XXI.

Probability of Judgments, ... 293

CHAPTER XXII.

Constitution of the Intellect, ... 311

章 ix

简写方法， 100

章 x

完美方法之条件， 117

章 xi

论次级命题， 124

章 xii

次级命题中的方法， 137

章 xiii

克拉克与斯宾诺莎， 143

章 xiv

分析示例， 169

章 xv

关于亚里士多德逻辑， 174

章 xvi

论概率的理论， 187

章 xvii

概率的一般方法， 194

章 xviii

初等例示， 211

章 xix

论统计条件， 227

章 xx

论原因问题， 247

章 xxi

判断的概率， 293

章 xxii

智力的构成， 311

NOTE.

In Prop. II., p. 261, by the "absolute probabilities" of the events $x, y, z..$ is meant simply what the probabilities of those events ought to be, in order that, regarding them as independent, and their probabilities as our only data, the calculated probabilities of the same events under the condition V should be $p, g, r..$ The statement of the appended problem of the urn must be modified in a similar way. The true solution of that problem, as actually stated, is $p' = cp, q' = cq$, in which c is the arbitrary probability of the condition that the ball drawn shall be either white, or of marble, or both at once.–See p. 270, CASE II.*

Accordingly, since by the logical reduction the solution of all questions in the theory of probabilities is brought to a form in which, from the probabilities of simple events, s, t, &c. under a given condition, V, it is required to determine the probability of some combination, A, of those events under the same condition, the principle of the demonstration in Prop. IV. is really the following:–"The probability of such combination A under the condition V must be calculated as if the events s, t, &c. were independent, and possessed of such probabilities as would cause the derived probabilities of the said events under the same condition V to be such as are assigned to them in the data." This principle I regard as axiomatic. At the same time it admits of indefinite verification, as well directly as through the results of the method of which it forms the basis. I think it right to add, that it was in the above form that the principle first presented itself to my mind, and that it is thus that I have always understood it, the error in the particular problem referred to having arisen from inadvertence in the choice of a material illustration.

注。

在第261页的命题II中，事件x, y, z..的"绝对概率"所指的，简单地说就是这些事件的概率应当是什么，以便在把它们视为独立、且其概率为我们唯一数据的条件下，同样这些事件在条件V之下的计算概率应为p, g, r..。所附的关于瓮的问题的陈述必须以类似方式加以修改。该问题按实际陈述而言的真正解答是$p' = cp, q' = cq$，其中c是所抽出的球为白球、或为大理石球、或二者兼是的条件的任意概率。--见第270页，情形II。*

因此，既然通过逻辑还原，概率论中一切问题的解答都被归结为这样一种形式：从简单事件s、t等在给定条件V下的概率出发，要求确定这些事件在同一条件下某一组合A的概率，那么命题IV中证明的原则实际上如下：--"这种组合A在条件V下的概率，必须按照事件s、t等相互独立、并具有这样的概率来计算，即这些概率会使上述事件在同一条件V下所得出的概率成为数据中所赋予它们的那些概率。"我把这一原则视为自明的。同时，它容许进行无限的验证，既可直接进行，也可通过以它为基的方法所得的结果来进行。我认为应当补充一点：这一原则正是以上述形式首次呈现于我心里的，而我也一直是这样理解它的；所提到的那个特定问题中的错误，源于在选择具体例证时的疏忽。

Chapter I

NATURE AND DESIGN OF THIS WORK.

1. The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed; to give expression to them in the symbolical language of a Calculus, and upon this foundation to establish the science of Logic and construct its method; to make that method itself the basis of a general method for the application of the mathematical doctrine of Probabilities; and, finally, to collect from the various elements of truth brought to view in the course of these inquiries some probable intimations concerning the nature and constitution of the human mind.

2. That this design is not altogether a novel one it is almost needless to remark, and it is well known that to its two main practical divisions of Logic and Probabilities a very considerable share of the attention of philosophers has been directed. In its ancient and scholastic form, indeed, the subject of Logic stands almost exclusively associated with the great name of Aristotle. As it was presented to ancient Greece in the partly technical, partly metaphysical disquisitions of the Organon, such, with scarcely any essential change, it has continued to the present day. The stream of original inquiry has rather been directed towards questions of general philosophy, which, though they have arisen among the disputes of the logicians, have outgrown their origin, and given to successive ages of speculation their peculiar bent and character. The eras of Porphyry and Proclus, of Anselm and Abelard, of Ramus, and of Descartes, together with the final protests of Bacon and Locke, rise up before the mind as examples of the remoter influences of the study upon the course of human thought, partly in suggesting topics fertile of discussion, partly in provoking remonstrance against its own undue pretensions. The history of the theory of Probabilities, on the other hand, has presented far more of that character of steady growth which belongs to science. In its origin the early genius of Pascal,–in its maturer stages of development the most recondite of all the mathematical speculations of Laplace,–were directed to its improvement; to omit here the mention of other names scarcely less distinguished than these. As the study of Logic has been remarkable for the kindred questions of Metaphysics to which it has given occasion, so that of Probabilities also has been remarkable for the impulse which it has bestowed upon the higher departments of mathematical

第一章

本书的性质与设计。

1. 以下论著的设计，在于研究那些据以进行推理的心灵活动的基本规律；以微积分的符号语言将它们表达出来，并在此基础上建立逻辑科学并构造其方法；使该方法本身成为一般方法的基础，以应用于概率的数学学说；最后，从这些探究过程中所呈现的真理的诸元素中，收集一些关于人类心灵的本性与构造的可能暗示。

2. 这一设计并非全然新颖，几乎无需赘言，且众所周知，其逻辑与概率两大实践部门已受到哲学家们相当多的关注。确实，在其古代与经院形式中，逻辑这一主题几乎专与亚里士多德这一伟大名字相联系。正如它通过《工具论》中部分技术性、部分形而上学的论述呈现给古希腊那样，它几乎未经任何本质变化而延续至今。原创性探究的潮流反而转向了一般哲学的问题，这些问题虽源于逻辑学家的争论，却已超出其起源，并赋予相继的思辨时代以其特有的倾向与特征。波菲利与普罗克洛、安瑟伦与阿伯拉尔、拉穆斯与笛卡尔的时代，连同培根与洛克的最终抗议，作为该研究对人类思想进程的较远影响的例证浮现于心灵之前，部分在于提示富有讨论成果的论题，部分在于激起对其自身不当僭越的反驳。另一方面，概率理论的历史则呈现出远为更多的那种属于科学的稳定成长特征。在其起源阶段，帕斯卡的早期天才--在其更成熟的发展阶段，拉普拉斯所有数学思辨中最深奥者--都指向其改进；此处省略提及几乎不逊于这些名字的其他人物。正如逻辑的研究因其所引发的形而上学的亲缘问题而引人注目，概率的研究也因其所赋予数学更高部门的推动而引人注目。

science. Each of these subjects has, moreover, been justly regarded as having relation to a speculative as well as to a practical end. To enable us to deduce correct inferences from given premises is not the only object of Logic; nor is it the sole claim of the theory of Probabilities that it teaches us how to establish the business of life assurance on a secure basis; and how to condense whatever is valuable in the records of innumerable observations in astronomy, in physics, or in that field of social inquiry which is fast assuming a character of great importance. Both these studies have also an interest of another kind, derived from the light which they shed upon the intellectual powers. They instruct us concerning the mode in which language and number serve as instrumental aids to the processes of reasoning; they reveal to us in some degree the connexion between different powers of our common intellect; they set before us what, in the two domains of demonstrative and of probable knowledge, are the essential standards of truth and correctness,–standards not derived from without, but deeply founded in the constitution of the human faculties. These ends of speculation yield neither in interest nor in dignity, nor yet, it may be added, in importance, to the practical objects, with the pursuit of which they have been historically associated. To unfold the secret laws and relations of those high faculties of thought by which all beyond the merely perceptive knowledge of the world and of ourselves is attained or matured, is an object which does not stand in need of commendation to a rational mind.

3. But although certain parts of the design of this work have been entertained by others, its general conception, its method, and, to a considerable extent, its results, are believed to be original. For this reason I shall offer, in the present chapter, some preparatory statements and explanations, in order that the real aim of this treatise may be understood, and the treatment of its subject facilitated.

It is designed, in the first place, to investigate the fundamental laws of those operations of the mind by which reasoning is performed. It is unnecessary to enter here into any argument to prove that the operations of the mind are in a certain real sense subject to laws, and that a science of the mind is therefore *possible*. If these are questions which admit of doubt, that doubt is not to be met by an endeavour to settle the point of dispute *à priori*, but by directing the attention of the objector to the evidence of actual laws, by referring him to an actual science. And thus the solution of that doubt would belong not to the introduction to this treatise, but to the treatise itself. Let the assumption be granted, that a science of the intellectual powers is possible, and let us for a moment consider how the knowledge of it is to be obtained.

4. Like all other sciences, that of the intellectual operations must primarily rest upon observation,–the subject of such observation being the very operations and processes of which we desire to determine the laws. But while the necessity of a foundation in experience is thus a condition common to all sciences, there are some special differences between the modes in which this principle becomes available for the determination of general truths when the subject of inquiry is the mind, and when the subject is external nature. To these it is necessary to direct attention.

科学。此外，这些主题中的每一个都被公正地视为既与思辨目的相关，也与实践目的相关。使我们能够从给定前提中推出正确推论，并非逻辑的唯一目标；概率理论也并非仅因其教导我们如何将人寿保险业务建立在稳固基础上，以及如何浓缩天文学、物理学或那个正迅速获得极大重要性特征的社会探究领域中无数观察记录中有价值的内容，而拥有其唯一主张。这两项研究还具有另一种旨趣，源自它们所投射于理智能力之上的光芒。它们教导我们语言与数如何作为推理过程的工具性辅助；它们在一定程度上向我们揭示我们共同理智的不同能力之间的联系；它们在证明性知识与或然性知识这两个领域中，向我们呈现真理与正确性的本质标准--这些标准并非来自外部，而是深深植根于人类诸能力的构造之中。这些思辨的目的，在旨趣、尊严，或许还可补充说，在重要性上，都不逊于那些它们在历史上与之相伴的实践目标。展开那些高级思维能力的秘密规律与关系--凭借这些能力，一切超出对世界与自身的单纯知觉知识的东西才得以获得或成熟--这是一个无需向理性心灵推荐的目标。

3. 然而，尽管本书设计的某些部分已被他人所考虑，但其总体概念、方法以及在相当程度上的结果，被认为是原创的。因此，我将在本章中提供一些预备性的陈述和解释，以便理解本论著的实际目标，并促进对其主题的处理。

其设计，首先在于研究那些据以进行推理的心灵活动的基本规律。此处无需进入任何论证以证明心灵活动在某种真实意义上服从规律，因而心灵的科学是可能的。如果这些是容许怀疑的问题，那么这种怀疑不应通过试图先验地解决争论之点来应对，而应通过将反对者的注意力引向实际规律的证据、引导他走向一门实际的科学来应对。因此，那种怀疑的解决将不属于本论著的导论，而属于论著本身。让我们暂且假定理智能力的科学是可能的，并让我们片刻考虑如何获得关于它的知识。

4. 像所有其他科学一样，关于理智活动的科学必须首先立足于观察--这种观察的对象正是我们意欲确定其规律的那些活动与过程。然而，尽管经验中的基础之必要性是 all 科学共有的条件，但当探究的主题是心灵时与当主题是外部自然时，这一原则可用于确定普遍真理的方式之间仍存在一些特殊差异。有必要将注意力引向这些差异。

The general laws of Nature are not, for the most part, immediate objects of perception. They are either inductive inferences from a large body of facts, the common truth in which they express, or, in their origin at least, physical hypotheses of a causal nature serving to explain phænomena with undeviating precision, and to enable us to predict new combinations of them. They are in all cases, and in the strictest sense of the term, *probable* conclusions, approaching, indeed, ever and ever nearer to certainty, as they receive more and more of the confirmation of experience. But of the character of probability, in the strict and proper sense of that term, they are never wholly divested. On the other hand, the knowledge of the laws of the mind does not require as its basis any extensive collection of observations. The general truth is seen in the particular instance, and it is not confirmed by the repetition of instances. We may illustrate this position by an obvious example. It may be a question whether that formula of reasoning, which is called the *dictum* of Aristotle, *de omni et nullo*, expresses a primary law of human reasoning or not; but it is no question that it expresses a general truth in Logic. Now that truth is made manifest in all its generality by reflection upon a single instance of its application. And this is both an evidence that the particular principle or formula in question is founded upon some general law or laws of the mind, and an illustration of the doctrine that the perception of such general truths is not derived from an induction from many instances, but is involved in the clear apprehension of a single instance. In connexion with this truth is seen the not less important one that our knowledge of the laws upon which the science of the intellectual powers rests, whatever may be its extent or its deficiency, is not probable knowledge. For we not only see in the particular example the general truth, but we see it also as a certain truth,–a truth, our confidence in which will not continue to increase with increasing experience of its practical verifications.

5. But if the general truths of Logic are of such a nature that when presented to the mind they at once command assent, wherein consists the difficulty of constructing the Science of Logic? Not, it may be answered, in collecting the materials of knowledge, but in discriminating their nature, and determining their mutual place and relation. All sciences consist of general truths, but of those truths some only are primary and fundamental, others are secondary and derived. The laws of elliptic motion, discovered by Kepler, are general truths in astronomy, but they are not its fundamental truths. And it is so also in the purely mathematical sciences. An almost boundless diversity of theorems, which are known, and an infinite possibility of others, as yet unknown, rest together upon the foundation of a few simple axioms; and yet these are all *general* truths. It may be added, that they are truths which to an intelligence sufficiently refined would shine forth in their own unborrowed light, without the need of those connecting links of thought, those steps of wearisome and often painful deduction, by which the knowledge of them is actually acquired. Let us define as fundamental those laws and principles from which all other general truths of science may be deduced, and into which they may all be again resolved. Shall we then err in regarding that as the true science of Logic which, laying down certain elementary laws, confirmed by the very testimony of the

自然界的普遍规律，大部分并不是感知的直接对象。它们或者是从大量事实中归纳出的推论，这些事实所表达的正是其中的共同真理；或者，至少在其起源上，是因果性质的物理假说，用以毫厘不爽地解释现象，并使我们能够预测它们的新组合。在一切情况下，并且按该词最严格的意义，它们都是概率结论，诚然，随着它们越来越多地得到经验的证实，它们会越来越接近确定性。但就概率的性质而言，按该词严格而本来的意义，它们从未完全摆脱。另一方面，关于心灵规律的知识并不需要以大量观察的收集作为其基础。普遍真理在个别实例中即可见及，并且它不是通过实例的重复而得到证实的。我们可以用一个明显的例子来说明这一立场。被称为亚里士多德*dictum*的*de omni et nullo*这一推理程式是否表达了人类推理的一条基本规律，这可能是一个问题；但它表达了逻辑中的一条普遍真理，这却不是一个问题。现在，这条真理通过对其应用的单一实例的反思，便在其全部普遍性中显现出来。这既是一个证据，证明所讨论的特定原理或程式建立在心灵的某条或某些普遍规律之上，也是对下述学说的说明：对这种普遍真理的感知并非来自对众多实例的归纳，而是包含在对单一实例的清晰领会之中。与这一真理相联系，还可以看到另一个同样重要的真理：我们关于理智能力之科学所依托的那些规律的知识，无论其范围如何或有何缺陷，都不是概率知识。因为我们不仅在个别例子中看到普遍真理，而且我们还把它看作一个确定的真理--一个我们对它的信心不会随着其实践验证经验的增加而继续增加的真理。

5. 但是，如果逻辑的普遍真理具有这样的性质，即当它们呈现于心灵时便立刻要求同意，那么建构逻辑科学的困难又在哪里呢?可以回答说，困难不在于收集知识的材料，而在于辨别其性质，并确定它们相互的位置和关系。一切科学都由普遍真理构成，但在这些真理中，有些只是原初的和基本的，另一些则是次级的和派生的。开普勒所发现的行星椭圆运动规律是天文学中的普遍真理，但它们不是天文学的基本真理。在纯粹数学科学中也是如此。已知的几乎无限多样的定理，以及无限可能的、尚不为人知的定理，共同建立在少数几条简单公理的基础之上；然而这些都是普遍真理。还可以补充说，这些真理对于一个足够精微的理智来说，会以其自身固有的光芒显现出来，而不需要那些思想的连接环节，那些令人疲倦且常常痛苦的演绎步骤，而实际上正是通过这些步骤才获得关于它们的知识。让我们把那些一切其他科学普遍真理都可从中演绎出来、并且它们又都可被重新分解为的那些规律和原理，定义为基本的。那么，如果我们把下述东西视为真正的逻辑科学，是否会出错呢?--它确立某些基本规律，这些规律由

mind, permits us thence to deduce, by uniform processes, the entire chain of its secondary consequences, and furnishes, for its practical applications, methods of perfect generality? Let it be considered whether in any science, viewed either as a system of truth or as the foundation of a practical art, there can properly be any other test of the completeness and the fundamental character of its laws, than the completeness of its system of derived truths, and the generality of the methods which it serves to establish. Other questions may indeed present themselves. Convenience, prescription, individual preference, may urge their claims and deserve attention. But as respects the question of what constitutes science in its abstract integrity, I apprehend that no other considerations than the above are properly of any value.

6. It is designed, in the next place, to give expression in this treatise to the fundamental laws of reasoning in the symbolical language of a Calculus. Upon this head it will suffice to say, that those laws are such as to suggest this mode of expression, and to give to it a peculiar and exclusive fitness for the ends in view. There is not only a close analogy between the operations of the mind in general reasoning and its operations in the particular science of Algebra, but there is to a considerable extent an exact agreement in the laws by which the two classes of operations are conducted. Of course the laws must in both cases be determined independently; any formal agreement between them can only be established *à posteriori* by actual comparison. To borrow the notation of the science of Number, and then assume that in its new application the laws by which its use is governed will remain unchanged, would be mere hypothesis. There exist, indeed, certain general principles founded in the very nature of language, by which the use of symbols, which are but the elements of scientific language, is determined. To a certain extent these elements are arbitrary. Their interpretation is purely conventional: we are permitted to employ them in whatever sense we please. But this permission is limited by two indispensable conditions,–first, that from the sense once conventionally established we never, in the same process of reasoning, depart; secondly, that the laws by which the process is conducted be founded exclusively upon the above fixed sense or meaning of the symbols employed. In accordance with these principles, any agreement which may be established between the laws of the symbols of Logic and those of Algebra can but issue in an agreement of processes. The two provinces of interpretation remain apart and independent, each subject to its own laws and conditions.

Now the actual investigations of the following pages exhibit Logic, in its practical aspect, as a system of processes carried on by the aid of symbols having a definite interpretation, and subject to laws founded upon that interpretation alone. But at the same time they exhibit those laws as identical in form with the laws of the general symbols of algebra, with this single addition, viz., that the symbols of Logic are further subject to a special law (Chap, II.), to which the symbols of quantity, as such, are not subject. Upon the nature and the evidence of this law it is not purposed here to dwell. These questions will be fully discussed in a future page. But as constituting the essential ground of difference between those forms of inference with which Logic is conversant, and those which present themselves in the particular science of Number, the law in

心灵本身的证词所确认，允许我们由此通过齐一的过程演绎出其全部次级结论的链条，并为其实际应用提供完全普遍的方法。让我们考虑一下，在任何一门科学中，无论把它视为真理的体系还是视为实践技艺的基础，除了其派生真理体系的完备性以及它用以建立的方法的普遍性之外，是否还能有别的什么检验其规律的完备性和基本性质的标准。当然，其他问题也可能出现。便利、惯例、个人偏好，都可能提出自己的要求并值得注意。但就什么构成抽象完整意义上的科学这一问题而言，我认为除上述考虑之外，没有别的考虑真正具有任何价值。

6. 其次，本书旨在用微积分的符号语言表达推理的基本规律。关于这一点，只需说：这些规律本身便提示了这种表达方式，并赋予它一种独特而排他的、适合所期目的的适用性。一般推理中的心灵活动与代数这门特定科学中的心灵活动之间，不仅存在密切的类比，而且在相当大的程度上，支配这两类活动的规律存在精确的一致。当然，在这两种情况下，规律都必须独立地确定；它们之间的任何形式一致，只能通过实际比较`*a posteriori*来建立。借用数这门科学的记号，然后假定在其新的应用中支配其使用的规律将保持不变，那只不过是假说。确实存在某些植根于语言本性之中的一般原则，符号--它们不过是科学语言的元素--的使用正是由这些原则决定的。在某种程度上，这些元素是任意的。它们的解释纯粹是约定的：我们被允许按我们喜欢的任何意义来使用它们。但这种许可受到两个不可或缺的条件的限制--第一，一旦约定地确立了某种意义，我们在同一推理过程中绝不离开它；第二，支配这一过程的规律必须完全建立在所用符号的上述固定意义或含义之上。根据这些原则，逻辑符号的规律与代数符号的规律之间所能建立的任何一致，都只能导致过程的一致。两个解释领域保持分离和独立，各自服从自己的规律和条件。

现在，以下各页的实际研究把逻辑在其实践方面展示为一个由符号辅助进行的过程体系，这些符号具有确定的解释，并服从仅建立在该解释之上的规律。但与此同时，这些研究又表明，那些规律在形式上与代数的一般符号的规律相同，只增加了一点，即：逻辑的符号还进一步服从一条特殊规律(第二章)，而量的符号本身并不服从这条规律。关于这条规律的性质和证据，此处不打算详述。这些问题将在后面的一页中充分讨论。但是，由于这条规律构成了逻辑所处理的那些推论形式与数这门特定科学中出现的那些推论形式之间本质差异的基础，这条规律在

question is deserving of more than a passing notice. It may be said that it lies at the very foundation of general reasoning,–that it governs those intellectual acts of conception or of imagination which are preliminary to the processes of logical deduction, and that it gives to the processes themselves much of their actual form and expression. It may hence be affirmed that this law constitutes the germ or seminal principle, of which every approximation to a general method in Logic is the more or less perfect development.

7. The principle has already been laid down (5) that the sufficiency and truly fundamental character of any assumed system of laws in the science of Logic must partly be seen in the perfection of the methods to which they conduct us. It remains, then, to consider what the requirements of a general method in Logic are, and how far they are fulfilled in the system of the present work.

Logic is conversant with two kinds of relations,–relations among things, and relations among facts. But as facts are expressed by propositions, the latter species of relation may, at least for the purposes of Logic, be resolved into a relation among propositions. The assertion that the fact or event A is an invariable consequent of the fact or event B may, to this extent at least, be regarded as equivalent to the assertion, that the truth of the proposition affirming the occurrence of the event B always implies the truth of the proposition affirming the occurrence of the event A. Instead, then, of saying that Logic is conversant with relations among things and relations among facts, we are permitted to say that it is concerned with relations among things and relations among propositions. Of the former kind of relations we have an example in the proposition–"All men are mortal;" of the latter kind in the proposition–"If the sun is totally eclipsed, the stars will become visible." The one expresses a relation between "men" and "mortal beings," the other between the elementary propositions–"The sun is totally eclipsed;" "The stars will become visible." Among such relations I suppose to be included those which affirm or deny existence with respect to things, and those which affirm or deny truth with respect to propositions. Now let those things or those propositions among which relation is expressed be termed the elements of the propositions by which such relation is expressed. Proceeding from this definition, we may then say that the *premises* of any logical argument express *given* relations among certain elements, and that the conclusion must express an *implied* relation among those elements, or among a part of them, i.e. a relation implied by or inferentially involved in the premises.

8. Now this being premised, the requirements of a general method in Logic seem to be the following:–

1st. As the conclusion must express a relation among the whole or among a part of the elements involved in the premises, it is requisite that we should possess the means of eliminating those elements which we desire not to appear in the conclusion, and of determining the whole amount of relation implied by the premises among the elements which we wish to retain. Those elements which do not present themselves in the conclusion are, in the language of the common Logic, called middle terms; and the species of elimination exemplified in treatises on Logic consists in deducing from two propositions, containing a

这个问题值得不止是顺便注意。可以说，它正是普遍推理的基础--它支配着那些先于逻辑演绎过程的理智的概念或想象活动，并且赋予这些过程本身以其实际形式和表达的大部分。因此可以断言，这条规律构成了胚芽或本源原则，而逻辑中任何对一般方法的接近都是这一原则或多或少完善的展开式。

7. 前面已经确立了这一原则(5)：任何假定的逻辑科学规律体系的充分性和真正基础性，必须部分地通过它们引导我们达到的方法的完善性来体现。因此，接下来需要考虑的是，逻辑中一般方法的要求是什么，以及在本著作的体系中这些要求得到了多大程度的满足。

逻辑学涉及两类关系--事物之间的关系与事实之间的关系。然而，既然事实是由命题来表达的，那么后一类关系至少就逻辑学的目的而言，可以化归为命题之间的关系。断言事实或事件A是事实或事件B的恒常结果，至少就此而言，可以等同于断言：肯定事件B发生的命题之真，总是蕴涵着肯定事件A发生的命题之真。于是，我们不说逻辑学涉及事物之间的关系与事实之间的关系，而可以说它关涉事物之间的关系与命题之间的关系。前一类关系，我们有一个例子见于命题--"所有人都是会死的"；后一类关系见于命题--"如果太阳被完全食，星辰就会变得可见"。前者表达"人"与"会死的存在者"之间的关系，后者表达基本命题--"太阳被完全食";"星辰会变得可见"--之间的关系。我设想这类关系之中包括那些就事物而言肯定或否定存在的关系，以及那些就命题而言肯定或否定真理的关系。现在，让那些在其中表达关系的事物或命题，被称为表达此种关系的命题的元素。从这一定义出发，我们便可以说，任何逻辑论证的前提都表达某些元素之间的给定关系，而结论则必定表达那些元素之间、或它们一部分之间的一个隐含关系，即一个由前提所蕴涵或在推理上包含于前提之中的关系。

8. 现在，以此为前提，逻辑中一般方法的要求似乎如下：--

1. 由于结论必须在前提所涉及的元素的全部或一部分之间表达一种关系，因此我们必须具备手段，以消去那些我们不愿在结论中出现的元素，并确定前提在我们希望保留的元素之间所隐含的全部关系量。那些不在结论中出现的元素，用普通逻辑的语言来说，称为中项；逻辑学论著中所例示的消元种类，在于从两个命题中推出一个

common element or middle term, a conclusion connecting the two remaining terms. But the problem of elimination, as contemplated in this work, possesses a much wider scope. It proposes not merely the elimination of one middle term from two propositions, but the elimination generally of middle terms from propositions, without regard to the number of either of them, or to the nature of their connexion. To this object neither the processes of Logic nor those of Algebra, in their actual state, present any strict parallel. In the latter science the problem of elimination is known to be limited in the following manner:–From two equations we can eliminate one symbol of quantity; from three equations two symbols; and, generally, from n equations $n - 1$ symbols. But though this condition, necessary in Algebra, seems to prevail in the existing Logic also, it has no essential place in Logic as a science. There, no relation whatever can be proved to prevail between the number of terms to be eliminated and the number of propositions from which the elimination is to be effected. From the equation representing a single proposition, any number of symbols representing terms or elements in Logic may be eliminated; and from any number of equations representing propositions, one or any other number of symbols of this kind may be eliminated in a similar manner. For such elimination there exists one general process applicable to all cases. This is one of the many remarkable consequences of that distinguishing law of the symbols of Logic, to which attention has been already directed.

2ndly. It should be within the province of a general method in Logic to express the final relation among the elements of the conclusion by any admissible *kind* of proposition, or in any selected *order* of terms. Among varieties of kind we may reckon those which logicians have designated by the terms categorical, hypothetical, disjunctive, &c. To a choice or selection in the order of the terms, we may refer whatsoever is dependent upon the appearance of particular elements in the subject or in the predicate, in the antecedent or in the consequent, of that proposition which forms the "conclusion." But waiving the language of the schools, let us consider what really distinct species of problems may present themselves to our notice. We have seen that the elements of the final or inferred relation may either be *things* or *propositions*. Suppose the former case; then it might be required to deduce from the premises a definition or description of some one thing, or class of things, constituting an element of the conclusion in terms of the other things involved in it. Or we might form the conception of some thing or class of things, involving more than one of the elements of the conclusion, and require its expression in terms of the other elements. Again, suppose the elements retained in the conclusion to be propositions, we might desire to ascertain such points as the following, viz., Whether, in virtue of the premises, any of those propositions, taken singly, are true or false?–Whether particular combinations of them are true or false?–Whether, assuming a particular proposition to be true, any consequences will follow, and if so, what consequences, with respect to the other propositions?–Whether any particular condition being assumed with reference to certain of the propositions, any consequences, and what consequences, will follow with respect to the others? and so on. I say that these are general questions, which

共同元素或中项的命题中推出一个连接其余两项的结论。但本书所设想的消元问题具有远为广阔的范围。它提出的不仅是从两个命题中消除一个中项，而是一般地从命题中消除中项，不论其数目多少，也不论其联系的性质如何。对于这一目标，无论是逻辑的还是代数的过程，在其实际状态下，都没有任何严格的平行对应。在后一门科学中，消元问题已知受到如下限制：--从两个方程我们可以消除一个量记号；从三个方程消除两个记号；一般地，从n个方程消除n – 1个记号。但是，尽管这一在代数中必要的条件似乎在现有逻辑中也盛行，它在作为一门科学的逻辑中并没有本质性的位置。在那里，无论什么关系都不能被证明在待消除的项的数目与据以进行消元的命题的数目之间盛行。从代表单一命题的方程中，任意数目的代表逻辑中的项或元素的记号可以被消除；而从代表命题的任意数目的方程中，一个或任何其他数目的这类记号可以以类似的方式被消除。对于这样的消元，存在一个适用于所有情况的一般方法。这是逻辑记号那条已经引起注意的区分性规律的许多显著结果之一。

第二，逻辑学中一般方法的范围应当包括：用任何可容许的种类的命题，或以任何选定的次序的项，来表达结论各元素之间的最终关系。在种类的多样性中，我们可以计入逻辑学家用直言、假言、选言等术语所命名的那些种类。至于项的顺序上的选择或挑选，我们可以把凡是依赖于以下情形的东西都归入其中：特定的元素出现在那构成"结论"的命题的主词或谓词中、前件或后件中。但撇开学派的语言，让我们考虑究竟有哪些真正不同种类的问题可能呈现于我们面前。我们已经看到，最终的或被推出的关系的元素既可能是事物，也可能是命题。假定是前一种情况；那么可能要求从前提推出某个事物或某类事物的定义或描述，该事物或该类事物构成结论的一个元素，并用其中所涉及的其他事物来表达。或者我们可能形成某个事物或某类事物的概念，它涉及结论中不止一个元素，并要求用其他元素来表达它。再假定保留在结论中的元素是命题，我们可能希望确定如下各点，即：凭借前提，那些命题中任何一个单独来看是真的还是假的?--它们的特定组合是真的还是假的?--假定某个特定命题为真，是否会推出任何后果，如果会，相对于其他命题是什么后果?--假定就某些命题设定了某个特定条件，是否会推出任何后果，以及相对于其他命题是什么后果?如此等等。我说这些都是一般问题，它们

it should fall within the scope or province of a general method in Logic to solve. Perhaps we might include them all under this one statement of the final problem of practical Logic. Given a set of premises expressing relations among certain elements, whether things or propositions: required explicitly the whole relation consequent among *any* of those elements under any proposed conditions, and in any proposed form. That this problem, under all its aspects, is resolvable, will hereafter appear. But it is not for the sake of noticing this fact, that the above inquiry into the nature and the functions of a general method in Logic has been introduced. It is necessary that the reader should apprehend what are the specific ends of the investigation upon which we are entering, as well as the principles which are to guide us to the attainment of them.

9. Possibly it may here be said that the Logic of Aristotle, in its rules of syllogism and conversion, sets forth the elementary processes of which all reasoning consists, and that beyond these there is neither scope nor occasion for a general method. I have no desire to point out the defects of the common Logic, nor do I wish to refer to it any further than is necessary, in order to place in its true light the nature of the present treatise. With this end alone in view, I would remark:–1st. That syllogism, conversion, &c., are not the ultimate processes of Logic. It will be shown in this treatise that they are founded upon, and are resolvable into, ulterior and more simple processes which constitute the real elements of method in Logic. Nor is it true in fact that all inference is reducible to the particular forms of syllogism and conversion.–*Vide* Chap. xv. 2ndly. If all inference were reducible to these two processes (and it has been maintained that it is reducible to syllogism alone), there would still exist the same necessity for a general method. For it would still be requisite to determine in what order the processes should succeed each other, as well as their particular nature, in order that the desired relation should be obtained. By the desired relation I mean that full relation which, in virtue of the premises, connects any elements selected out of the premises at will, and which, moreover, expresses that relation in any desired form and order. If we may judge from the mathematical sciences, which are the most perfect examples of method known, this *directive* function of Method constitutes its chief office and distinction. The fundamental processes of arithmetic, for instance, are in themselves but the elements of a possible science. To assign their nature is the first business of its method, but to arrange their succession is its subsequent and higher function. In the more complex examples of logical deduction, and especially in those which form a basis for the solution of difficult questions in the theory of Probabilities, the aid of a directive method, such as a Calculus alone can supply, is indispensable.

10. Whence it is that the ultimate laws of Logic are mathematical in their form; why they are, except in a single point, identical with the general laws of Number; and why in that particular point they differ;–are questions upon which it might not be very remote from presumption to endeavour to pronounce a positive judgment. Probably they lie beyond the reach of our limited faculties. It may, perhaps, be permitted to the mind to attain a knowledge of the laws to which it is itself subject, without its being also given to it to understand their ground and origin, or even, except in a very limited degree, to comprehend their

它应当落在逻辑学中一般方法的范围或领域之内以求解。或许我们可以把它们全都归入关于实践逻辑最终问题的这一陈述之下。给定一组表达某些元素之间关系的前提，无论这些元素是事物还是命题：要求明确地给出在任意提出的条件下、以任意提出的形式，那些元素中任何之间的全部关系结果。这一问题在其一切方面都是可解的，这一点将在以后显现。但上述对逻辑学中一般方法的性质与函数的探讨，并非为了注意到这一事实而引入。读者必须领会我们即将进入的这项研究的具体目标是什么，以及引导我们达到这些目标的原则是什么。

9. 也许在此可以说，亚里士多德逻辑学在其三段论和换位规则中，阐述了所有推理所由构成的基本过程，而除此之外，既无一般方法的余地，也无其必要。我无意指出通常逻辑学的缺陷，也不想超出必要范围去涉及它，只是为了如实阐明本书的性质。仅以此为目的，我要指出：第一，三段论、换位等并非逻辑学的终极过程。本书将表明，它们建立在更深远、更简单的过程之上，并可分解为这些过程，而这些过程构成了逻辑学中方法的真正元素。事实上，也并非所有推论都可还原为三段论和换位的特定形式。-- 参见第十五章。第二，即使所有推论都可还原为这两种过程(也有人主张仅可还原为三段论)，一般方法的必要性依然存在。因为仍然需要确定这些过程应以何种顺序相继进行，以及它们各自的特定性质，以便获得所期望的关系。我所说的所期望的关系，是指那种凭借前提将任意从前提中选出的元素联系起来的完整关系，并且还以任何期望的形式和顺序表达该关系。如果我们从数学科学--已知方法中最完善的范例--来判断，方法的这种指导功能构成了它的主要职责和特征。例如，算术的基本过程本身只是一个可能科学的元素。规定其性质是该方法的首要任务，但安排其顺序则是其后续的更高功能。在逻辑演绎的更复杂例子中，尤其是在那些为概率论中难题的解决提供基础的例子中，一种指导方法的帮助，如只有微积分才能提供的，是不可或缺的。

10. 由此可知，逻辑学的终极规律为何在形式上是数学的；为何它们除了一点之外与数的一般规律相同；以及为何在那一点上它们有所不同；--这些问题是若试图作出确定判断，或许离僭妄并不很远。它们大概超出我们有限官能所及的范围。也许可以允许心灵获得关于它自身所服从的规律的知识，而不同时也被赋予理解这些规律的根据与起源，甚至除了在极为有限的程度上，理解它们

fitness for their end, as compared with other and conceivable systems of law. Such knowledge is, indeed, unnecessary for the ends of science, which properly concerns itself with what is, and seeks not for grounds of preference or reasons of appointment. These considerations furnish a sufficient answer to all protests against the exhibition of Logic in the form of a Calculus. It is not because we choose to assign to it such a mode of manifestation, but because the ultimate laws of thought render that mode possible, and prescribe its character, and forbid, as it would seem, the perfect manifestation of the science in any other form, that such a mode demands adoption. It is to be remembered that it is the business of science not to create laws, but to discover them. We do not originate the constitution of our own minds, greatly as it may be in our power to modify their character. And as the laws of the human intellect do not depend upon our will, so the forms of the science, of which they constitute the basis, are in all essential regards independent of individual choice.

11. Beside the general statement of the principles of the above method, this treatise will exhibit its application to the analysis of a considerable variety of propositions, and of trains of propositions constituting the premises of demonstrative arguments. These examples have been selected from various writers, they differ greatly in complexity, and they embrace a wide range of subjects. Though in this particular respect it may appear to some that too great a latitude of choice has been exercised, I do not deem it necessary to offer any apology upon this account. That Logic, as a science, is susceptible of very wide applications is admitted; but it is equally certain that its ultimate forms and processes are mathematical. Any objection *à priori* which may therefore be supposed to lie against the adoption of such forms and processes in the discussion of a problem of morals or of general philosophy must be founded upon misapprehension or false analogy. It is not of the essence of mathematics to be conversant with the ideas of number and quantity. Whether as a general habit of mind it would be desirable to apply symbolical processes to moral argument, is another question. Possibly, as I have elsewhere observed,[1] the perfection of the method of Logic may be chiefly valuable as an evidence of the speculative truth of its principles. To supersede the employment of common reasoning, or to subject it to the rigour of technical forms, would be the last desire of one who knows the value of that intellectual toil and warfare which imparts to the mind an athletic vigour, and teaches it to contend with difficulties, and to rely upon itself in emergencies. Nevertheless, cases may arise in which the value of a scientific procedure, even in those things which fall confessedly under the ordinary dominion of the reason, may be felt and acknowledged. Some examples of this kind will be found in the present work.

12. The general doctrine and method of Logic above explained form also the basis of a theory and corresponding method of Probabilities. Accordingly, the development of such a theory and method, upon the above principles, will constitute a distinct object of the present treatise. Of the nature of this application it may be desirable to give here some account, more especially as

[1] Mathematical Analysis of Logic. London : G. Bell. 1847.

与其他可设想的规律体系相比，对其目的的适合性。这样的知识对于科学的目的而言确实是不必要的，科学恰当地关注的是实际存在的东西，而不寻求偏好之根据或指定之理由。这些考虑为一切反对以演算形式展示逻辑学的抗议提供了充分的回答。并不是因为我们选择赋予它这样一种表现形式，而是因为思维的终极规律使这种形式成为可能，并规定了它的特性，并且看来还禁止以任何其他形式完善地展示这门科学，所以这种形式才要求被采用。应当记住，科学的事务不是创造规律，而是发现规律。我们并非创造自己心灵的构造，尽管我们大有能力改变其特性。而且，正如人类理智的规律不依赖于我们的意志，以这些规律为基础的科学的形式，在一切本质方面也独立于个人的选择。

11. 除对上述方法原理的一般陈述之外，本论著还将展示其应用于分析相当多样的命题，以及构成论证性前提的命题序列。这些例子选自各种作者，它们在复杂性上差异很大，并涵盖广泛的主题。尽管在这一特定方面，有些人可能认为选择的范围过于宽泛，但我认为没有必要为此作任何辩解。逻辑学作为一门科学能够有非常广泛的应用，这是被承认的；但同样确定的是，它的终极形式与过程是数学的。因此，任何先验的反对意见，若被认为存在于在讨论道德或一般哲学问题时采用这样的形式与过程，必定建立在误解或虚假类比之上。数学的本质并不在于通晓数与量的观念。作为一种普遍的心灵习惯，将符号过程应用于道德论证是否可取，是另一个问题。也许，正如我在别处所观察到的，[1]逻辑学方法的完善性，其主要价值也许在于作为其原理思辨真理的证据。取代通常推理的运用，或使其服从技术形式的严格性，将是一个深知那种理智劳作与论战之价值的人的最后愿望，那种劳作与论战赋予心灵强健的活力，教导它与困难搏斗，并在紧急情况下依靠自身。然而，可能会出现这样的情况：科学程序的价值，即使在那些公认属于理性通常支配之下的事情中，也可能被感受到并得到承认。在本著作中将找到一些这类例子。

12. 上述解释的逻辑的一般学说和方法，也构成了概率论及其相应方法的基础。因此，基于上述原则对这样一种理论和方法进行展开，将构成本论著的一个独立目标。关于这一应用的性质，在此作一些说明或许是可取的，尤其是因为

1 《逻辑的数学分析》。伦敦：G.Bell。1847年。

regards the character of the solutions to which it leads. In connexion with this object some further detail will be requisite concerning the forms in which the results of the logical analysis are presented.

The ground of this necessity of a prior method in Logic, as the basis of a theory of Probabilities, may be stated in a few words. Before we can determine the mode in which the expected frequency of occurrence of a particular event is dependent upon the known frequency of occurrence of any other events, we must be acquainted with the mutual dependence of the events themselves. Speaking technically, we must be able to express the event whose probability is sought, as a function of the events whose probabilities are given. Now this explicit determination belongs in all instances to the department of Logic. Probability, however, in its mathematical acceptation, admits of numerical measurement. Hence the subject of Probabilities belongs equally to the science of Number and to that of Logic. In recognising the co-ordinate existence of both these elements, the present treatise differs from all previous ones; and as this difference not only affects the question of the possibility of the solution of problems in a large number of instances, but also introduces new and important elements into the solutions obtained, I deem it necessary to state here, at some length, the peculiar consequences of the theory developed in the following pages.

13. The measure of the probability of an event is usually defined as a fraction, of which the numerator represents the number of cases favourable to the event, and the denominator the whole number of cases favourable and unfavourable; all cases being supposed equally likely to happen. That definition is adopted in the present work. At the same time it is shown that there is another aspect of the subject (shortly to be referred to) which might equally be regarded as fundamental, and which would actually lead to the same system of methods and conclusions. It may be added, that so far as the received conclusions of the theory of Probabilities extend, and so far as they are consequences of its fundamental definitions, they do not differ from the results (supposed to be equally correct in inference) of the method of this work.

Again, although questions in the theory of Probabilities present themselves under various aspects, and may be variously modified by algebraical and other conditions, there seems to be one general type to which all such questions, or so much of each of them as truly belongs to the theory of Probabilities, may be referred. Considered with reference to the *data* and the *quæsitum*, that type may be described as follows:—1st. The data are the probabilities of one or more given events, each probability being either that of the absolute fulfilment of the event to which it relates, or the probability of its fulfilment under given supposed conditions. 2ndly. The *quæsitum*, or object sought, is the probability of the fulfilment, absolutely or conditionally, of some other event differing in expression from those in the data, but more or less involving the same elements. As concerns the data, they are either *causally given*,—as when the probability of a particular throw of a die is deduced from a knowledge of the constitution of the piece,—or they are derived from observation of repeated instances of the success or failure of events. In the latter case the probability of an event may be defined as the limit toward which the ratio of the favourable to the whole number

涉及到它所导向的解的性质。与此目标相关，还需要进一步详细说明逻辑分析结果所呈现的形式。

逻辑中先验方法作为概率论基础的必要性之根据，可以用几句话来说明。在我们能够确定一个特定事件的预期发生频率依赖于任何其他事件的已知发生频率的方式之前，我们必须熟悉这些事件本身之间的相互依赖关系。用术语来说，我们必须能够将所求概率的事件表达为已知概率的事件的函数。然而，这种明确的确定在所有情况下都属于逻辑学的范畴。概率，在其数学意义上，允许数值的度量。因此，概率的主题同等地属于数的科学和逻辑的科学。在承认这两个元素的协同存在时，本论著与所有先前的论著不同；并且由于这种差异不仅影响大量情况下问题可解性的问题，而且在所获得的解中引入了新的和重要的元素，我认为有必要在此较为详细地陈述以下各页中所发展的理论的特殊后果。

13. 一个事件的概率的度量通常被定义为一个分数，其中分子表示有利于该事件的情况数，分母表示有利和不利情况的全部数目；所有情况被假定为同等可能发生。本著作采用该定义。同时表明，主题还有另一个方面(稍后将提及)，它可以同样被视为基本的，并且实际上会导致相同的方法和结论体系。可以补充说，就概率论公认的结论所及的范围而言，并且就它们是基本原理定义的推论而言，它们与本著作方法的结果(在推论中被假定为同样正确)并无不同。

再者，尽管概率论中的问题以各种面貌呈现，并可能被代数的和其他条件以各种方式修改，似乎存在一个所有这类问题--或每个问题中真正属于概率论的部分--都可被归入的一般类型。参照数据和所求来考虑，该类型可描述如下：-1. 数据是一个或多个给定事件的概率，每个概率要么是其所属事件的绝对实现的概率，要么是在给定假设条件下其实现的概率。2. 所求，或寻求的目标，是某个其他事件的实现(绝对地或条件地)的概率，该事件在表达上不同于数据中的事件，但或多或少涉及相同的元素。就数据而言，它们要么是因果地给定的,--如当骰子特定投掷的概率从对该骰子构造的知识中推导出来时，--要么它们是从对事件成功或失败的重复实例的观察中得出的。在后一种情况下，一个事件的概率可以被定义为有利数与全部数目之比所趋向的极限

of observed cases approaches (the uniformity of nature being presupposed) as the observations are indefinitely continued. Lastly, as concerns the nature or relation of the events in question, an important distinction remains. Those events are either *simple* or *compound*. By a compound event is meant one of which the expression in language, or the conception in thought, depends upon the expression or the conception of other events, which, in relation to it, may be regarded as *simple* events. To say "it rains," or to say "it thunders," is to express the occurrence of a simple event; but to say "it rains and thunders," or to say "it either rains or thunders," is to express that of a compound event. For the expression of that event depends upon the elementary expressions, "it rains," "it thunders." The criterion of simple events is not, therefore, any supposed simplicity in their nature. It is founded solely on the mode of their expression in language or conception in thought.

14. Now one general problem, which the existing theory of Probabilities enables us to solve, is the following, viz.:—Given the probabilities of any simple events: required the probability of a given compound event, i.e. of an event compounded in a given manner out of the given simple events. The problem can also be solved when the compound event, whose probability is required, is subjected to given conditions, i.e. to conditions dependent also in a given manner on the given simple events. Beside this general problem, there exist also particular problems of which the principle of solution is known. Various questions relating to *causes* and *effects* can be solved by known methods under the particular hypothesis that the causes are mutually exclusive, but apparently not otherwise. Beyond this it is not clear that any advance has been made toward the solution of what may be regarded as the general problem of the science, viz.: Given the probabilities of any events, simple or compound, conditioned or unconditioned: required the probability of any other event equally arbitrary in expression and conception. In the statement of this question it is not even postulated that the events whose probabilities are given, and the one whose probability is sought, should involve some common elements, because it is the office of a method to determine whether the data of a problem are sufficient for the end in view, and to indicate, when they are not so, wherein the deficiency consists.

This problem, in the most unrestricted form of its statement, is resolvable by the method of the present treatise; or, to speak more precisely, its theoretical solution is completely given, and its practical solution is brought to depend only upon processes purely mathematical, such as the resolution and analysis of equations. The order and character of the general solution may be thus described.

15. In the first place it is always possible, by the preliminary method of the Calculus of Logic, to express the event whose probability is sought as a logical function of the events whose probabilities are given. The result is of the following character: Suppose that X represents the event whose probability is sought, A, B, C, &c. the events whose probabilities are given, those events being either simple or compound. Then the *whole* relation of the event X to the events A, B, C, &c. is deduced in the form of what mathematicians term a *development*,

在观察到的案例中，随着观察无限地继续下去，趋近于(自然界的齐一性是被预设的)。最后，就所讨论事件的性质或关系而言，还剩下一个重要的区分。那些事件要么是简单的，要么是复合的。所谓复合事件，是指其在语言中的表达或在思想中的概念依赖于其他事件的表达或概念的事件，而这些其他事件相对于它而言可以被视为简单的事件。说"下雨"，或者说"打雷"，是表达一个简单事件的发生；但说"下雨且打雷"，或者说"要么下雨要么打雷"，是表达一个复合事件的发生。因为那个事件的表达依赖于基本表达"下雨"、"打雷"。因此，简单事件的判准并不是其本性中任何被设想的简单性。它完全建立在它们在语言中的表达方式或在思想中的概念方式之上。

14. 现在，现有概率论使我们能够解决的一个一般问题如下，即：给定任何简单事件的概率：求一个给定复合事件的概率，即一个以给定方式由给定简单事件复合而成的事件的概率。当所求其概率的复合事件受制于给定条件，即受制于也以给定方式依赖于给定简单事件的条件时，该问题也可以解决。除这个一般问题之外，还存在一些其解法原理已知的特殊问题。与原因和结果有关的各种问题，在原因互斥这一特殊假说下可以用已知方法解决，但显然不能以其他方式解决。除此之外，对于可被视为该科学的一般问题--即：给定任何事件(简单的或复合的、受条件的或不受条件的)的概率：求任何其他在表达和概念上同样任意的事件的概率--的解决，是否已取得任何进展，尚不清楚。在这个问题的陈述中，甚至没有设定：其概率被给定的事件与所求其概率的那个事件应当涉及某些共同元素，因为一种方法的职责就是判定一个问题的数据对于所考虑的目的而言是否充分，并在不充分时指出缺陷之所在。

这个问题，在其最无限制的陈述形式中，可由本论著的方法解决；或者，更确切地说，其理论上的解答已完全给出，而其实际上的解答则仅取决于纯粹数学的过程，例如方程的消解与分析。一般解的顺序与性质可如此描述。

15. 首先，通过逻辑演算的预备方法，总是可以把所求其概率的事件表达为那些其概率被给定的事件的逻辑函数。结果具有如下性质：设X表示所求其概率的事件，A、B、C等表示其概率被给定的事件，那些事件要么是简单的，要么是复合的。于是，事件X与事件A、B、C等之间的整个关系，被推导为数学家所称的展开式的形式，

consisting, in the most general case, of four distinct classes of terms. By the first class are expressed those combinations of the events A, B, C, which both necessarily accompany and necessarily indicate the occurrence of the event X; by the second class, those combinations which necessarily accompany, but do not necessarily imply, the occurrence of the event X; by the third class, those combinations whose occurrence in connexion with the event X is impossible, but not otherwise impossible; by the fourth class, those combinations whose occurrence is impossible under any circumstances. I shall not dwell upon this statement of the result of the logical analysis of the problem, further than to remark that the elements which it presents are precisely those by which the expectation of the event X, as dependent upon our knowledge of the events A, B, C, is, or alone can be, affected. General reasoning would verify this conclusion; but general reasoning would not usually avail to disentangle the complicated web events and circumstances from which the solution above described must be evolved. The attainment of this object constitutes the first step towards the complete solution of the question I proposed. It is to be noted that thus far the process of solution is logical, i. e. conducted by symbols of logical significance, and resulting in an equation interpretable into a *proposition*. Let this result be termed the *final logical equation*.

The second step of the process deserves attentive remark. From the final logical equation to which the previous step has conducted us, are deduced, by inspection, a series of algebraic equations implicitly involving the complete solution of the problem proposed. Of the mode in which this transition is effected let it suffice to say, that there exists a definite relation between the laws by which the probabilities of events are expressed as algebraic functions of the probabilities of other events upon which they depend, and the laws by which the logical connexion of the events is itself expressed. This relation, like the other coincidences of formal law which have been referred to, is not founded upon hypothesis, but is made known to us by observation (I.4), and reflection. If, however, its reality were assumed *à priori* as the basis of the very definition of Probability, strict deduction would thence lead us to the received numerical definition as a necessary consequence. The Theory of Probabilities stands, as it has already been remarked (I.12), in equally close relation to Logic and to Arithmetic; and it is indifferent, so far as results are concerned, whether we regard it as springing out of the latter of these sciences, or as founded in the mutual relations which connect the two together.

16. There are some circumstances, interesting perhaps to the mathematician, attending the general solutions deduced by the above method, which it may be desirable to notice.

1st. As the method is independent of the number and the nature of the data, it continues to be applicable when the latter are insufficient to render determinate the value sought. When such is the case, the final expression of the solution will contain terms with arbitrary constant coefficients. To such terms there will correspond terms in the final logical equation (I. 15), the interpretation of which will inform us what new data are requisite in order to determine the values of those constants, and thus render the numerical solution complete.

在最一般的情形下，它由四个不同类别的项组成。第一类表达的是事件A、B、C的那些组合，它们既必然伴随又必然指示事件X的发生；第二类表达的是那些必然伴随但并不必然蕴涵事件X发生的组合；第三类表达的是那些与事件X相关联时的发生是不可能的、但并非在其他方面不可能的合；第四类表达的是那些在任何情况下其发生都是不可能的合。我将不再详述这一对问题的逻辑分析结果的陈述，只进一步指出：它所呈现的元素，恰恰就是那些据以能够或唯独能够影响对事件X的期望(该期望依赖于我们对事件A、B、C的知识)的元素。一般推理会证实这个结论；但一般推理通常不足以理清那错综复杂的事件与情况之网，而上述解法必须从其中演化出来。达到这一目标构成了对我所提出的问题的完整解决的第一步。应当注意，迄今为止，解决的过程是逻辑的，即由具有逻辑意义的记号进行，并导致一个可解释为命题的方程。让这个结果被称为最终逻辑方程。

过程的第二步值得仔细注意。从先前步骤引导我们得到的最终逻辑方程出发，通过审视，推导出一系列代数方程，这些方程隐含地包含着所提出问题的完整解决。关于实现这一过渡的方式，只需说：在把事件的概率表达为它们所依赖的其他事件的概率的代数函数的那些定律，与事件的逻辑联系本身据以表达的那些定律之间，存在着一种确定的关系。这种关系，如同已经提到的其他形式定律的吻合一样，并非建立在假说之上，而是通过观察(I.4)和反思为我们所知的。然而，如果它的实在性被先天地假定为概率之定义本身的基础，那么严格的演绎就会由此引导我们得出公认的数值定义，作为其必然结果。概率论正如已经指出的(I.12)，与逻辑学和算术保持着同样密切的关系；而且就结果而言，我们把它视为从后一门科学中产生，还是视为建立在把二者联系起来的相互关系中，都是无关紧要的。

16. 在由上述方法推出的一般解中，有一些或许令数学家感兴趣的情况，值得予以注意。

1. 由于该方法独立于数据的数目和性质，当后者不足以确定所求之值时，它仍然适用。在这种情况下，解的最终表达式将包含带有任意常数系数的项。与这些项相对应，最终的逻辑方程(I.15)中也将出现相应的项，对这些项的解释将告知我们需要哪些新数据才能确定那些常数的值，从而使数值解完整。

If such data are not to be obtained, we can still, by giving to the constants their limiting values 0 and 1, determine the limits within which the probability sought must lie independently of all further experience. When the event whose probability is sought is *quite* independent of those whose probabilities are given, the limits thus obtained for its value will be 0 and 1, as it is evident that they ought to be, and the interpretation of the constants will only lead to a re-statement of the original problem.

2ndly. The expression of the final solution will in all cases involve a particular element of quantity, determinable by the solution of an algebraic equation. Now when that equation is of an elevated degree, a difficulty may seem to arise as to the selection of the proper root. There are, indeed, cases in which both the elements given and the element sought are so obviously restricted by necessary conditions that no choice remains. But in complex instances the discovery of such conditions, by unassisted force of reasoning, would be hopeless. A distinct method is requisite for this end,—a method which might not appropriately be termed the Calculus of Statistical Conditions, into the nature of this method I shall not here further enter than to say, that, like the previous method, it is based upon the employment of the "final logical equation," and that it definitely assigns, 1st, the conditions which must be fulfilled among the numerical elements of the data, in order that the problem may be real, i.e. derived from a *possible experience*; 2ndly, the numerical limits, within which the probability sought must have been confined, if, instead of being determined by theory, it had been deduced directly by observation from the same system of phænomena from which the data were derived. It is clear that these limits will be actual limits of the probability sought. Now, on supposing the data subject to the conditions above assigned to them, it appears in every instance which I have examined that there exists one root, and only one root, of the final algebraic equation which is subject to the required limitations. Every source of ambiguity is thus removed. It would even seem that new truths relating to the theory of algebraic equations are thus incidentally brought to light. It is remarkable that the special element of quantity, to which the previous discussion relates, depends only upon the *data*, and not at all upon the *quæsitum* of the problem proposed. Hence the solution of each particular problem unties the knot of difficulty for a system of problems, viz., for that system of problems which is marked by the possession of common data, independently of the nature of their *quæsita*. This circumstance is important whenever from a particular system of data it is required to deduce a series of connected conclusions. And it further gives to the solutions of particular problems that character of relationship, derived from their dependence upon a central and fundamental unity, which not unfrequently marks the application of general methods.

17. But though the above considerations, with others of a like nature, justify the assertion that the method of this treatise, for the solution of questions in the theory of Probabilities, is a general method, it does not thence follow that we are relieved in all cases from the necessity of recourse to hypothetical grounds. It has been observed that a solution may consist entirely of terms affected by arbitrary constant coefficients,—may, in fact, be wholly indefinite. The application of

如果无法获得这样的数据，我们仍然可以通过赋予常数其极限值0和1，来确定所寻求的概率必定位于其间的界限，而不依赖于一切进一步的经验。当所寻求其概率的事件与那些其概率已给出的事件完全无关时，如此为其值所获得的界限将是0和1，正如它们显然应当如此，而常数的解释将只会导致对原问题的重新陈述。

第二，最终解的表达式在所有情况下都将包含一个特定的量的元素，该元素可由一个代数方程的解来确定。当该方程的次数较高时，关于如何选取恰当的根，似乎会产生困难。诚然，在某些情形中，所给元素与所求元素都显然受到必要条件的限制，以致别无选择。但在复杂的情形中，单凭推理的力量去发现这类条件，将是毫无希望的。为此需要一种独特的方法--一种或许不无理由地可称为统计条件演算的方法。关于这种方法的本性，我在此不再进一步论述，只须说明：它如同前述方法一样，建立在运用"最终逻辑方程"的基础之上，并且它明确地指定：第一，数据的数值元素之间必须满足哪些条件，才能使问题成为真实的，即源自一种可能的经验；第二，所求概率必定被限制于其内的数值界限--假如它不是由理论所确定，而是直接从数据所由导出的同一现象系统中通过观察推演出来的话。显然，这些界限将是所求概率的实际界限。现在，假定数据服从上述指定给它们的条件，则在我所考察的每一个实例中，最终代数方程似乎都存在一个根，而且只有一个根，服从所要求的限制。一切歧义的来源由此被消除。甚至可以说，与代数方程理论有关的新真理就这样被附带地揭示出来。值得注意的是，前述讨论所涉及的那个特定的量的元素，仅仅依赖于数据，而完全不依赖于所提问题的所求者。因此，每一个特定问题的解，都为一系列问题解开了困难之结，即，为那一系列以拥有共同数据为标志、而与其所求者的性质无关的问题解开了困难之结。每当需要从一组特定的数据推演出一系列相互关联的结论时，这一情形都是重要的。而且，它还赋予特定问题的解以那种由其对某个中心的和基本的统一体的依赖而来的关系特性--这种特性常常标志着一般方法的应用。

17. 然而，尽管上述考虑以及其他类似性质的考虑，足以证成本论著中用于解决概率论问题的方法是一般方法，但并不能由此推出我们在所有情况下都免于诉诸假设性根据的必要性。已经指出，一个解可能完全由带有任意常数系数的项组成，--事实上可能完全是不定的。

the method of this work to some of the most important questions within its range would–were the data of experience alone employed–present results of this character. To obtain a *definite* solution it is necessary, in such cases, to have recourse to hypotheses possessing more or less of independent probability, but incapable of exact verification. Generally speaking, such hypotheses will differ from the immediate results of experience in partaking of a logical rather than of a numerical character; in prescribing the conditions under which phænomena occur, rather than assigning the relative frequency of their occurrence. This circumstance is, however, unimportant. Whatever their nature may be, the hypotheses assumed must thenceforth be regarded as belonging to the actual data, although tending, as is obvious, to give to the solution itself somewhat of a hypothetical character. With this understanding as to the possible sources of the data actually employed, the method is perfectly general, but for the correctness of the hypothetical elements introduced it is of course no more responsible than for the correctness of the numerical data derived from experience.

In illustration of these remarks we may observe that the theory of the reduction of astronomical observations[2] rests, in part, upon hypothetical grounds. It assumes certain positions as to the nature of error, the equal probabilities of its occurrence in the form of excess or defect, &c., without which it would be impossible to obtain any *definite* conclusions from a system of conflicting observations. But granting such positions as the above, the residue of the investigation falls strictly within the province of the theory of Probabilities. Similar observations apply to the important problem which proposes to deduce from the records of the majorities of a deliberative assembly the mean probability of correct judgment in one of its members. If the method of this treatise be applied to the mere numerical data, the solution obtained is of that wholly indefinite kind above described. And to show in a more eminent degree the insufficiency of those data by themselves, the interpretation of the arbitrary constants (I. 16) which appear in the solution, merely produces a re-statement of the original problem. Admitting, however, the hypothesis of the independent formation of opinion in the individual mind, either absolutely, as in the speculations of Laplace and Poisson, or under limitations imposed by the actual data, as will be seen in this treatise, Chap. XXI., the problem assumes a far more definite character. It will be manifest that the ulterior value of the theory of Probabilities must depend very much upon the correct formation of such mediate hypotheses, where the purely experimental data are insufficient for *definite* solution, and where that further experience indicated by the interpretation of the final logical equation is unattainable. Upon the other hand, an undue readiness to form hypotheses in subjects which from their very nature are placed beyond human ken, must re-act upon the credit of the theory of Probabilities, and tend to throw doubt in the general mind over its most legitimate conclusions.

18. It would, perhaps, be premature to speculate here upon the question

[2]The author designs to treat this subject either in a separate work or in a future Appendix. In the present treatise he avoids the use of the integral calculus.

将本书的方法应用于其范围内一些最重要的问题--如果仅仅使用经验材料--就会产生这种性质的结果。在此类情形下，要获得一个确定的解答，就必须求助于或多或少具有独立概率、但无法精确验证的假说。一般而言，这类假说与经验的直接结果不同，它们具有逻辑的而非数值的性质；它们规定现象发生的条件，而不是指定现象发生的相对频率。然而，这种情况并不重要。无论其性质如何，所假定的假说从此必须被视为属于实际材料的一部分，尽管显而易见，这会使解答本身带有某种假说的性质。在这样理解实际所用材料的可能来源之后，该方法便是完全一般的，但对于所引入的假说要素的正确性，它当然并不比对于从经验中获得的数值材料的正确性负有更多责任。

为说明这些评论，我们可以指出，天文观测的归算理论[2]部分地建立在假说的基础之上。它假定了一些关于误差性质、误差以过大或过小形式出现的相等概率等等的命题，没有这些命题，就不可能从一组相互冲突的观测中得出任何确定的结论。但是，一旦承认上述命题，其余的研究就严格属于概率论的领域。类似的评论也适用于那个重要问题，即试图从审议会议多数派的记录中推导出其成员之一作出正确判断的平均概率。如果将本论著的方法应用于单纯的数值材料，所得到的解答就是上述那种完全不确定的解答。而为了更突出地表明这些材料本身的不足，对解答中出现的任意常数(I.16)的解释，仅仅是对原问题的重新陈述。然而，如果承认个体心灵中意见独立形成的假说，无论是像拉普拉斯和泊松的思辨中那样绝对地承认，还是在实际材料所施加的限制之下承认(如本论著第二十一章将要表明的那样)，问题就呈现出远为确定的性质。显而易见，概率论的最终价值必定在很大程度上取决于这类中介假说的正确形成，因为在这些情形下，纯粹实验的材料不足以获得确定的解答，而通过对最终逻辑方程的解释所指示的那种进一步经验又是无法获得的。另一方面，对于那些从其本性上就超出人类理解范围的题材，过于轻率地形成假说，必定会反过来损害概率论的信誉，并倾向于在一般心灵中对其最合法的结论投下怀疑。

18. 在此推测这个问题或许为时过早

2 作者计划在另一部著作或未来的附录中处理这一主题。在本论著中，他避免使用积分学。

whether the methods of abstract science are likely at any future day to render service in the investigation of social problems at all commensurate with those which they have rendered in various departments of physical inquiry. An attempt to resolve this question upon pure *à priori* grounds of reasoning would be very likely to mislead us. For example, the consideration of human free-agency would seem at first sight to preclude the idea that the movements of the social system should ever manifest that character of orderly evolution which we are prepared to expect under the reign of a physical necessity. Yet already do the researches of the statist reveal to us facts at variance with such an anticipation. Thus the records of crime and pauperism present a degree of regularity unknown in regions in which the disturbing influence of human wants and passions is unfelt. On the other hand, the distemperature of seasons, the eruption of volcanoes, the spread of blight in the vegetable, or of epidemic maladies in the animal kingdom, things apparently or chiefly the product of natural causes, refuse to be submitted to regular and apprehensible laws. "Fickle as the wind," is a proverbial expression. Reflection upon these points teaches us in some degree to correct our earlier judgments. We learn that we are not to expect, under the dominion of necessity, an order perceptible to human observation, unless the play of its producing causes is sufficiently simple; nor, on the other hand, to deem that free agency in the individual is inconsistent with regularity in the motions of the system of which he forms a component unit. Human freedom stands out as an apparent fact of our consciousness, while it is also, I conceive, a highly probable deduction of analogy (Chap, XXII.) from the nature of that portion of the mind whose scientific constitution we are able to investigate. But whether accepted as a fact reposing on consciousness, or as a conclusion sanctioned by the reason, it must be so interpreted as not to conflict with an established result of observation, viz.: that phænomena, in the production of which large masses of men are concerned, do actually exhibit a very remarkable degree of regularity, enabling us to collect in each succeeding age the elements upon which the estimate of its state and progress, so far as manifested in outward results, must depend. There is thus no sound objection *à priori* against the possibility of that species of data which is requisite for the experimental foundation of a science of social statistics. Again, whatever other object this treatise may accomplish, it is presumed that it will leave no doubt as to the existence of a system of abstract principles and of methods founded upon those principles, by which any collective body of social data may be made to yield, in an explicit form, whatever information they implicitly involve. There may, where the data are exceedingly complex, be very great difficulty in obtaining this information,—difficulty due not to any imperfection of the theory, but to the laborious character of the analytical processes to which it points. It is quite conceivable that in many instances that difficulty may be such as only united effort could overcome. But that we possess theoretically in all cases, and practically, so far as the requisite labour of calculation may be supplied, the means of evolving from statistical records the seeds of general truths which lie buried amid the mass of figures, is a position which may, I conceive, with perfect safety be affirmed.

抽象科学的方法是否有可能在未来的某一天，在社会问题的研究中发挥与它们在各个物理探究部门中所发挥的作用完全相称的作用。试图在纯粹的`*a priori*推理依据上解决这个问题，很可能会使我们误入歧途。例如，考虑人类自由能动性，乍看之下似乎会排除这样一种观念：社会系统的运动竟然会表现出我们在物理必然性统治下所期待的那种有序进化的特征。然而，统计学家的研究已经向我们揭示了与这种预期相悖的事实。因此，犯罪和贫困的记录呈现出一种在人类需求和激情的干扰性影响未被感受到的地区所未知的规律性。另一方面，季节的失调、火山的喷发、植物枯萎病的蔓延或动物界流行病的传播--这些显然或主要是自然原因产物的现象--却拒绝服从于有规律的、可理解的法则。"像风一样多变"是一句谚语。对这些问题的反思在某种程度上教导我们纠正早先的判断。我们认识到，在必然性的支配下，我们不应期待一种人类观察可感知的秩序，除非其产生原因的作用足够简单；另一方面，也不应认为个体的自由能动性与他所构成的系统运动的规律性不相容。人类自由作为我们意识的一个明显事实而凸显出来，同时，据我理解，它也是从我们能够研究其科学构造的那部分心灵的本性出发，通过类比(第二十二章)得出的一个高度可能的演绎结论。但无论它是作为基于意识的事实被接受，还是作为理性所认可的结论被接受，都必须以不与观察的既定结果相冲突的方式来解释，即：有大量人群参与产生的现象，实际上确实表现出非常显著的规律性程度，使我们能够在每个相继的时代收集到那些要素，对其状态和进步的估计--就其在外部结果中所表现而言--必须依赖于这些要素。因此，不存在任何健全的`*a priori*反对意见来否定那种为社会统计学之实验基础所必需的数据类型的可能性。再者，无论本论著可能达成其他什么目标，可以推定它将使以下一点不再有疑问：存在一个抽象原理的体系以及建立在这些原理之上的方法体系，凭借它们，任何社会数据的集合体都可以被使得以明确的形式产出其所隐含的任何信息。在数据极其复杂的情况下，获取这些信息可能存在极大的困难--这种困难并非源于理论的任何不完善，而是源于它所指向的分析过程的繁重性质。完全可以想象，在许多情况下这种困难可能只有通过联合的努力才能克服。但是，我们在理论上于所有情况下都拥有，并且在实践上--只要所需的计算劳动能够提供--也拥有从统计记录中引出埋藏在大量数字之中的一般真理之种子的手段，这一立场，据我理解，可以完全有把握地予以肯定。

19. But beyond these general positions I do not venture to speak in terms of assurance. Whether the results which might be expected from the application of scientific methods to statistical records, over and above those the discovery of which requires no such aid, would so far compensate for the labour involved as to render it worth while to institute such investigations upon a proper scale of magnitude, is a point which could, perhaps, only be determined by experience. It is to be desired, and it might without great presumption be expected, that in this, as in other instances, the abstract doctrines of science should minister to more than intellectual gratification. Nor, viewing the apparent order in which the sciences have been evolved, and have successively contributed their aid to the service of mankind, does it seem very improbable that a day may arrive in which similar aid may accrue from departments of the field of knowledge yet more intimately allied with the elements of human welfare. Let the speculations of this treatise, however, rest at present simply upon their claim to be regarded as true.

20. I design, in the last place, to endeavour to educe from the scientific results of the previous inquiries some general intimations respecting the nature and constitution of the human mind. Into the grounds of the possibility of this species of inference it is not necessary to enter here. One or two general observations may serve to indicate the track which I shall endeavour to follow. It cannot but be admitted that our views of the science of Logic must materially influence, perhaps mainly determine, our opinions upon the nature of the intellectual faculties. For example, the question whether reasoning consists merely in the application of certain first or necessary truths, with which the mind has been originally imprinted, or whether the mind is itself a seat of law, whose operation is as manifest and as conclusive in the particular as in the general formula, or whether, as some not undistinguished writers seem to maintain, all reasoning is of particulars; this question, I say, is one which not merely affects the science of Logic, but also concerns the formation of just views of the constitution of the intellectual faculties. Again, if it is concluded that the mind is by original constitution a seat of law, the question of the nature of its subjection to this law,—whether, for instance, it is an obedience founded upon necessity, like that which sustains the revolutions of the heavens, and preserves the order of Nature,—or whether it is a subjection of some quite distinct kind, is also a matter of deep speculative interest. Further, if the mind is truly determined to be a subject of law, and if its laws also are truly assigned, the question of their probable or necessary influence upon the course of human thought in different ages is one invested with great importance, and well deserving a patient investigation, as matter both of philosophy and of history. These and other questions I propose, however imperfectly, to discuss in the concluding portion of the present work. They belong, perhaps, to the domain of probable or conjectural, rather than to that of positive, knowledge. But it may happen that where there is not sufficient warrant for the certainties of science, there may be grounds of analogy adequate for the suggestion of highly probable opinions. It has seemed to me better that this discussion should be entirely reserved for the sequel of the main business of this treatise,—

19. 但超出这些一般立场之外，我不敢以确信的措辞来谈论。将科学方法应用于统计记录所可能预期的结果--超出那些无需此类辅助即可发现的结果之上--是否足以补偿所涉及的劳动，使得以适当规模进行此类研究值得为之，这一点或许只能由经验来确定。值得期望，而且或许可以不失大胆地预期，在这一事例中如同在其他事例中一样，科学的抽象学说应当服务于超越理智满足之上的目的。而且，鉴于各门科学被演化出来并相继为人类服务贡献其助力的明显次序，似乎并非很不可能有一天会到来，届时类似的助力可能从知识领域中与人类福祉要素更为密切相关的部门中产生。然而，让本论著的思辨目前仅仅立足于其应被视为真理的主张之上。

20. 我最后打算从先前探究的科学结果中，努力引出一些关于人类心灵之本性和构造的一般性提示。关于这种推论之可能性的依据，此处无需深入探讨。一两条一般性观察或许足以指示我将努力遵循的路径。不得不承认，我们对逻辑科学的看法必然会在很大程度上影响、或许主要决定我们对理智能力之本性的意见。例如，推理是否仅仅在于应用某些心灵原初被印刻于其上的最初或必然真理，还是心灵本身就是一个法则的所在，其运作在特殊公式中如同在一般公式中一样明显而确凿，还是如某些并非不杰出的作者似乎主张的那样，一切推理都是关于特称的；这个问题，我说，不仅影响逻辑科学，而且关乎对理智能力之构造形成正确看法。再者，如果得出结论认为心灵按其原初构造是一个法则的所在，那么关于它服从这一法则之性质的问题--例如，它是一种基于必然性的服从，如同维持天体运行并保持自然秩序的那种服从，还是一种完全不同类型的服从--也是一个具有深刻思辨意义的问题。此外，如果心灵确实被判定为法则的主体，如果其法则也确实被指定，那么它们对不同时代人类思想进程之或然或必然影响的问题，就是一个具有重大重要性、值得耐心探究的问题，既是哲学的问题也是历史的问题。这些以及其他问题，我打算在本论著的结尾部分加以讨论，尽管可能不够完善。它们或许属于或然的或推测的知识领域，而非确实的知识领域。但可能发生的情况是，在没有充分根据达到科学确定性之处，可能存在足够的类比依据来提出高度或然的意见。在我看来，将这一讨论完全保留为本论著主要事务的续篇似乎更好--

which is the investigation of scientific truths and laws. Experience sufficiently instructs us that the proper order of advancement in all inquiries after truth is to proceed from the known to the unknown. There are parts, even of the philosophy and constitution of the human mind, which have been placed fully within the reach of our investigation. To make a due acquaintance with those portions of our nature the basis of all endeavours to penetrate amid the shadows and uncertainties of that conjectural realm which lies beyond and above them, is the course most accordant with the limitations of our present condition.

即对科学真理和法则的探究。经验充分教导我们，在一切追寻真理的探究中，前进的恰当次序是从已知到未知。即使关于人类心灵之哲学和构造，也有部分已被完全置于我们探究的范围之内。以对我们本性中那些部分的适当认识作为一切努力的基础，去穿透那位于它们之上和之外的推测领域的阴影与不确定性，是最符合我们当前状况之限度的做法。

Chapter II

OF SIGNS IN GENERAL, AND OF THE SIGNS APPROPRIATE TO THE SCIENCE OF LOGIC IN PARTICULAR; ALSO OF THE LAWS TO WHICH THAT CLASS OF SIGNS ARE SUBJECT.

1. That Language is an instrument of human reason, and not merely a medium for the expression of thought, is a truth generally admitted. It is proposed in this chapter to inquire what it is that renders Language thus subservient to the most important of our intellectual faculties. In the various steps of this inquiry we shall be led to consider the constitution of Language, considered as a system adapted to an end or purpose; to investigate its elements; to seek to determine their mutual relation and dependence; and to inquire in what manner they contribute to the attainment of the end to which, as co-ordinate parts of a system, they have respect.

In proceeding to these inquiries, it will not be necessary to enter into the discussion of that famous question of the schools, whether Language is to be regarded as an *essential* instrument of reasoning, or whether, on the other hand, it is possible for us to reason without its aid. I suppose this question to be beside the design of the present treatise, for the following reason, viz., that it is the business of Science to investigate laws; and that, whether we regard signs as the representatives of things and of their relations, or as the representatives of the conceptions and operations of the human intellect, in studying the laws of signs, we are in effect studying the manifested laws of reasoning. If there exists a difference between the two inquiries, it is one which does not affect the scientific expressions of formal law, which are the object of investigation in the present stage of this work, but relates only to the mode in which those results are presented to the mental regard. For though in investigating the laws of signs, *à posteriori*, the immediate subject of examination is Language, with the rules which govern its use; while in making the internal processes of thought the direct object of inquiry, we appeal in a more immediate way to our personal consciousness,—it will be found that in both cases the results obtained are formally equivalent. Nor could we easily conceive, that the unnumbered tongues and dialects of the earth should have preserved through a long succession of ages so much that is common and universal, were we not assured of the existence of

第二章

论一般的记号，以及特别适于逻辑科学的记号；亦论该类记号所服从的规律。

1. 语言是人类理性的工具，而不仅仅是表达思想的媒介，这是一条普遍承认的真理。本章拟探究究竟是什么使语言如此服务于我们最重要的理智能力。在这一探究的各个步骤中，我们将被引导去考虑语言的构造--将其视为一个适应于某个目的或目标的系统；去研究其元素；去寻求确定它们相互的关系和依赖；并去探究它们以何种方式有助于达成那个作为系统之各协调部分它们所指向的目的。

在进行这些探讨时，没有必要进入那个著名的经院问题的讨论，即语言是否应被视为推理的本质工具，或者另一方面，我们是否可能在没有其帮助的情况下进行推理。我认为这个问题与本文的目的无关，理由如下，即：科学的事业在于研究规律；而且，无论我们将符号视为事物及其关系的代表，还是视为人类理智的概念和心灵活动的代表，在研究符号的规律时，我们实际上是在研究推理的表现规律。如果这两种探讨之间存在差异，那也是一种不影响形式规律的科学表达的差异，而形式规律正是本工作现阶段的研究对象，它只涉及那些结果呈现给心灵关注的方式。因为尽管在研究符号的规律时，`*a posteriori*，直接考察的对象是语言及其使用规则；而在将思维的内在过程作为直接探究对象时，我们以更直接的方式诉诸个人的意识--但将会发现，在两种情况下所获得的结果在形式上是等价的。我们也难以设想，地球上无数的语言和方言竟能在漫长的世代更迭中保留如此之多共同而普遍的东西，若非我们确信存在

some deep foundation of their agreement in the laws of the mind itself.

2. The elements of which all language consists are signs or symbols. Words are signs. Sometimes they are said to represent things; sometimes the operations by which the mind combines together the simple notions of things into complex conceptions; sometimes they express the relations of action, passion, or mere quality, which we perceive to exist among the objects of our experience; sometimes the emotions of the perceiving mind. But words, although in this and in other ways they fulfil the office of signs, or representative symbols, are not the only signs which we are capable of employing. Arbitrary marks, which speak only to the eye, and arbitrary sounds or actions, which address themselves to some other sense, are equally of the nature of signs, provided that their representative office is defined and understood. In the mathematical sciences, letters, and the symbols +, −, =, &c., are used as signs, although the term "sign" is applied to the latter class of symbols, which represent operations or relations, rather than to the former, which represent the elements of number and quantity. As the real import of a sign does not in any way depend upon its particular form or expression, so neither do the laws which determine its use. In the present treatise, however, it is with written signs that we have to do, and it is with reference to these exclusively that the term "sign" will be employed. The essential properties of signs are enumerated in the following definition.

Definition.—A sign is an arbitrary mark, having a fixed interpretation, and susceptible of combination with other signs in subjection to fixed laws dependent upon their mutual interpretation.

3. Let us consider the particulars involved in the above definition separately.

(1.) In the first place, a sign is an *arbitrary* mark. It is clearly indifferent what particular word or token we associate with a given idea, provided that the association once made is permanent. The Romans expressed by the word "civitas" what we designate by the word "state." But both they and we might equally well have employed any other word to represent the same conception. Nothing, indeed, in the nature of Language would prevent us from using a mere letter in the same sense. Were this done, the laws according to which that letter would require to be used would be essentially the same with the laws which govern the use of "civitas" in the Latin, and of "state" in the English language, so far at least as the use of those words is regulated by any general principles common to all languages alike.

(2.) In the second place, it is necessary that each sign should possess, within the limits of the same discourse or process of reasoning, a fixed interpretation. The necessity of this condition is obvious, and seems to be founded in the very nature of the subject. There exists, however, a dispute as to the precise nature of the representative office of words or symbols used as names in the processes of reasoning. By some it is maintained, that they represent the conceptions of the mind alone; by others, that they represent things. The question is not of great importance here, as its decision cannot affect the laws according to which signs are employed. I apprehend, however, that the general answer to this and such like questions is, that in the processes of reasoning, signs stand in the place and

他们一致性的某种深层基础在于心灵本身的规律之中。

2. 一切语言由以构成的元素是符号或记号。词是符号。有时据说它们代表事物；有时代表心灵将事物的简单观念结合为复杂概念的活动；有时它们表达我们在经验对象中所感知的行动、承受或单纯性质的关系；有时表达感知心灵的种种情感。但是，词虽然以这种或那种方式履行符号或代表记号的职能，却并非我们所能使用的唯一符号。只对眼睛说话的任意标记，以及诉诸其他感官的任意声音或动作，同样具有符号的性质，只要其代表职能被明确规定并得到理解。在数学科学中，字母以及+、−、=等符号被用作符号，尽管"符号"一词更多地用于后一类代表运算或关系的记号，而非前一类代表数和量的元素的记号。正如一个符号的真正含义绝不取决于其特定形式或表达方式，决定其使用的规律亦然。然而在本文中，我们所处理的是书写符号，而"符号"一词将专就这些书写符号而言。符号的本质属性在以下定义中被列举。

定义。--记号是一种任意的标记，具有固定的解释，并且能够与其他记号相结合，服从依赖于它们相互解释的固定规律。

3. 让我们分别考察上述定义中所涉及的各个要点。(1.)首先，符号是一种任意的标记。我们为某一给定观念关联何种特定的词或标记，显然是无关紧要的，只要这种关联一经建立便保持恒定。罗马人用"civitas"一词来表达我们所称的"state"。但他们和我们同样完全可以采用任何其他词来代表同一概念。事实上，语言的本质中没有任何东西会阻止我们仅仅用一个字母来表达同样的含义。倘若如此，该字母所须遵循的使用规律，与拉丁语中"civitas"和英语中"state"的使用规律，至少在那些为一切语言所共有的普遍原则所支配的范围内，本质上将是相同的。

(2.)其次，每个符号必须在同一论述或推理过程的范围内具有固定的解释。这一条件的必要性是显而易见的，并且似乎植根于主题的本性之中。然而，关于在推理过程中用作名称的词或符号的代表职能的确切性质，存在着争论。有些人主张，它们仅代表心灵的概念；另一些人则主张，它们代表事物。这个问题在此并不十分重要，因为它的解决不可能影响符号使用所遵循的规律。不过我认为，对这个问题及诸如此类问题的一般回答是：在推理过程中，符号代替并

fulfil the office of the conceptions and operations of the mind; but that as those conceptions and operations represent things, and the connexions and relations of things, so signs represent things with their connexions and relations; and lastly, that as signs stand in the place of the conceptions and operations of the mind, they are subject to the laws of those conceptions and operations. This view will be more fully elucidated in the next chapter; but it here serves to explain the third of those particulars involved in the definition of a sign, viz., its subjection to fixed laws of combination depending upon the nature of its interpretation.

4. The analysis and classification of those signs by which the operations of reasoning are conducted will be considered in the following Proposition:

PROPOSITION I.

All the operations of Language, as an instrument of reasoning, may be conducted by a system of signs composed of the following elements, viz.:

1st. *Literal symbols, as x, y, &c., representing things as subjects of our conceptions.*

2nd. *Signs of operation, as $+$, $-$, $\times$, standing for those operations of the mind by which the conceptions of things are combined or resolved so as to form new conceptions involving the same elements.*

3rd. *The sign of identity, $=$.*

And these symbols of Logic are in their use subject to definite laws, partly agreeing with and partly differing from the laws of the corresponding symbols in the science of Algebra.

Let it be assumed as a criterion of the true elements of rational discourse, that they should be susceptible of combination in the simplest forms and by the simplest laws, and thus combining should generate all other known and conceivable forms of language; and adopting this principle, let the following classification be considered.

CLASS I.

5. *Appellative or descriptive signs, expressing either the name of a thing, or some quality or circumstance belonging to it.*

To this class we may obviously refer the substantive proper or common, and the adjective. These may indeed be regarded as differing only in this respect, that the former expresses the substantive existence of the individual thing or things to which it refers; the latter implies that existence. If we attach to the adjective the universally understood subject "being" or "thing," it becomes virtually a substantive, and may for all the essential purposes of reasoning be replaced by the substantive. Whether or not, in every particular of the mental regard, it is the same thing to say, "Water is a fluid thing," as to say, "Water is fluid;" it is at least equivalent in the expression of the processes of reasoning.

It is clear also, that to the above class we must refer any sign which may conventionally be used to express some circumstance or relation, the detailed exposition of which would involve the use of many signs. The epithets of poetic diction are very frequently of this kind. They are usually

履行心灵的概念和心灵活动的职能；但由于那些概念和心灵活动代表事物以及事物的联系和关系，所以符号代表事物及其联系和关系；最后，由于符号代替心灵的概念和心灵活动，它们服从那些概念和心灵活动的规律。这一观点将在下一章中得到更充分的阐明；但在此它有助于解释符号定义中所涉及的第三个要点，即符号服从于依其解释的性质而定的固定组合规律。

4. 对那些借以进行推理活动的记号的分析与分类，将在下述命题中予以考虑：

命题一。

语言作为推理工具的一切活动，均可由由以下元素构成的记号系统来进行，即：

1. 字面记号，如 、 、等，表示作为我们概念之主词的事物。

2. 运算记号，如 + 、 、 ，代表那些心灵活动，通过它们，事物的概念被组合或分解，以形成包含相同元素的新概念。

3. 同一性的符号， = 。

而这些逻辑记号在其使用中服从确定的规律，这些规律部分地与代数学中相应记号的规律一致，部分地与之不同。

让我们设定以下原则作为理性话语之真正元素的判准：它们应当能够以最简单的形式和最简单的规律相结合，并且如此结合应当生成一切其他已知的和可设想的语言形式；采纳这一原则，让我们考虑以下分类。

第i类。

5. 称谓记号或描述记号，表达事物的名称，或属于该事物的某种性质或情况。

我们显然可以把实体性的专名或通名以及形容词归入这一类。确实，这些可以被认为仅仅在这一点上有所不同：前者表达其所指称的个体事物或诸事物的实体性存在；后者则蕴含那种存在。如果我们给形容词附加上普遍理解的主词"存在者"或"事物"，它实际上就变成了一个实体词，并且可以为推理的一切本质目的而由实体词来替代。说"水是一种流体的事物"与说"水是流体的"，在心灵的每一个特殊考虑方面是否相同，这至少在推理过程的表达中是等价的。

同样清楚的是，我们必须将任何可能按约定用于表达某种情况或关系的记号归入上述类别，对该情况或关系的详细阐述将涉及许多记号的使用。诗歌措辞中的修饰语非常频繁地属于此类。它们通常是

compounded adjectives, singly fulfilling the office of a many-worded description. Homer's "deep-eddying ocean" embodies a virtual description in the single word βαθυδίνης. And conventionally any other description addressed either to the imagination or to the intellect might equally be represented by a single sign, the use of which would in all essential points be subject to the same laws as the use of the adjective "good" or "great." Combined with the subject "thing," such a sign would virtually become a substantive; and by a single substantive the combined meaning both of thing and quality might be expressed.

6. Now, as it has been defined that a sign is an arbitrary mark, it is permissible to replace all signs of the species above described by letters. Let us then agree to represent the class of individuals to which a particular name or description is applicable, by a single letter, as x. If the name is "men," for instance, let x represent "all men," or the class "men." By a class is usually meant a collection of individuals, to each of which a particular name or description may be applied; but in this work the meaning of the term will be extended so as to include the case in which but a single individual exists, answering to the required name or description, as well as the cases denoted by the terms "nothing" and "universe," which as "classes" should be understood to comprise respectively "no beings," "all beings." Again, if an adjective, as "good," is employed as a term of description, let us represent by a letter, as y, all things to which the description "good" is applicable, i.e. "all good things," or the class "good things." Let it further be agreed, that by the combination xy shall be represented that class of things to which the names or descriptions represented by x and y are simultaneously applicable. Thus, if x alone stands for "white things," and y for "sheep," let xy stand for "white sheep;" and in like manner, if z stand for "horned things," and x and y retain their previous interpretations, let zxy represent "horned white sheep," i.e. that collection of things to which the name "sheep," and the descriptions "white" and "horned" are together applicable.

Let us now consider the laws to which the symbols x, y, &c., used in the above sense, are subject.

7. First, it is evident, that according to the above combinations, the order in which two symbols are written is indifferent. The expressions xy and yx equally represent that class of things to the several members of which the names or descriptions x and y are together applicable. Hence we have,

$$xy = yx. \tag{1}$$

In the case of x representing white things, and y sheep, either of the members of this equation will represent the class of "white sheep." There may be a difference as to the order in which the conception is formed, but there is none as to the individual things which are comprehended under it. In like manner, if x represent "estuaries," and y "rivers," the expressions xy and yx will indifferently represent "rivers that are estuaries," or "estuaries that are rivers," the combination in this case being in ordinary language that of two substantives, instead of that of a substantive and an adjective as in the previous instance.

复合形容词，单独履行多词描述的职能。荷马的"深涡的大洋"在单个词 βαθυδίνης 中体现了一种实质的描述。而习惯上，任何其他面向想象或理智的描述同样可以用一个单一符号来代表，其使用在一切本质要点上将服从与形容词"好"或"大"的使用相同的规律。与主词"事物"相结合，这样一个符号实际上就变成了一个实体词；而通过一个单一的实体词，事物和性质两者的联合含义都可以被表达出来。

6. 现在，既然已经定义符号是任意的标记，那么允许用字母来替换上述种类的一切符号。那么让我们约定，用一个单一字母，如 x，来代表某个特定名称或描述所适用的个体类。例如，如果名称是"人"，就让 x 代表"所有人"，或类"人"。所谓类，通常指个体的集合，其中每一个体都可以被应用某个特定名称或描述；但在本著作中，该词项的含义将被扩展，以致包括只存在一个个体符合所需名称或描述的情形，以及由词项"无"和"宇宙"所表示的情形，这些作为"类"应被理解为分别包含"无存在者"、"一切存在者"。再者，如果一个形容词，如"好"，被用作描述的词项，让我们用一个字母，如 y，来代表"好"这一描述所适用的一切事物，即"一切好的事物"，或类"好的事物"。进一步约定，通过组合 xy 应代表由 x 和 y 所代表的名称或描述同时适用的那一类事物。因此，如果 x 单独代表"白的事物"，而 y 代表"羊"，就让 xy 代表"白的羊"；同样地，如果 z 代表"有角的事物"，而 x 和 y 保持其先前的解释，就让 zxy 代表"有角的白的羊"，即名称"羊"以及描述"白的"和"有角的"共同适用的那一集合的事物。

现在让我们考虑以上述意义使用的记号 x、y 等所服从的规律。

7. 首先，显而易见的是，按照上述组合方式，两个记号书写的顺序是无关紧要的。表达式 xy 和 yx 同样地代表那样一类事物，名称或描述 x 和 y 共同适用于该类事物的各个成员。因此我们有，

$$xy = yx. \tag{1}$$

在 x 代表白的事物、y 代表羊的情形下，此方程中任一成员都将代表"白的羊"这一类。在概念形成所依据的顺序上可能存在差异，但在其所统摄的个体事物方面则没有差异。同样地，如果 x 代表"河口"，而 y 代表"河流"，则表达式 xy 和 yx 将无差别地代表"是河口的河流"，或"是河流的河口"，此处的组合在普通语言中是两个实体词的组合，而非如前例中实体词与形容词的组合。

Let there be a third symbol, as z, representing that class of things to which the term "navigable" is applicable, and any one of the following expressions,

$$zxy,\ zyx,\ xyz,\ \&c.,$$

will represent the class of "navigable rivers that are estuaries."

If one of the descriptive terms should have some implied reference to another, it is only necessary to include that reference expressly in its stated meaning, in order to render the above remarks still applicable. Thus, if x represent "wise" and y "counsellor," we shall have to define whether x implies wisdom in the absolute sense, or only the wisdom of counsel. With such definition the law $xy = yx$ continues to be valid.

We are permitted, therefore, to employ the symbols x, y, z, &c., in the place of the substantives, adjectives, and descriptive phrases subject to the rule of interpretation, that any expression in which several of these symbols are written together shall represent all the objects or individuals to which their several meanings are together applicable, and to the law that the order in which the symbols succeed each other is indifferent.

As the rule of interpretation has been sufficiently exemplified, I shall deem it unnecessary always to express the subject "things" in defining the interpretation of a symbol used for an adjective. When I say, let x represent "good," it will be understood that x only represents "good" when a subject for that quality is supplied by another symbol, and that, used alone, its interpretation will be "good things."

8. Concerning the law above determined, the following observations, which will also be more or less appropriate to certain other laws to be deduced hereafter, may be added.

First, I would remark, that this law is a law of thought, and not, properly speaking, a law of things. Difference in the order of the qualities or attributes of an object, apart from all questions of causation, is a difference in conception merely. The law (1) expresses as a general truth, that the same thing may be conceived in different ways, and states the nature of that difference; and it does no more than this.

Secondly, As a law of thought, it is actually developed in a law of Language, the product and the instrument of thought. Though the tendency of prose writing is toward uniformity, yet even there the order of sequence of adjectives absolute in their meaning, and applied to the same subject, is indifferent, but poetic diction borrows much of its rich diversity from the extension of the same lawful freedom to the substantive also. The language of Milton is peculiarly distinguished by this species of variety. Not only does the substantive often precede the adjectives by which it is qualified, but it is frequently placed in their midst. In the first few lines of the invocation to Light, we meet with such examples as the following:

"*Offspring of heaven first-born.*"
"The rising world of *waters dark and deep.*"
"Bright effluence of *bright essence increate.*"

设有第三个记号，如 z，代表"可航行的"一词所适用的那类事物，则以下任一表达式，

$$zxy,\ zyx,\ xyz,\ \&c.,$$

将表示"可航行的、属于河口湾的河流"这一类。

如果其中一个描述性词项对另一个有某种隐含的指涉，只需将该指涉明确地包含在其陈述的意义中，即可使上述论述仍然适用。例如，如果 x 代表"智慧的"而 y 代表"顾问"，我们就必须界定 x 是意味着绝对意义上的智慧，还是仅指顾问的智慧。有了这样的定义，规律 xy = yx 继续有效。

因此，我们被允许使用记号 、 、 等，以代替受解释规则约束的实词、形容词和描述性短语，即任何将其中若干记号写在一起的表达式，应代表其各自含义共同适用的一切对象或个体；并且服从这样的规律：记号彼此相继的顺序是无关紧要的。

由于解释规则已被充分例示，我认为在定义用于形容词的记号的解释时，不必总是表达主词"事物"。当我说，令 x 代表"好的"时，应当理解为 x 仅当该性质的承担者由另一记号提供时才代表"好的"，而单独使用时，其解释将是"好的事物"。

8. 关于上述确定的规律，可以补充以下观察，这些观察对以后将要推演的某些其他规律也或多或少适用。

首先，我要指出，这条规律是一条思维规律，而非严格意义上的事物规律。一个对象的性质或属性在顺序上的差异，撇开一切因果关系问题不谈，仅仅是概念上的差异。这条规律(1)作为一条普遍真理表达的是，同一事物可以以不同方式被设想，并陈述了那种差异的性质；它不过如此而已。

其次，作为思维规律，它实际上在语言的规律中得到发展，语言是思维的产物和工具。尽管散文写作的趋势是趋于一致，然而即便在那里，含义绝对的、应用于同一主词的形容词的先后顺序也是无关紧要的，但诗歌的措辞借由将同样的合法自由也扩展到实体词而获得了其丰富多样的许多特色。弥尔顿的语言尤其以这种多样性而著称。实体词不仅常常先于限定它的形容词，而且常常被置于它们中间。在向光呼吁的最初几行中，我们就遇到如下这样的例子：

" 上天的长子。 "
"幽暗深邃的水域那升起的世界。""明亮本质那未受造的光辉流溢。"

Now these inverted forms are not simply the fruits of a poetic license. They are the natural expressions of a freedom sanctioned by the intimate laws of thought, but for reasons of convenience not exercised in the ordinary use of language.

Thirdly, The law expressed by (1) may be characterized by saying that the literal symbols x, y, z, are *commutative, like the symbols of Algebra.* In saying this, it is not affirmed that the process of multiplication in Algebra, of which the fundamental law is expressed by the equation

$$xy = yx,$$

possesses in itself any analogy with that process of logical combination which xy has been made to represent above; but only that if the arithmetical and the logical process are expressed in the same manner, their symbolical expressions will be subject to the same formal law. The evidence of that subjection is in the two cases quite distinct.

9. As the combination of two literal symbols in the form xy expresses the whole of that class of objects to which the names or qualities represented by x and y are together applicable, it follows that if the two symbols have exactly the same signification, their combination expresses no more than either of the symbols taken alone would do. In such case we should therefore have

$$xy = x.$$

As y is, however, supposed to have the same meaning as x, we may replace it in the above equation by x, and we thus get

$$xx = x.$$

Now in common Algebra the combination xx is more briefly represented by x^2. Let us adopt the same principle of notation here; for the mode of expressing a particular succession of mental operations is a thing in itself quite as arbitrary as the mode of expressing a single idea or operation (II. 3). In accordance with this notation, then, the above equation assumes the form

$$x^2 = x, \tag{2}$$

and is, in fact, the expression of a second general law of those symbols by which names, qualities, or descriptions, are symbolically represented.

The reader must bear in mind that although the symbols x and y in the examples previously formed received significations distinct from each other, nothing prevents us from attributing to them precisely the same signification. It is evident that the more nearly their actual significations approach to each other, the more nearly does the class of things denoted by the combination xy approach to identity with the class denoted by x, as well as with that denoted by y. The case supposed in the demonstration of the equation (2) is that of *absolute* identity of meaning. The law which it expresses is practically exemplified in language. To say "good, good," in relation to any subject, though a cumbrous

现在，这些倒装的表达形式并非仅仅是诗的特许的产物。它们是思维的内在规律所认可的自由的天然表达，只是出于便利的考虑，在语言的日常使用中未被运用。

第三，(1)所表达的规律可以这样来描述：文字记号 x、y、z 是可交换的，如同代数学中的记号。这样说，并不是断言代数学中的乘法过程--其基本规律由方程

$$xy = yx,$$

所表达--本身与上面使 xy 所代表的逻辑结合过程有任何类比；而只是说，如果算术过程和逻辑过程以相同方式表达，它们的符号表达式将服从相同的形式规律。这种服从的证据在两种情形中是截然不同的。

9. 由于两个文字记号以 xy 形式的结合表达了 x 和 y 所代表的名称或性质共同适用的那一整类对象，由此推出，如果两个记号具有完全相同的意义，它们的结合所表达的并不比其中任一记号单独使用时更多。因此在这种情形下我们应有

$$xy = x.$$

然而，由于 y 被假定与 x 具有相同的意义，我们可以在上述方程中用 x 替换它，于是得到

$$xx = x.$$

现在在普通代数学中，组合 xx 被更简洁地表示为 x^2。让我们在此采用同样的记法原则；因为表达某一特定系列心灵活动的方式，本身与表达单一观念或活动的方式同样任意(II.3)。按照这一记法，上述方程便采取如下形式

$$x^2 = x, \tag{2}$$

并且事实上，这就是那些用以符号化地代表名称、性质或描述的记号的第二条一般规律的表达式。

读者必须牢记，尽管在前述所构成的例子中，记号x和y被赋予了彼此不同的含义，但没有任何东西阻止我们赋予它们完全相同的含义。显然，它们的实际含义彼此越接近，由组合xy所表示的那类事物就越接近于与x所表示的类以及y所表示的类同一。方程(2)的证明中所设的情形正是absolute含义同一的情形。它所表达的那条规律在语言中得到了实际的例证。就任何主词而言，说"好，好"，虽然是一种累赘

and useless pleonasm, is the same as to say "good." Thus "good, good" men, is equivalent to "good" men. Such repetitions of words are indeed sometimes employed to heighten a quality or strengthen an affirmation. But this effect is merely secondary and conventional; it is not founded in the intrinsic relations of language and thought. Most of the operations which we observe in nature, or perform ourselves, are of such a kind that their effect is augmented by repetition, and this circumstance prepares us to expect the same thing in language, and even to use repetition when we design to speak with emphasis. But neither in strict reasoning nor in exact discourse is there any just ground for such a practice.

10. We pass now to the consideration of another class of the signs of speech, and of the laws connected with their use.

CLASS II.

11. *Signs of those mental operations whereby we collect parts into a whole, or separate a whole into its parts.*

We are not only capable of entertaining the conceptions of objects, as characterized by names, qualities, or circumstances, applicable to each individual of the group under consideration, but also of forming the aggregate conception of a group of objects consisting of partial groups, each of which is separately named or described. For this purpose we use the conjunctions "and," "or," &c. "Trees and minerals," "barren mountains, or fertile vales," are examples of this kind. In strictness, the words "and," "or," interposed between the terms descriptive of two or more classes of objects, imply that those classes are quite distinct, so that no member of one is found in another. In this and in all other respects the words "and" "or" are analogous with the sign + in algebra, and their laws are identical. Thus the expression "men and women" is, conventional meanings set aside, equivalent with the expression "women and men." Let x represent "men," y, "women;" and let + stand for "*and*" and "*or*," then we have

$$x + y = y + x, \tag{3}$$

an equation which would equally hold true if x and y represented *numbers*, and + were the sign of arithmetical addition.

Let the symbol z stand for the adjective "European," then since it is, in effect, the same thing to say "European men and women," as to say "European men and European women," we have

$$z\,(x + y) = zx + zy. \tag{4}$$

And this equation also would be equally true were x, y, and z symbols of number, and were the juxtaposition of two literal symbols to represent their algebraic product, just as in the logical signification previously given, it represents the class of objects to which both the epithets conjoined belong.

The above are the laws which govern the use of the sign +, here used to denote the positive operation of aggregating parts into a whole. But the very

而无用的赘语，却等同于说"好"。因此，"好，好"的人，等价于"好"的人。这种词的重复确实有时被用来提高一种性质或加强一个肯定。但这种结果仅仅是次要的和约定俗成的；它并非建立在语言和思维的内在关系之上。我们在自然界中所观察到或我们自己所作的大多数操作，都属于这样一种：其结果因重复而增强，这一情况使我们预期语言中也有同样的事情，甚至当我们打算强调地说话时就使用重复。但无论在严格的推理中还是在精确的论述中，这种做法都没有任何正当的依据。

10. 我们现在转而考察言语记号中的另一类，以及与它们的使用相关的规律。

第二类。

11. 那些心灵活动的记号，我们借以将部分集合为整体，或将整体分解为部分。

我们不仅能够持有关于对象的概念--这些对象以适用于所考察的群体中每个个体的名称、性质或情况为特征--而且能够形成关于一个由部分群体组成的对象群体的聚合概念，其中每个部分群体都被单独命名或描述。为此我们使用连词"和"、"或"等。"树和矿物"、"荒山或沃谷"，就是这类例子。严格说来，置于描述两个或更多类对象的项之间的"和"、"或"这两个词，意味着那些类是完全有别的，以致于一个类中没有成员出现在另一个类中。在这方面以及在所有其他方面，"和""或"这两个词与代数中的符号+类似，它们的规律是相同的。因此，"男人和女人"这一表达式，撇开约定含义不谈，等价于"女人和男人"这一表达式。设x表示"男人",y表示"女人"；并设+代表"和"以及"或"，于是我们有

$$x + y = y + x, \tag{3}$$

一个方程，如果x和y表示数，且+是算术加法的记号，它也同样成立。

令记号z代表形容词"欧洲的"，那么，由于说"欧洲的男人和女人"与说"欧洲的男人和欧洲的女人"实际上是一回事，我们有

$$z\,(x + y) = zx + zy. \tag{4}$$

如果x、y和z是数的记号，并且两个文字记号的并置代表它们的代数乘积，正如在先前给出的逻辑意义中，它代表两个相连形容词所属对象的类，那么这个方程也同样成立。

以上是支配记号+使用的规律，此处用它表示将部分聚合为整体的肯定活动。但正是

idea of an operation effecting some positive change seems to suggest to us the idea of an opposite or negative operation, having the effect of undoing what the former one has done. Thus we cannot conceive it possible to collect parts into a whole, and not conceive it also possible to separate a part from a whole. This operation we express in common language by the sign except, as, "All men *except* Asiatics," "All states *except* those which are monarchical." Here it is implied that the things excepted form a part of the things from which they are excepted. As we have expressed the operation of aggregation by the sign $+$, so we may express the negative operation above described by $-$ minus. Thus if x be taken to represent men, and y, Asiatics, i. e. Asiatic men, then the conception of "All men except Asiatics" will be expressed by $x - y$. And if we represent by x, "states," and by y the descriptive property "having a monarchical form," then the conception of "All states except those which are monarchical" will be expressed by $x - xy$.

As it is indifferent for all the *essential* purposes of reasoning whether we express excepted cases first or last in the order of speech, it is also indifferent in what order we write any series of terms, some of which are affected by the sign $-$. Thus we have, as in the common algebra,

$$x - y = -y + x. \tag{5}$$

Still representing by x the class "men," and by y "Asiatics," let z represent the adjective "white." Now to apply the adjective "white" to the collection of men expressed by the phrase "Men except Asiatics," is the same as to say, "White men, except white Asiatics." Hence we have

$$z\,(x - y) = zx - zy. \tag{6}$$

This is also in accordance with the laws of ordinary algebra.

The equations (4) and (6) may be considered as exemplification of a single general law, which may be stated by saying, *that the literal symbols, x, y, z, &c. are distributive in their operation.* The general fact which that law expresses is this, viz.:—If any quality or circumstance is ascribed to all the members of a group, formed either by aggregation or exclusion of partial groups, the resulting conception is the same as if the quality or circumstance were first ascribed to each member of the partial groups, and the aggregation or exclusion effected afterwards. That which is ascribed to the members of the whole is ascribed to the members of all its parts, howsoever those parts are connected together.

CLASS III.

12. *Signs by which relation is expressed, and by which we form propositions.*

Though all verbs may with propriety be referred to this class, it is sufficient for the purposes of Logic to consider it as including only the substantive verb *is* or *are*, since every other verb may be resolved into this element, and one of the signs included under Class I. For as those signs are used to express quality or circumstance of every kind, they may be employed to express the active or

关于一种实现某种肯定性改变的操作的观念，似乎向我们暗示出一种相反或否定的操作观念，其结果是撤销前一种操作所做之事。因此，我们不可能设想把部分集合成一个整体，而不也设想把部分从整体中分离出来。这一操作我们在日常语言中用符号"除……之外"来表达，例如，"所有人除亚洲人之外","所有国家除君主制国家之外"。这里隐含的是，被排除的事物构成它们所从被排除的那些事物的一部分。正如我们用符号+表达了聚合的操作，我们也可以用–减来表达上述否定的操作。因此，若取x表示人，y表示亚洲人，即亚洲的人，那么"所有人除亚洲人之外"这一概念将由x – y来表达。而若我们用x表示"国家"，用y表示描述性性质"具有君主制的形式"，那么"所有国家除君主制国家之外"这一概念将由x – xy来表达。

正如在言语顺序中，我们将例外情况放在前面或后面表达，对于推理的一切本质目的而言都无关紧要，同样，我们书写任何一系列项的顺序也无关紧要，其中一些项受到记号–的影响。因此，正如在普通代数中，我们有

$$x - y = -y + x. \tag{5}$$

仍以x表示"人"这个类，以y表示"亚洲人"，令z代表形容词"白的"。现在，将形容词"白的"应用于由短语"除亚洲人以外的人"所表达的人的集合，等同于说"白人，除白种亚洲人以外"。因此我们有

$$z\,(x - y) = zx - zy. \tag{6}$$

这也符合普通代数的规律。

方程(4)和(6)可以被视为一条单一普遍规律的例证，这条规律可以这样陈述，即文字记号，，，等在其运算中是分配性的。这条规律所表达的一般事实是这样的，即：如果任何一种性质或情况被赋予一个群体的所有成员，而这个群体是通过部分群体的聚合或排除而形成的，那么所得出的概念就与先把该性质或情况赋予各部分群体的每个成员、然后再进行聚合或排除所得的概念相同。被赋予整体之成员的东西，被赋予其所有部分的成员，无论那些部分如何联结在一起。

第三类。

12. 关系借以表达、并借以构成命题的符号。虽然一切动词都可恰当地归入此类，但为逻辑的目的，只需考虑它仅包含实体动词 is 或*are*，因为其他一切动词都可分解为此元素以及第一类所含符号之一。既然这些符号用于表达各种性质或情况，它们便可用来表达动词主词的主动或

passive relation of the subject of the verb, considered with reference either to past, to present, or to future time. Thus the Proposition, "Cæsar conquered the Gauls," may be resolved into "Cæsar is he who conquered the Gauls." The ground of this analysis I conceive to be the following:—Unless we understand what is meant by having conquered the Gauls, i.e. by the expression "One who conquered the Gauls," we cannot understand the sentence in question. It is, therefore, truly an element of that sentence; another element is "Cæsar," and there is yet another required, the copula *is* to show the connexion of these two. I do not, however, affirm that there is no other mode than the above of contemplating the relation expressed by the proposition, "Cæsar conquered the Gauls;" but only that the analysis here given is a correct one for the particular point of view which has been taken, and that it suffices for the purposes of logical deduction. It may be remarked that the passive and future participles of the Greek language imply the existence of the principle which has been asserted, viz.: that the sign *is* or *are* may be regarded as an element of every personal verb.

13. The above sign, *is* or *are* may be expressed by the symbol =. The laws, or as would usually be said, the axioms which the symbol introduces, are next to be considered.

Let us take the Proposition, "The stars are the suns and the planets," and let us represent stars by x, suns by y, and planets by z; we have then

$$x = y + z. \tag{7}$$

Now if it be true that the stars are the suns and the planets, it will follow that the stars, except the planets, are suns. This would give the equation

$$x - z = y, \tag{8}$$

which must therefore be a deduction from (7). Thus a term z has been removed from one side of an equation to the other by changing its sign. This is in accordance with the algebraic rule of transposition.

But instead of dwelling upon particular cases, we may at once affirm the general axioms:—

1st. If equal things are added to equal things, the wholes are equal.

2nd. If equal things are taken from equal things, the remainders are equal.

And it hence appears that we may add or subtract equations, and employ the rule of transposition above given just as in common algebra.

Again: If two classes of things, x and y, be identical, that is, if all the members of the one are members of the other, then those members of the one class which possess a given property z will be identical with those members of the other which possess the same property z. Hence if we have the equation

$$x = y;$$

then whatever class or property z may represent, we have also

$$zx = zy.$$

被动关系，而不论所考虑的是过去、现在还是将来的时间。例如命题"Cæsar conquered the Gauls"可分解为"Cæsar is he who conquered the Gauls"。我认为这一分析的根据如下：--除非我们理解"having conquered the Gauls"即"One who conquered the Gauls"这一表达所意指的东西，我们就不能理解所讨论的句子。因此它确实是该句的一个元素；另一个元素是"Cæsar"，此外还需要一个元素，即系词*is*，以表明这两者的联系。然而我并不断言，除了上述方式之外，就没有别的方式可以考察命题"Cæsar conquered the Gauls"所表达的关系；我只是说，就所采取的特定观点而言，这里给出的分析是正确的，并且它足以满足逻辑演绎的目的。可以指出，希腊语的被动分词和将来分词暗示了已断言的原则的存在，即：符号*is*或*are*可被视为每个人称动词的一个元素。

13.上述记号，是或是可以用符号=来表示。接下来要考虑的是该符号所引入的规律，或者通常所说的公理。

让我们取命题"星辰是太阳和行星"，并以x表示星辰，以y表示太阳，以z表示行星；于是我们有

$$x = y + z. \tag{7}$$

现在，如果星辰是太阳和行星为真，那么由此可得，除行星以外的星辰是太阳。这将给出方程

$$x - z = y, \tag{8}$$

因此这必定是从(7)演绎出来的。这样，项z通过改变符号从方程的一边移到了另一边。这与代数的移项规则一致。但与其详述个别情况，我们可以直接断言一般公理：-

第一，若相等之物加于相等之物，则整体相等。

2. 若从相等之物中取去相等之物，则余下者相等。由此看来，我们可以加减方程，并像在普通代数中那样运用上述移项规则。再者：若两类事物 x 与 y 同一，即若一类的所有成员都是另一类的成员，则一类中具有给定性质 z 的那些成员，将与另一类中具有同一性质 z 的那些成员同一。因此，若我们有方程

$$x = y;$$

那么无论 z 可能代表什么类别或属性，我们也有

$$zx = zy.$$

This is formally the same as the algebraic law:—If both members of an equation are multiplied by the same quantity, the products are equal.

In like manner it may be shown that if the corresponding members of two equations are multiplied together, the resulting equation is true.

14. Here, however, the analogy of the present system with that of algebra, as commonly stated, appears to stop. Suppose it true that those members of a class x which possess a certain property z are identical with those members of a class y which possess the same property z, it does not follow that the members of the class x universally are identical with the members of the class y. Hence it cannot be inferred from the equation

$$zx = zy,$$

that the equation

$$x = y$$

is also true. In other words, the axiom of algebraists, that both sides of an equation may be divided by the same quantity, has no formal equivalent here. I say no *formal equivalent*, because, in accordance with the general spirit of these inquiries, it is not even sought to determine whether the mental operation which is represented by removing a logical symbol, z, from a combination zx, is in itself analogous with the operation of division in Arithmetic. That mental operation is indeed identical with what is commonly termed Abstraction, and it will hereafter appear that its laws are dependent upon the laws already deduced in this chapter. What has now been shown is, that there does not exist among those laws anything analogous in *form* with a commonly received axiom of Algebra.

But a little consideration will show that even in common algebra that axiom does not possess the generality of those other axioms which have been considered. The deduction of the equation $x = y$ from the equation $zx = zy$ is only valid when it is known that z is not equal to 0. If then the value $z = 0$ is supposed to be admissible in the algebraic system, the axiom above stated ceases to be applicable, and the analogy before exemplified remains at least unbroken.

15. However, it is not with the symbols of quantity generally that it is of any importance, except as a matter of speculation, to trace such affinities. We have seen (II. 9) that the symbols of Logic are subject to the special law,

$$x^2 = x.$$

Now of the symbols of Number there are but two, viz. 0 and 1, which are subject to the same formal law. We know that $0^2 = 0$, and that $1^2 = 1$; and the equation $x^2 = x$, considered as algebraic, has no other roots than 0 and 1. Hence, instead of determining the measure of formal agreement of the symbols of Logic with those of Number generally, it is more immediately suggested to us to compare them with symbols of quantity *admitting only of the values* 0 *and* 1. Let us conceive, then, of an Algebra in which the symbols x, y, z, etc.

这在形式上与代数定律相同：--若方程的两边同乘以同一量，则乘积相等。

以同样的方式可以表明，若将两个方程的对应边相乘，所得方程亦为真。

14. 然而，现行系统与通常所陈述的代数系统之间的类比，似乎到此为止。假定某一类x中具有某种性质z的那些成员，与某一类y中具有同一性质z的那些成员是同一的，并不能由此推出类x的成员普遍地与类y的成员是同一的。因此，不能从方程

$$zx = zy,$$

该方程

$$x = y$$

也是真的。换言之，代数学家的公理，即方程两边可以除以同一个量，在这里没有形式上的等价物。我说没有形式上的等价物，是因为按照这些探究的一般精神，甚至并不试图确定，从一个组合 zx 中移除一个逻辑记号 z 所代表的心理操作，本身是否与算术中的除法操作类似。那个心理操作确实与通常所称的抽象是同一回事，并且下文将表明，它的规律依赖于本章已经推导出的规律。现在所表明的是，在这些规律之中，不存在任何在形式上与一个通常接受的代数公理类似的东西。

但稍加考虑就会表明，即使在普通代数中，那条公理也不具备已经考虑过的那些其他公理的那种普遍性。从方程 $zx = zy$ 推出方程 $x = y$，只有在已知 z 不等于0时才有效。那么，如果假定值 $z = 0$ 在代数体系中是可容许的，上述公理就不再适用，而前面所例示的类比至少仍然未被打破。

15. 然而，追踪这类亲缘关系，除了作为思辨之事以外，对于一般的量之记号并无任何重要性。我们已看到(II.9)，逻辑的记号服从于特殊的规律，

$$x^2 = x.$$

现在，数的符号只有两个，即0和1，它们服从相同的形式规律。我们知道$0^2 = 0$，且$1^2 = 1$；而方程 $x^2 = x$，作为代数的方程来看，除0和1之外没有别的根。因此，与其一般地确定逻辑的记号与数的记号之间形式一致的程度，不如更直接地提示我们，将它们与只容许取值0 和1的量的符号相比较。那么，让我们设想一种代数，其中的记号 x、y、z 等等。

admit indifferently of the values 0 and 1, and of these values alone. The laws, the axioms, and the processes, of such an Algebra will be identical in their whole extent with the laws, the axioms, and the processes of an Algebra of Logic. Difference of interpretation will alone divide them. Upon this principle the method of the following work is established.

16. It now remains to show that those constituent parts of ordinary language which have not been considered in the previous sections of this chapter are either resolvable into the same elements as those which have been considered, or are subsidiary to those elements by contributing to their more precise definition.

The substantive, the adjective, and the verb, together with the particles *and*, *except*, we have already considered. The pronoun may be regarded as a particular form of the substantive or the adjective. The adverb modifies the meaning of the verb, but does not affect its nature. Prepositions contribute to the expression of circumstance or relation, and thus tend to give precision and detail to the meaning of the literal symbols. The conjunctions *if*, *either*, *or*, are used chiefly in the expression of relation among propositions, and it will hereafter be shown that the same relations can be completely expressed by elementary symbols analogous in interpretation, and identical in form and law with the symbols whose use and meaning have been explained in this Chapter. As to any remaining elements of speech, it will, upon examination, be found that they are used either to give a more definite significance to the terms of discourse, and thus enter into the interpretation of the literal symbols already considered, or to express some emotion or state of feeling accompanying the utterance of a proposition, and thus do not belong to the province of the understanding, with which alone our present concern lies. Experience of its use will testify to the sufficiency of the classification which has been adopted.

可无差别地取0和1这两个值，且仅取这两个值。这样一种代数的规律、公理和过程，在其全部范围内，将与逻辑代数的规律、公理和过程完全相同。唯有解释的不同才会将它们区分开来。本书的方法即建立于此原则之上。

16. 现在尚需表明，普通语言中那些在本章前几节未曾考察的组成部分，要么可以分解为与那些已被考察的部分相同的元素，要么从属于那些元素，有助于对它们作出更精确的定义。

名词、形容词和动词，连同虚词和、除……之外，我们已经考虑过了。代词可以被视为名词或形容词的一种特殊形式。副词修饰动词的意义，但不影响其性质。介词有助于表达情况或关系，从而倾向于使字面记号的意义更加精确和详细。连词如果、要么、或者，主要用于表达命题之间的关系，并且下文将表明，同样的关系可以完全由解释上类似、在形式和规律上与本章已解释其用法和意义的记号相同的初等记号来表达。至于言语的任何其余要素，经考察就会发现，它们要么用于给论说的项以更确定的含义，从而进入已经考虑过的字面记号的解释，要么用于表达伴随命题说出时的某种情绪或感情状态，从而不属于知性的领域，而我们的关注仅在于知性。对其使用的经验将证明所采用的分类是充分的。

Chapter III

DERIVATION OF THE LAWS OF THE SYMBOLS OF LOGIC FROM THE LAWS OF THE OPERATIONS OF THE HUMAN MIND.

1. The object of science, properly so called, is the knowledge of laws and relations. To be able to distinguish what is essential to this end, from what is only accidentally associated with it, is one of the most important conditions of scientific progress. I say, to *distinguish* between these elements, because a consistent devotion to science does not require that the attention should be altogether withdrawn from other speculations, often of a metaphysical nature, with which it is not unfrequently connected. Such questions, for instance, as the existence of a sustaining ground of phænomena, the reality of cause, the propriety of forms of speech implying that the successive states of things are connected by *operations*, and others of a like nature, may possess a deep interest and significance in relation to science, without being essentially scientific. It is indeed scarcely possible to express the conclusions of natural science without borrowing the language of these conceptions. Nor is there necessarily any practical inconvenience arising from this source. They who believe, and they who refuse to believe, that there is more in the relation of cause and effect than an invariable order of succession, agree in their interpretation of the conclusions of physical astronomy. But they only agree because they recognise a common element of scientific truth, which is independent of their particular views of the nature of causation.

2. If this distinction is important in physical science, much more does it deserve attention in connexion with the science of the intellectual powers. For the questions which this science presents become, in expression at least, almost necessarily mixed up with modes of thought and language, which betray a metaphysical origin. The idealist would give to the laws of reasoning one form of expression; the sceptic, if true to his principles, another. They who regard the phænomena with which we are concerned in this inquiry as the mere successive *states* of the thinking subject devoid of any causal connexion, and they who refer them to the *operations* of an active intelligence, would, if consistent, equally differ in their modes of statement. Like difference would also result from a difference of classification of the mental faculties. Now the

第三章

从人类心灵活动的规律推导逻辑记号之规律。

1. 真正所谓的科学，其对象是关于规律和关系的知识。能够把对这一目的而言本质的东西，与只是偶然与之相关联的东西区分开来，是科学进步最重要的条件之一。我说区分这些要素，是因为对科学的一贯献身并不要求把注意力完全从其他思辨中撤回，这些思辨往往是形而上学性质的，而科学并非不常与它们相关联。例如，诸如现象是否存在一个支撑性的根据、原因是否真实、暗示事物的相继状态由操作相联系的言语形式是否恰当等问题，以及诸如此类的问题，可能对科学具有深切的兴趣和意义，而本身并不本质上是科学的。确实，要表达自然科学的结论，几乎不可能不借用这些概念的语言。由此也未必产生任何实际的不便。那些相信在因果关系中不止有一种不变的相继秩序的人，和那些拒绝相信这一点的人，在对物理天文学结论的解释上是一致的。但他们之所以一致，只是因为他们承认一种共同的科学真理要素，这种要素独立于他们对因果性本质的特殊看法。

2.如果这一区分在物理科学中是重要的，那么在涉及理智能力的科学中就更值得注意。因为这门科学所提出的问题，至少在表述上，几乎必然与那些带有形而上学起源印记的思维方式和语言混杂在一起。唯心主义者会给予推理规律一种表述形式；怀疑论者，如果忠于其原则，则会给予另一种。那些将我们在本探究中所关注的现象视为思维主体的纯粹相继状态、不具任何因果联系的人，与那些将这些现象归因于能动理智的活动的人，如果前后一致，同样会在其陈述方式上产生分歧。类似的差异也会因对心智能力的分类不同而产生。现在，

principle which I would here assert, as affording us the only ground of confidence and stability amid so much of seeming and of real diversity, is the following, viz., that if the laws in question are really deduced from observation, they have a real existence as laws of the human mind, independently of any metaphysical theory which may seem to be involved in the mode of their statement. They contain an element of truth which no ulterior criticism upon the nature, or even upon the reality, of the mind's operations, can essentially affect. Let it even be granted that the mind is but a succession of states of consciousness, a series of fleeting impressions uncaused from without or from within, emerging out of nothing, and returning into nothing again,—the last refinement of the sceptic intellect,—still, as laws of succession, or at least of a past succession, the results to which observation had led would remain true. They would require to be interpreted into a language from whose vocabulary all such terms as cause and effect, operation and subject, substance and attribute, had been banished; but they would still be valid as scientific truths.

Moreover, as any statement of the laws of thought, founded upon actual observation, must thus contain scientific elements which are independent of metaphysical theories of the nature of the mind, the practical application of such elements to the construction of a system or method of reasoning must also be independent of metaphysical distinctions. For it is upon the scientific elements involved in the statement of the laws, that any practical application will rest, just as the practical conclusions of physical astronomy are independent of any theory of the cause of gravitation, but rest only on the knowledge of its phænomenal effects. And, therefore, as respects both the determination of the laws of thought, and the practical use of them when discovered, we are, for all really scientific ends, unconcerned with the truth or falsehood of any metaphysical speculations whatever.

3. The course which it appears to me to be expedient, under these circumstances, to adopt, is to avail myself as far as possible of the language of common discourse, without regard to any theory of the nature and powers of the mind which it may be thought to embody. For instance, it is agreeable to common usage to say that we converse with each other by the communication of ideas, or conceptions, such communication being the office of words; and that with reference to any particular ideas or conceptions presented to it, the mind possesses certain powers or faculties by which the mental regard maybe fixed upon some ideas, to the exclusion of others, or by which the given conceptions or ideas may, in various ways, be combined together. To those faculties or powers different names, as Attention, Simple Apprehension, Conception or Imagination, Abstraction, &c., have been given,—names which have not only furnished the titles of distinct divisions of the philosophy of the human mind, but passed into the common language of men. Whenever, then, occasion shall occur to use these terms, I shall do so without implying thereby that I accept the theory that the mind possesses such and such powers and faculties as distinct elements of its activity. Nor is it indeed necessary to inquire whether such powers of the understanding have a distinct existence or not. We may merge these different titles under the one generic name of *Operations* of the

我在此所要主张的原则，作为在如此多表面与真实的分歧之中给予我们信心与稳定性的唯一根据，如下，即：如果所讨论的规律确实是从观察中推导出来的，那么它们作为人类心灵的规律具有真实的存在，独立于其陈述方式中可能似乎涉及的任何形而上学理论。它们包含着一个真理的元素，任何对心灵活动的性质、甚至对其实在性的进一步批评，都不能在本质上影响这个元素。即便承认心灵只不过是一系列意识状态的相继，一系列既非来自外部亦非来自内部的、无原因的短暂印象，从无中涌现又复归于无--这是怀疑论理智的最后精炼--然而，作为相继的规律，或至少是过去相继的规律，观察所导致的结果仍将保持为真。它们将需要被翻译成一种语言，从其词汇中所有诸如原因与结果、活动与主体、实体与属性这样的术语都已被清除；但它们作为科学真理仍将有效。

此外，由于任何基于实际观察的思维规律的陈述都必然包含独立于关于心灵性质的形而上学理论的科学元素，这些元素对构建推理体系或方法的实际应用也必然独立于形而上学区分。因为任何实际应用都将立足于规律陈述中所涉及的科学元素，正如物理天文学的实际结论独立于任何关于引力原因的理论，而仅立足于对其现象结果的知识。因此，就思维规律的确定以及发现后对它们的实际运用而言，对于一切真正科学的目的，我们都不关心任何形而上学思辨的真假。

3.在这种情况下，我认为适宜采取的做法是尽可能利用日常话语的语言，而不顾及它可能被认为体现了何种关于心灵性质与能力的理论。例如，按照通常用法，我们说我们通过交流观念或概念而彼此交谈，这种交流是词语的职能；并且，相对于呈现给它的任何特定观念或概念，心灵具有某些能力或功能，借此心理关注可以固定于某些观念而排除其他观念，或者借此给定的概念或观念可以以各种方式组合在一起。这些功能或能力被赋予了不同的名称，如注意、简单领会、概念或想象、抽象等--这些名称不仅构成了人类心灵哲学不同分支的标题，而且进入了人们的日常语言。因此，每当有机会使用这些术语时，我将如此使用，而不因此暗示我接受心灵拥有这样一些能力和功能作为其活动的不同元素的理论。而且，确实没有必要探究这些知性能力是否具有独立的存在。我们可以将这些不同的名称归入一个属名，即活动--

human mind, define these operations so far as is necessary for the purposes of this work, and then seek to express their ultimate laws. Such will be the general order of the course which I shall pursue, though reference will occasionally be made to the names which common agreement has assigned to the particular states or operations of the mind which may fall under our notice.

It will be most convenient to distribute the more definite results of the following investigation into distinct Propositions.

Proposition I.

4. *To deduce the laws of the symbols of Logic from a consideration of those operations of the mind which are implied in the strict use of language as an instrument of reasoning.*

In every discourse, whether of the mind conversing with its own thoughts, or of the individual in his intercourse with others, there is an assumed or expressed limit within which the subjects of its operation are confined. The most unfettered discourse is that in which the words we use are understood in the widest possible application, and for them the limits of discourse are co-extensive with those of the universe itself. But more usually we confine ourselves to a less spacious field. Sometimes, in discoursing of men we imply (without expressing the limitation) that it is of men only under certain circumstances and conditions that we speak, as of civilized men, or of men in the vigour of life, or of men under some other condition or relation. Now, whatever may be the extent of the field within which all the objects of our discourse are found, that field may properly be termed the universe of discourse.

5. Furthermore, this universe of discourse is in the strictest sense the ultimate *subject* of the discourse. The office of any name or descriptive term employed under the limitations supposed is not to raise in the mind the conception of all the beings or objects to which that name or description is applicable, but only of those which exist within the supposed universe of discourse. If that universe of discourse is the actual universe of things, which it always is when our words are taken in their real and literal sense, then by men we mean *all men that exist*; but if the universe of discourse is limited by any antecedent implied understanding, then it is of men under the limitation thus introduced that we speak. It is in both cases the business of the word *men* to direct a certain operation of the mind, by which, from the proper universe of discourse, we select or fix upon the individuals signified.

6. Exactly of the same kind is the mental operation implied by the use of an adjective. Let, for instance, the universe of discourse be the actual Universe. Then, as the word *men* directs us to select mentally from that Universe all the beings to which the term "men" is applicable; so the adjective "good," in the combination "good men," directs us still further to select mentally from the class of *men* all those who possess the further quality "good;" and if another adjective were prefixed to the combination "good men," it would direct a further operation of the same nature, having reference to that further quality which it might be chosen to express.

人类心灵，就本书目的所必需的范围而言，对这些活动作出界定，然后寻求表达它们的终极规律。这将是我所要遵循的一般进程次序，尽管偶尔会提及共同约定赋予那些可能落入我们注意范围的特定心灵状态或活动的名称。

将以下研究较为确定的结果分列为各个命题，将最为便利。

命题一。

4. 从严格使用语言作为推理工具时所隐含的那些心灵活动的考察中，推导逻辑记号的规律。

在一切话语中，无论是心灵与自身思想的交谈，还是个人与他人的交往，都存在一个被假定或表达的界限，话语运作的主词被限制于其中。最无拘束的话语是其中我们使用的词语在尽可能最广泛的应用中被理解，而对话语而言，其界限与宇宙本身的界限同延。但更通常地，我们将自己限制在一个不那么广阔的范围之内。有时，在谈论人时，我们暗示(而未表达这一限制)我们只谈论在某些境况和条件下的人，如文明人，或处于生命旺盛期的人，或处于某种其他状况或关系中的人。那么，无论我们话语的一切对象所存在于其中的范围有多大，那个范围都可以恰当地被称为论域。

5. 此外，这个论域在最严格的意义上是话语的终极主词。在假定限制下所使用的任何名称或描述性项，其职责不是在心灵中唤起该名称或描述所适用的所有存在者或对象的概念，而只是唤起那些存在于所设论域之内的存在者或对象的概念。如果那个论域是事物的实际宇宙--当我们的语词在其真实和字面的意义上被理解时，它总是如此--那么"人"意指所有存在的人；但如果论域被任何先行的隐含理解所限制，那么我们所说的就是在此引入的限制之下的人。在两种情况下，"人"这个词的任务都是引导某种心灵活动，借此从适当的论域中选取或确定所指称的个体。

6. 形容词的使用所隐含的心灵活动恰恰属于同一种类。例如，设论域为实际的宇宙。那么，正如人这个词引导我们从那个宇宙中在心灵上选取"人"这个项所适用的一切存在者；同样，"善"这个形容词在"善人"这个组合中进一步引导我们从人这个类中在心灵上选取所有具有进一步性质"善"的那些人；如果再有一个形容词被加在"善人"这个组合之前，它就会引导一种相同性质的进一步活动，关涉到它可能被选择来表达的那种进一步的性质。

It is important to notice carefully the real nature of the operation here described, for it is conceivable, that it might have been different from what it is. Were the adjective simply *attributive* in its character, it would seem, that when a particular set of beings is designated by *men*, the prefixing of the adjective *good* would direct us to attach mentally to all those beings the quality of goodness. But this is not the real office of the adjective. The operation which we really perform is one of *selection according to a prescribed principle or idea*. To what faculties of the mind such an operation would be referred, according to the received classification of its powers, it is not important to inquire, but I suppose that it would be considered as dependent upon the two faculties of Conception or Imagination, and Attention. To the one of these faculties might be referred the formation of the general conception; to the other the fixing of the mental regard upon those individuals within the prescribed universe of discourse which answer to the conception. If, however, as seems not improbable, the power of Attention is nothing more than the power of continuing the exercise of any other faculty of the mind, we might properly regard the whole of the mental process above described as referrible to the mental faculty of Imagination or Conception, the first step of the process being the conception of the Universe itself, and each succeeding step limiting in a definite manner the conception thus formed. Adopting this view, I shall describe each such step, or any definite combination of such steps, as a *definite act of conception*. And the use of this term I shall extend so as to include in its meaning not only the conception of classes of objects represented by particular names or simple attributes of quality, but also the combination of such conceptions in any manner consistent with the powers and limitations of the human mind; indeed, any intellectual operation short of that which is involved in the structure of a sentence or proposition. The general laws to which such operations of the mind are subject are now to be considered.

7. Now it will be shown that the laws which in the preceding chapter have been determined *à posteriori* from the constitution of language, for the use of the literal symbols of Logic, are in reality the laws of that definite mental operation which has just been described. We commence our discourse with a certain understanding as to the limits of its subject, i.e. as to the limits of its Universe. Every name, every term of description that we employ, directs him whom we address to the performance of a certain mental operation upon that subject. And thus is thought communicated. But as each name or descriptive term is in this view but the representative of an intellectual operation, that operation being also prior in the order of nature, it is clear that the laws of the name or symbol must be of a derivative character,—must, in fact, originate in those of the operation which they represent. That the laws of the symbol and of the mental process are identical in expression will now be shown.

8. Let us then suppose that the universe of our discourse is the actual universe, so that words are to be used in the full extent of their meaning, and let us consider the two mental operations implied by the words “white” and “men.” The word “men” implies the operation of selecting in thought from its subject, the universe, all men; and the resulting conception, *men*, becomes the

仔细注意此处所描述的活动之真正性质是重要的，因为可以设想，它本可能不同于其实际所是。如果形容词在其性质上仅仅是归属性的，那么看来，当某一特定的一群存在者被人所指称时，加上形容词善就会引导我们在心灵上把善的性质附加到所有那些人身上。但这不是形容词的真正职责。我们真正执行的活动是按照一个规定的原则或观念进行选取。按照公认的心灵能力分类，这样一种活动应归于何种能力，探究它并不重要，但我设想它会被认为依赖于概念或想象与注意这两种能力。其中之一可以被归于一般概念的形成；另一种则被归于将心灵的目光固定于规定论域之内那些符合该概念的个人身上。然而，如果--这似乎并非不可能--注意的能力不过是继续运用心灵任何其他能力的能力，那么我们完全可以认为上述整个心灵过程应归于想象或概念的心灵能力，该过程的第一步是论域本身的概念，而每一个后续步骤都以确定的方式限制如此形成的概念。采取这一观点，我将把每一个这样的步骤，或任何确定的此类步骤的组合，描述为一个确定的概念活动。而我将扩展这个术语的使用，使其含义不仅包括由特定名称或性质的简单属性所代表的各类对象的概念，而且包括这些概念以任何与人类心灵的能力和限制相一致的方式的组合；实际上，包括任何不及句子或命题结构所涉及的那种程度的理智活动。此类心灵活动所服从的一般规律现在将要加以考察。

7. 现在将要表明，前一章中`*a posteriori*从语言的构造为逻辑的字面记号的使用所确定的那些规律，实际上是刚刚所描述的那种确定的心灵活动的规律。我们以关于其主词之界限的某种理解，即关于其论域之界限的某种理解，来开始我们的论述。我们所使用的每一个名称、每一个描述性项，都引导我们对其说话的人在那个主词上执行某种心灵活动。思想就是这样被传达的。但既然从这一观点看，每个名称或描述性项都只是某种理智活动的代表，而那种活动在自然秩序中也是在先的，那么显然，名称或记号的规律必定具有派生的性质--事实上必定起源于它们所代表的那种活动的规律。记号的规律与心灵过程的规律在表达上是同一的，这一点现在将要表明。

8. 那么，让我们假定我们论域的全域是实际的全域，从而词语应在其意义的全部范围内使用，并让我们考察“white”和“men”这两个词所隐含的两种心灵活动。“men”一词隐含了在思想中从其主词即全域中选取一切人的活动；而由此产生的概念，*men*，便成为

subject of the next operation. The operation implied by the word "white" is that of selecting from its subject, "men," all of that class which are white. The final resulting conception is that of "white men." Now it is perfectly apparent that if the operations above described had been performed in a converse order, the result would have been the same. Whether we begin by forming the conception of "*men*," and then by a second intellectual act limit that conception to "white men," or whether we begin by forming the conception of "white objects," and then limit it to such of that class as are "men," is perfectly indifferent so far as the result is concerned. It is obvious that the order of the mental processes would be equally indifferent if for the words "white" and "men" we substituted any other descriptive or appellative terms whatever, provided only that their meaning was fixed and absolute. And thus the indifference of the order of two successive acts of the faculty of Conception, the one of which furnishes the subject upon which the other is supposed to operate, is a general condition of the exercise of that faculty. It is a law of the mind, and it is the real origin of that law of the literal symbols of Logic which constitutes its formal expression (1) Chap. II.

9. It is equally clear that the mental operation above described is of such a nature that its effect is not altered by repetition. Suppose that by a definite act of conception the attention has been fixed upon men, and that by another exercise of the same faculty we limit it to those of the race who are white. Then any further repetition of the latter mental act, by which the attention is limited to white objects, does not in any way modify the conception arrived at, viz., that of white men. This is also an example of a general law of the mind, and it has its formal expression in the law ((2) Chap, II.) of the literal symbols.

10. Again, it is manifest that from the conceptions of two distinct classes of things we can form the conception of that collection of things which the two classes taken together compose; and it is obviously indifferent in what order of position or of priority those classes are presented to the mental view. This is another general law of the mind, and its expression is found in (3) Chap. II.

11. It is not necessary to pursue this course of inquiry and comparison. Sufficient illustration has been given to render manifest the two following positions, viz.:

First, That the operations of the mind, by which, in the exercise of its power of imagination or conception, it combines and modifies the simple ideas of things or qualities, not less than those operations of the reason which are exercised upon truths and propositions, are subject to general laws.

Secondly, That those laws are mathematical in their form, and that they are actually developed in the essential laws of human language. Wherefore the laws of the symbols of Logic are deducible from a consideration of the operations of the mind in reasoning.

12. The remainder of this chapter will be occupied with questions relating to that law of thought whose expression is $x^2 = x$ (II. 9), a law which, as has been implied (II. 15), forms the characteristic distinction of the operations of the mind in its ordinary discourse and reasoning, as compared with its operations when occupied with the general algebra of quantity. An important part of the

下一个活动的主词。"白"这个词所隐含的活动是从其主词"人"中选取属于该类的所有白的个体。最终得到的概念是"白人"的概念。现在十分明显，如果上述活动以相反的顺序进行，结果也会相同。无论我们是先形成"人"的概念，然后通过第二次理智活动将该概念限制为"白人"，还是先形成"白物"的概念，然后将其限制为该类中那些是"人"的个体，就结果而言都完全无关紧要。显然，如果我们用任何其他描述性或称谓性的项来替换"白"和"人"这两个词，只要它们的含义是固定和绝对的，心灵过程的顺序同样无关紧要。因此，概念能力的两个相继活动的顺序的无关紧要性--其中一个活动提供另一个活动被假定在其上运作的主词--是该能力之运用的一个一般条件。它是心灵的一条规律，并且它是逻辑字面记号的那条构成其形式表达的规律的真正起源。

(1) 第二章

9. 同样清楚的是，上述心灵活动具有这样的性质：其结果不因重复而改变。假定通过一次确定的概念活动，注意力已固定于"人"之上，又通过同一能力的另一次运用，我们将其限制于这一种族中那些白的人。那么，对后一心灵活动的任何进一步重复--即把注意力限制于白的对象--都丝毫不会改变所达到的概念，即白的人的概念。这也是心灵的一条普遍规律的例证，并且它在字面记号的法律((2) 第二章)中具有其形式表达。

10. 再者，显然从两个不同事物类的概念，我们可以形成由这两个类合在一起所构成的那一事物集合的概念；而这两个类以何种位置次序或优先次序呈现于心灵视野，显然是无关紧要的。这是心灵的又一条一般规律，其表达式见于(3)第二章。

11. 不必继续这一探究与比较的进程。已给出充分的说明，使以下两个论点得以显明，即：

第一，心灵的活动--在运用其想象力或概念能力时，它组合并改变事物或性质的简单观念，这些活动不亚于那些施用于真理和命题的理性活动--服从普遍规律。第二，那些规律在形式上是数学的，并且它们实际上是在人类语言的本质规律中被展开的。因此，逻辑的记号的法律可以从对推理中心灵活动的考察中推导出来。

12. 本章余下的部分将用于讨论与那条思维规律相关的问题，该规律的表达式为$x^2 = x$(II.9)，正如已隐含指出的(II.15)，这条规律构成了心灵在其日常言说与推理中的活动，与心灵从事一般量之代数时的活动相比，所具有的特征性区别。以下探究的一个重要部分将在于

following inquiry will consist in proving that the symbols 0 and 1 occupy a place, and are susceptible of an interpretation, among the symbols of Logic; and it may first be necessary to show how particular symbols, such as the above, may with propriety and advantage be employed in the representation of distinct systems of thought.

The ground of this propriety cannot consist in any community of interpretation. For in systems of thought so truly distinct as those of Logic and Arithmetic (I use the latter term in its widest sense as the science of Number), there is, properly speaking, no community of subject. The one of them is conversant with the very conceptions of things, the other takes account solely of their numerical relations. But inasmuch as the forms and methods of any system of reasoning depend immediately upon the laws to which the symbols are subject, and only mediately, through the above link of connexion, upon their interpretation, there may be both propriety and advantage in employing the same symbols in different systems of thought, provided that such interpretations can be assigned to them as shall render their formal laws identical, and their use consistent. The ground of that employment will not then be community of interpretation, but the community of the formal laws, to which in their respective systems they are subject. Nor must that community of formal laws be established upon any other ground than that of a careful observation and comparison of those results which are seen to flow independently from the interpretations of the systems under consideration.

These observations will explain the process of inquiry adopted in the following Proposition. The literal symbols of Logic are universally subject to the law whose expression is $x^2 = x$. Of the symbols of Number there are two only, 0 and 1, which satisfy this law. But each of these symbols is also subject to a law peculiar to itself in the system of numerical magnitude, and this suggests the inquiry, what interpretations must be given to the literal symbols of Logic, in order that the same peculiar and formal laws may be realized in the logical system also.

Proposition II

13. *To determine the logical value and significance of the symbols* 0 *and* 1.
The symbol 0, as used in Algebra, satisfies the following formal law,

$$0 \times y = 0, \text{ or } 0y = 0, \tag{1}$$

whatever *number* y may represent. That this formal law may be obeyed in the system of Logic, we must assign to the symbol 0 such an interpretation that the class represented by $0y$ may be identical with the class represented by 0, whatever the class y may be. A little consideration will show that this condition is satisfied if the symbol 0 represent Nothing. In accordance with a previous definition, we may term Nothing a class. In fact, Nothing and Universe are the two limits of class extension, for they are the limits of the possible interpretations of general names, none of which can relate to fewer individuals than are comprised in Nothing, or to more than are comprised in the Universe.

证明记号0和1在逻辑的记号中占有一席之地，并且可以接受一种解释；或许首先有必要表明，诸如上述那样的特殊记号，如何可以恰当而有利地用于表示不同的思想系统。

这种恰当性的根据不可能在于任何解释的共同性。因为在像逻辑和算术(我使用后一术语在其最广的意义上，即数的科学)那样真正不同的思想体系中，严格说来，并没有主词的共同性。其中一个与事物的概念本身打交道，另一个则仅仅考虑它们的数值关系。但是，既然任何推理体系的形式和方法直接取决于记号所服从的规律，并且只是间接地、通过上述联系环节，取决于它们的解释，那么在不同的思想体系中采用相同的记号就可能既有恰当性又有益处，只要能够赋予它们这样的解释，使得它们的形式规律相同，并且它们的使用一致。那么，这种采用的根据将不是解释的共同性，而是形式规律的共同性，在它们各自的体系中它们服从这些规律。也不得根据任何其他理由来确立这种形式规律的共同性，而只能根据对所考察的体系的解释中独立地产生出来的那些结果进行仔细的观察和比较。

这些观察将解释以下命题中所采用的探究过程。逻辑的字面记号普遍服从其表达为 $x^2 = x$ 的规律。在数的记号中，只有两个，0 和 1，满足这一规律。但是，这些记号中的每一个在数值量体系中还服从一个为其自身所特有的规律，这提示了这样的探究：必须赋予逻辑的字面记号以什么解释，以便同样的特有形式和形式规律也能在逻辑体系中实现。

命题 II

13. 确定记号 0 和 1 的逻辑价值与意义。记号0，如代数中所用，满足以下形式规律，

$$0 \times y = 0, \text{ or } 0y = 0, \tag{1}$$

无论数 y 可能代表什么。为了使这一形式规律在逻辑体系中得到遵守，我们必须赋予记号 0 这样的解释，使得由 0 y 所代表的类与由 0 所代表的类相同，无论类 y 可能是什么。稍加思考就会表明，如果记号 0 代表无，则这一条件得到满足。按照先前的定义，我们可以把无称为一个类。事实上，无和宇宙是类外延的两个极限，因为它们是普通名称的可能解释的极限，其中没有一个能关涉少于无中所包含的个体，或多于宇宙中所包含的个体。

Now whatever the class y may be, the individuals which are common to it and to the class "Nothing" are identical with those comprised in the class "Nothing," for they are none. And thus by assigning to 0 the interpretation Nothing, the law (1) is satisfied; and it is not otherwise satisfied consistently with the perfectly general character of the class y.

Secondly, The symbol 1 satisfies in the system of Number the following law, viz.,

$$1 \times y = y, \text{ or } 1y = y,$$

whatever number y may represent. And this formal equation being assumed as equally valid in the system of this work, in which 1 and y represent classes, it appears that the symbol 1 must represent such a class that all the individuals which are found in *any* proposed class y are also all the individuals $1y$ that are common to that class y and the class represented by 1. A little consideration will here show that the class represented by 1 must be "the Universe," since this is the only class in which are found *all* the individuals that exist in *any* class. Hence the respective interpretations of the symbols 0 and 1 in the system of Logic are *Nothing* and *Universe*.

14. As with the idea of any class of objects as "men," there is suggested to the mind the idea of the contrary class of beings which are not men; and as the whole Universe is made up of these two classes together, since of every individual which it comprehends we may affirm either that it is a man, or that it is not a man, it becomes important to inquire how such contrary names are to be expressed. Such is the object of the following Proposition.

PROPOSITION III.

If x represent any class of objects, then will $1 - x$ represent the contrary or supplementary class of objects., i.e. the class including all objects which are not comprehended in the class x.

For greater distinctness of conception let x represent the class men, and let us express, according to the last Proposition, the Universe by 1; now if from the conception of the Universe, as consisting of "men" and "not-men," we exclude the conception of "men," the resulting conception is that of the contrary class, "not-men." Hence the class "not-men" will be represented by $1 - x$. And, in general, whatever class of objects is represented by the symbol x, the contrary class will be expressed by $1 - x$.

15. Although the following Proposition belongs in strictness to a future chapter of this work, devoted to the subject of *maxims* or *necessary truths*, yet, on account of the great importance of that law of thought to which it relates, it has been thought proper to introduce it here.

PROPOSITION IV.

That axiom of metaphysicians which is termed the principle of contradiction, and which affirms that it is impossible for any being to possess a quality, and at the same time not to possess it, is a consequence of the fundamental law of thought, whose expression is $x^2 = x$.

现在，无论类y可能是什么，它与类"Nothing"所共有的个体，与包含在类"Nothing"中的那些个体是同一的，因为它们一个也没有。于是，通过赋予0以Nothing这一解释，规律(1)得到满足；而若要与类y的完全一般的性质相一致，它便不能以别的方式得到满足。

Secondly, The symbol 1 satisfies in the system of Number the following law, viz.,

$$1 \times y = y, \text{ or } 1y = y,$$

无论数y可能代表什么。并且，这个形式方程在本著作的系统中被假定为同样有效，其中1和y代表类，看来符号1必须代表这样一个类，即凡在任何提出的类y中发现的一切个体，也都是那个类y与由1所代表的类所共有的全部个体1y。稍加思考即可表明，由1所代表的类必须是"论域"，因为这是在其中发现存在于任何类中的全部个体的唯一一个类。因此，符号0和1在逻辑系统中的各自解释是无和论域。

14. 正如随着任何一类对象如"men"的观念，心灵中便浮现出与之相反的、不是人的存在者之类的观念；又正如整个全域是由这两个类合在一起构成的，因为对于它所包含的每一个个体，我们都可以断言它是一个人，或者断言它不是一个人，因此，探究这类相反的名称应如何表达，便变得重要起来。这便是以下命题的目的。

命题三。

若表示任何一类对象，则 1 – x 将表示与之相反或补充的对象类，即包括一切未被包含在类中的对象的类。

为了概念更加清晰，让x代表人类这个类，并且按照上一个命题，我们用1来表示论域；现在，如果从由"人"和"非人"组成的论域概念中排除"人"的概念，那么得到的概念就是相反的类，"非人"。因此，"非人"这个类将由1–x来表示。并且，一般来说，无论符号x代表哪一类对象，相反的类都将由1–x来表示。

15. 尽管以下命题严格说来属于本书后面论述准则或必然真理的那一章，然而，鉴于它所涉及的那条思维规律极为重要，此处引入它被认为是恰当的。

命题四。

形而上学家称之为矛盾律的那条公理，它断言任何存在者不可能具有一种性质同时又不同时具有它，乃是思维基本规律的一个推论，该规律的表达式为 $x^2 = x$ 。

Let us write this equation in the form

$$x - x^2 = 0,$$

whence we have

$$x\,(1-x) = 0; \tag{1}$$

both these transformations being justified by the axiomatic laws of combination and transposition (II. 13). Let us, for simplicity of conception, give to the symbol x the particular interpretation of *men*, then $1 - x$ will represent the class: of "not-men" (Prop. III.) Now the formal product of the expressions of two classes represents that class of individuals which is common to them both (II. 6). Hence $x\,(1-x)$ will represent the class whose members are at once "men," and "not men," and the equation (1) thus express the principle, *that a class whose members are at the same time men and not men does not exist.* In other words, that *it is impossible for the same individual to be at the same time a man and not a man.* Now let the meaning of the symbol x be extended from the representing of "men," to that of any class of beings characterized by the possession of any quality whatever; and the equation (1) will then express that it is impossible for a being to possess a quality and not to possess that quality at the same time. But this is identically that "principle of contradiction" which Aristotle has described as the fundamental axiom of all philosophy. "It is impossible that the same quality should both belong and not belong to the same thing.. . . This is the most certain of all principles.. . . Wherefore they who demonstrate refer to this as an ultimate opinion. For it is by nature the source of all the other axioms."[1]

The above interpretation has been introduced not on account of its immediate value in the present system, but as an illustration of a significant fact in the philosophy of the intellectual powers, viz., that what has been commonly regarded as the fundamental axiom of metaphysics is but the consequence of a law of thought, mathematical in its form. I desire to direct attention also to the circumstance that the equation (1) in which that fundamental law of thought is expressed is an equation of the second degree.[2] Without speculating

[1]Τὸ γὰρ αὐτὸ ἅμα ὑπάρχειν τε καὶ μὴ ὑπάρχειν ἀδύνατον τῷ αὐτῷ καὶ κατὰ τὸ αὐτό. . . Αὕτη δὴ πασῶν ἐστὶ βεβαιοτάτη τῶν ἀρχῶν. . . Διὸ πάντες οἱ ἀποδεικνύντες εἰς ταύτην ἀνάγουσιν ἐσχάτην δόξαν· φύσει γὰρ ἀρξὴ καὶ τῶν ἄλλων ἀξεωμάτων αὕτη πάντων.—*Metaphysica*, III, 3.

[2]Should it here be said that the existence of the equation $x^2 = x$ necessitates also the existence of the equation $x^3 = x$, which is of the third degree, and then inquired whether that equation does not indicate a process of *trichotomy*; the answer is, that the equation $x^3 = x$ is not interpretable in the system of logic. For writing it in either of the forms

$$x\,(1-x)\,(1+x) = 0, \tag{2}$$
$$x\,(1-x)\,(-1-x) = 0, \tag{3}$$

we see that its interpretation, if possible at all, must involve that of the factor $1 + x$, or of the factor $-1 - x$. The former is not interpretable, because we cannot conceive of the addition of any class x to the universe 1; the latter is not interpretable, because the symbol -1 is not subject to the law $x(1 - x) = 0$, to which all class symbols are subject. Hence the equation $x^3 = x$ admits of no interpretation analogous to that of the equation $x^2 = x$. Were the former equation, however, true independently of the latter, i.e. were that act of the mind which is

让我们把这个方程写成如下形式

$$x - x^2 = 0,$$

由此我们得到 have

$$x\,(1-x) = 0; \tag{1}$$

这两个变换都由组合与换位的公理定律(II.13)证明为正当。为了概念的简单性，让我们给符号x以人这一特定解释，那么1–x将代表"非人"这个类(命题III)。现在，两个类的表达式的形式乘积代表它们二者所共有的那个个体类(II.6)。因此，x(1–x)将代表其成员同时是"人"和"非人"的那个类，而方程(1)因此表达了这一原理：一个其成员同时是人和非人的类并不存在。换言之，同一个个体不可能同时既是人又不是人。现在，让符号x的意义从代表"人"扩展到代表任何以具有某种性质为特征的存在物的类；于是方程(1)将表达：一个存在物不可能同时具有一种性质又不具有那种性质。但这恰恰就是亚里士多德所描述为一切哲学之基本公理的"矛盾原理"。"同一性质不可能既属于又不属于同一事物.....这是一切原理中最确定的.....因此，进行证明的人把它当作最终的意见来诉诸。因为它在本性上就是其他一切公理的根源。"[1]

上述解释之所以引入，并不是因为它在当前系统中具有直接价值，而是作为理智能力哲学中一个有意义事实的例证，即：通常被视为形而上学基本公理的东西，只不过是一条思维规律的结果，而这条规律在形式上是数学的。我还希望提请注意这一情况：表达那条思维基本规律的方程(1)是一个二次方程。[2]不作推测

1Τὸ γὰρ αὐτὸ ἅμα ὑπάρχειν τε καὶ μὴ ὑπάρχειν ἀδύνατον τ˜ψ αὐτ˜ψ καὶ κατὰ τὸ αὐτό...Αὕτη δὴ πασνῶν ἐστὶ βεβαιοτάτη τῶν ἀρχῶν...Διὸ πάντες οἱ ἀποδεικνύντες εἰς ταύτην ἀνάγουσιν ἐσχάτην δόξαν· φύσει γὰρ ἀρξὴ καὶ τῶν ἄλλων ἀξεωμάτων αὕτη πάντων.-Metaphysica, III, 3. 2如果这里有人说，方程x2 = x的存在也必然导致方程x3 = x的存在，而后者是三次方程，然后追问该方程是否指示一个三分过程；回答是，方程x3 = x在逻辑系统中不可解释。因为把它写成以下两种形式之一

$$x\,(1-x)\,(1+x) = 0, \tag{2}$$
$$x\,(1-x)\,(-1-x) = 0, \tag{3}$$

我们看到，它的解释，如果终究可能的话，必定涉及因子1 + x的解释，或因子– 1 – x的解释。前者不可解释，因为我们无法设想将任何类x加到全集1上；后者不可解释，因为记号– 1不服从定律x(1 – x) = 0，而所有类记号都服从该定律。因此方程x 3 = x不容有类似于方程x 2 = x的解释。然而，如果前一方程独立于后一方程而为真，也就是说，如果那种心灵活动

at all in this chapter upon the question, whether that circumstance is necessary in its own nature, we may venture to assert that if it had not existed, the whole procedure of the understanding would have been different from what it is. Thus it is a consequence of the fact that the fundamental equation of thought is of the second degree, that we perform the operation of analysis and classification, by division into pairs of opposites, or, as it is technically said, by *dichotomy.* Now if the equation in question had been of the third degree, still admitting of interpretation as such, the mental division must have been threefold in character, and we must have proceeded by a species of *trichotomy*, the real nature of which it is impossible for us, with our existing faculties, adequately to conceive, but the laws of which we might still investigate as an object of intellectual speculation.

16. The law of thought expressed by the equation (1) will, for reasons which are made apparent by the above discussion, be occasionally referred to as the "law of duality."

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denoted by the symbol x, such that its second repetition should reproduce the result of a single operation, but not its first or mere repetition, it is presumable that we should be able to interpret one of the forms (1), (2), which under the actual conditions of thought we cannot do. There exist operations, known to the mathematician, the law of which may be adequately expressed by the equation $x^3 = x$. But they are of a nature altogether foreign to the province of general reasoning.

In saying that it is conceivable that the law of thought might have been different from what it is, I mean only that we can frame such an hypothesis, and study its consequences. The possibility of doing this involves no such doctrine as that the actual law of human reason is the product either of chance or of arbitrary will.

在本章中完全取决于这样一个问题，即那种情况在其本性上是否是必要的，那么我们可以冒昧地断言，如果它不曾存在，知性的整个程序就会与现在不同。因此，思维的基本方程是二次的这一事实的一个后果是，我们通过划分为成对的对立面，或者按术语所说，通过二分法，来进行分析和分类的操作。现在，如果所讨论的方程是三次的，仍然容许作为这样的解释，那么心理的划分在性质上必定是三重的，我们就必定通过一种三分法来进行，其真实性质对于我们凭借现有的能力来说是不可能充分设想的，但我们仍然可以将其规律作为理智思辨的对象来研究。

16. 由方程(1)所表达的思维规律，由于上述讨论已使之显明的理由，将偶尔被称为"对偶律"。

由记号x表示，使得它的第二次重复应当再现一次操作的结果，但不是其第一次或仅仅一次重复的结果，那么可以推定，我们应当能够解释形式(1)、(2)之一，而在思维的实际条件下我们不能这样做。存在着数学家所知道的某些操作，其规律可以由方程$x^3 = x$充分表达。但它们具有一种与一般推理的领域完全无关的性质。

在说思维规律可以设想为不同于其实际所是时，我仅仅是指我们能够构造这样一个假说，并研究其后果。这样做的可能性并不包含这样的学说，即人类理性的实际规律是偶然或任意意志的产物。

Chapter IV

OF THE DIVISION OF PROPOSITIONS INTO THE TWO CLASSES OF "PRIMARY" AND "SECONDARY;" OF THE CHARACTERISTIC PROPERTIES OF THOSE CLASSES, AND OF THE LAWS OF THE EXPRESSION OF PRIMARY PROPOSITIONS.

1. The laws of those mental operations which are concerned in the processes of Conception or Imagination having been investigated, and the corresponding laws of the symbols by which they are represented explained, we are led to consider the practical application of the results obtained: first, in the expression of the complex terms of propositions; secondly, in the expression of propositions; and lastly, in the construction of a general method of deductive analysis. In the present chapter we shall be chiefly concerned with the first of these objects, as an introduction to which it is necessary to establish the following Proposition:

PROPOSITION I.

All logical propositions may be considered as belonging to one or the other of two great classes, to which the respective names of "Primary" or "Concrete Propositions," and "Secondary" or "Abstract Propositions," may be given.

Every assertion that we make may be referred to one or the other of the two following kinds. Either it expresses a relation among *things*, or it expresses, or is equivalent to the expression of, a relation among *propositions*. An assertion respecting the properties of things, or the phænomena which they manifest, or the circumstances in which they are placed, is, properly speaking, the assertion of a relation among things. To say that "snow is white," is for the ends of logic equivalent to saying, that "snow is a white thing." An assertion respecting facts or events, their mutual connexion and dependence, is, for the same ends, generally equivalent to the assertion, that such and such propositions concerning those events have a certain relation to each other as respects their mutual truth or falsehood. The former class of propositions, relating to *things*, I call "Primary;" the latter class, relating to *propositions*, I call "Secondary." The

第四章

论命题划分为"初级"与

"次级的；"特征

这些类的性质，以及初级命题之表达式的规律。

1. 在研究了那些与概念或想象过程有关的心灵操作的规律，并解释了表示它们的记号的相应规律之后，我们被引导去考虑所获得结果的实际应用：首先，在命题的复杂项的表达中；其次，在命题的表达中；最后，在演绎分析的一般方法的构造中。在本章中，我们将主要关注这些对象中的第一个，作为它的导论，有必要建立以下命题：

命题一。

一切逻辑命题都可以被认为属于两个大类之一，这两个大类可分别命名为"初级"或"具体命题"，以及"次级"或"抽象命题"。

我们所做的每一个断言都可以归入以下两种之一。它要么表达事物之间的关系，要么表达--或等价于表达--命题之间的关系。关于事物的属性、或它们所显现的现象、或它们所处的情况的断言，确切地说，是关于事物之间关系的断言。说"雪是白的"，就逻辑的目的而言，等价于说"雪是一种白的东西"。关于事实或事件、它们的相互联系和依赖的断言，就同样的目的而言，通常等价于这样的断言：关于那些事件的如此这般的命题，就它们相互的真或假而言，彼此具有某种关系。前一类命题，涉及事物，我称之为"初级的"；后一类命题，涉及命题，我称之为"次级的"。

distinction is in practice nearly but not quite co-extensive with the common logical distinction of propositions as categorical or hypothetical.

For instance, the propositions, "The sun shines," "The earth is warmed," are primary; the proposition, "If the sun shines the earth is warmed," is secondary. To say, "The sun shines," is to say, "The sun is that which shines," and it expresses a relation between two classes of things, viz., "the sun" and "things which shine." The secondary proposition, however, given above, expresses a relation of dependence between the two primary propositions, "The sun shines," and "The earth is warmed." I do not hereby affirm that the relation between these propositions is, like that which exists between the facts which they express, a relation of causality, but only that the relation among the propositions so implies, and is so implied by, the relation among the facts, that it may for the ends of logic be used as a fit representative of that relation.

2. If instead of the proposition, "The sun shines," we say, "It is true that the sun shines," we then speak not directly of things, but of a proposition concerning things, viz., of the proposition, "The sun shines." And, therefore, the proposition in which we thus speak is a secondary one. Every primary proposition may thus give rise to a secondary proposition, viz., to that secondary proposition which asserts its truth, or declares its falsehood.

It will usually happen, that the particles *if*, *either*, *or*, will indicate that a proposition is secondary; but they do not necessarily imply that such is the case. The proposition, "Animals are either rational or irrational," is primary. It cannot be resolved into "Either animals are rational or animals are irrational," and it does not therefore express a relation of dependence between the two propositions connected together in the latter disjunctive sentence. The particles, *either*, *or*, are in fact no *criterion* of the nature of propositions, although it happens that they are more frequently found in secondary propositions. Even the conjunction *if* may be found in primary propositions. "Men are, if wise, then temperate," is an example of the kind. It cannot be resolved into "If all men are wise, then all men are temperate."

3. As it is not my design to discuss the merits or defects of the ordinary division of propositions, I shall simply remark here, that the principle upon which the present classification is founded is clear and definite in its application, that it involves a real and fundamental distinction in propositions, and that it is of essential importance to the development of a general method of reasoning. Nor does the fact that a primary proposition may be put into a form in which it becomes secondary at all conflict with the views here maintained. For in the case thus supposed, it is not of the things connected together in the primary proposition that any direct account is taken, but only of the proposition itself considered as *true* or as *false*.

4. In the expression both of primary and of secondary propositions, the same symbols, subject, as it will appear, to the same laws, will be employed in this work. The difference between the two cases is a difference not of form but of interpretation. In both cases the actual relation which it is the object of the proposition to express will be denoted by the sign =. In the expression of primary propositions, the members thus connected will usually represent the

这一区分在实践上几乎但不完全与通常逻辑上命题分为直言命题与假言命题的区分相重合。

例如，“太阳照耀”、“大地变暖”是初级命题；“如果太阳照耀，那么大地变暖”是次级命题。说“太阳照耀”，就是说“太阳是那个照耀的东西”，它表达了两类事物之间的关系，即“太阳”与“照耀的东西”之间的关系。然而，上面给出的次级命题表达的是两个初级命题“太阳照耀”和“大地变暖”之间的依赖关系。我在此并不断言这些命题之间的关系就像它们所表达的事实之间的关系那样是一种因果关系，而只是说，命题之间的关系如此蕴涵着事实之间的关系、并且如此被事实之间的关系所蕴涵，以至于为了逻辑的目的，它可以被用作那种关系的合适代表。

2. 如果我们不说“The sun shines”这个命题，而说“It is true that the sun shines”，那么我们便不是直接谈论事物，而是谈论一个关于事物的命题，即“The sun shines”这个命题。因此，我们如此言说的命题是一个次级命题。每一个初级命题都可以由此产生一个次级命题，即那个断言其真或宣布其假的次级命题。

通常会出现这样的情况：虚词*if*、*either*、*or*会指示一个命题是次级命题；但它们并不必然意味着情况就是如此。“动物要么是理性的要么是非理性的”这个命题是初级命题。它不能分解为“要么动物是理性的，要么动物是非理性的”，因此它并不表达后一个析取句中连接在一起的两个命题之间的依赖关系。虚词*either*、*or*实际上并不是命题性质的标准，尽管它们更频繁地出现在次级命题中。甚至连词*if*也可能出现在初级命题中。“人如果明智，那么就是节制的”就是这类例子。它不能分解为“如果所有人都是明智的，那么所有人都是节制的”。

3. 由于我的目的并非讨论命题的通常划分的优劣，我在此只需说明：当前分类所依据的原则在其应用中清晰而明确，它涉及命题中一种真实而根本的区别，并且对于一种一般推理方法的发展具有本质的重要性。初级命题可以被置入一种形式而成为次级命题这一事实，也完全不与这里所主张的观点相冲突。因为在如此设想的情形中，被考虑的并不是初级命题中连接在一起的事物本身的任何直接说明，而只是命题本身被看作真或看作假。

4. 在初级命题与次级命题的表达中，本书将使用相同的记号，它们如将要表明的那样，服从相同的规律。两种情形之间的差别不是形式上的差别，而是解释上的差别。在两种情形中，命题所要表达的实际关系都将用符号=来表示。在初级命题的表达中，如此联结起来的各项通常将表示

"terms" of a proposition, or, as they are more particularly designated, its subject and predicate.

Proposition II.

5. *To deduce a general method, founded upon the enumeration of possible varieties, for the expression of any class or collection of things, which may constitute a "term" of a Primary Proposition.*

First, If the class or collection of things to be expressed is defined only by names or qualities common to all the individuals of which it consists, its expression will consist of a single term, in which the symbols expressive of those names or qualities will be combined without any connecting sign, as if by the algebraic process of multiplication. Thus, if x represent opaque substances, y polished substances, z stones, we shall have,

xyz = opaque polished stones;

$xy(1-z)$ = opaque polished substances which are not stones;

$x(1-y)(1-z)$ = opaque substances which are not polished, and are not stones;

and so on for any other combination. Let it be observed, that each of these expressions satisfies the same law of duality, as the individual symbols which it contains. Thus,

$$xyz \times xyz = xyz;$$
$$xy(1-z) \times xy(1-z) = xy(1-z);$$

and so on. Any such term as the above we shall designate as a "class term," because it expresses a class of things by means of the common properties or names of the individual members of such class.

Secondly, If we speak of a collection of things, different portions of which are defined by different properties, names, or attributes, the expressions for those different portions must be separately formed, and then connected by the sign $+$. But if the collection of which we desire to speak has been formed by excluding from some wider collection a defined portion of its members, the sign $-$ must be prefixed to the symbolical expression of the excluded portion. Respecting the use of these symbols some further observations may be added.

6. Speaking generally, the symbol $+$ is the equivalent of the conjunctions "and," "or," and the symbol $-$, the equivalent of the preposition "except." Of the conjunctions "and" and "or," the former is usually employed when the collection to be described forms the subject, the latter when it forms the predicate, of a proposition. "The scholar *and* the man of the world desire happiness," may be taken as an illustration of one of these cases. "Things possessing utility are *either* productive of pleasure *or* preventive of pain," may exemplify the other. Now whenever an expression involving these particles

命题的"项"，或者更具体地称之为命题的主词和谓词。

命题二。

5. 推导出一种一般方法，其基础在于列举可能的各种变体，用以表达任何类或事物的集合，而这类或集合可以构成一个特称命题的"项"。

首先，如果要表达的类或事物集合仅仅由该集合中所有个体共有的名称或性质来定义，那么它的表达将只包含一个项，其中表达那些名称或性质的记号将不带任何连接符号地组合在一起，如同代数的乘法过程一样。因此，如果x表示不透明物质，y表示抛光物质，z表示石头，我们就会有，

xyz = 不透明抛光石；

xy(1 – z) = 不透明的抛光物质，它们不是石头；x(1 – y)(1 – z) = 不透明物质，它们不是抛光的，并且不是石头；

对于任何其他组合也是如此。请注意，这些表达中的每一个都满足与其所包含的各个记号相同的对偶律。因此，

$$xyz \times xyz = xyz;$$
$$xy(1-z) \times xy(1-z) = xy(1-z);$$

如此等等。任何如上所述的项，我们将称之为"类项"，因为它通过该类中各个成员的共同性质或名称来表达一个事物的类。

其次，如果我们谈论的是事物的一个集合，其不同部分由不同的性质、名称或属性来定义，那么那些不同部分的表达式必须分别形成，然后用符号+连接起来。但是，如果我们想要谈论的那个集合是通过从某个更广泛的集合中排除其成员的一个确定部分而形成的，那么符号–必须加在被排除部分的符号表达式之前。关于这些符号的使用，还可以补充一些进一步的观察。

6. 一般地说，符号+等价于连词"and"、"or"，而符号–等价于介词"except"。在连词"and"和"or"中，前者通常用于所要描述的集合构成命题的主词时，后者用于它构成命题的谓词时。"学者*and*世故之人欲求幸福"可以作为其中一种情形的例证。"具有效用的事物*either*产生快乐*or*防止痛苦"可以例示另一种情形。现在，每当一个涉及这些虚词的表达式

presents itself in a primary proposition, it becomes very important to know whether the groups or classes separated in thought by them are intended to be quite distinct from each other and mutually exclusive, or not. Does the expression, "Scholars and men of the world," include or exclude those who are both? Does the ex-pression, "Either productive of pleasure or preventive of pain," include or exclude things which possess both these qualities? I apprehend that in strictness of meaning the conjunctions "and," "or," do possess the power of separation or exclusion here referred to; that the formula, "All x's are either y's or z's," rigorously interpreted, means, "All x's are either y's, but not z's," or, "z's but not y's." But it must at the same time be admitted, that the "jus et norma loquendi" seems rather to favour an opposite interpretation. The expression, "Either y's or z's," would generally be understood to include things that are y's and z's at the same time, together with things which come under the one, but not the other. Remembering, however, that the symbol $+$ does possess the separating power which has been the subject of discussion, we must resolve any disjunctive expression which may come before us into elements really separated in thought, and then connect their respective expressions by the symbol $+$.

And thus, according to the meaning implied, the expression, "Things which are either x's or y's," will have two different symbolical equivalents. If we mean, "Things which are x's, but not y's, or y's, but not x's," the expression will be

$$x(1-y)+y(1-x);$$

the symbol x standing for x's, y for y's. If, however, we mean, "Things which are either x's, or, if not x's, then y's," the expression will be

$$x+y(1-x).$$

This expression supposes the admissibility of things which are both x's and y's at the same time. It might more fully be expressed in the form

$$xy+x(1-y)+y(1-x);$$

but this expression, on addition of the two first terms, only reproduces the former one.

Let it be observed that the expressions above given satisfy the fundamental law of duality (III. 16). Thus we have

$$\begin{aligned}\{x(1-y)+y(1-x)\}^2 &= x(1-y)+y(1-x),\\ \{x+(1-x)\}^2 &= x+y(1-x).\end{aligned}$$

It will be seen hereafter, that this is but a particular manifestation of a general law of expressions representing "classes or collections of things."

7. The results of these investigations may be embodied in the following rule of expression.

当它出现在一个主词命题中时，了解它们在思想中所分离的群或类是否被意图为彼此完全区别且相互排斥，就变得非常重要。“学者和世俗之人”这一表达，是包括还是排除那些两者兼有的人?“要么产生快乐，要么防止痛苦”这一表达，是包括还是排除同时具有这两种性质的事物?我认为，在严格的意义上，连词“和”、“或”确实具有这里所指的分离或排除的力量；公式“所有 x 要么是 y 要么是 z”，严格解释为，“所有 x 要么是 y，但不是 z”，或者，“z 但不是 y”。但必须同时承认，“jus et norma loquendi”似乎更倾向于相反的解释。“要么 y 要么 z”这一表达，通常会被理解为包括同时是 y 和 z 的事物，以及属于其一但不属于另一的事物。然而，记住记号 + 确实具有所讨论的分离力量，我们必须将任何可能出现在我们面前的析取表达分解为在思想中真正分离的元素，然后用记号 + 连接它们各自的表达。

因此，根据所隐含的意义，表达式“要么是x要么是y的事物”将有两种不同的符号等价物。如果我们的意思是“是x但不是y的事物，或者是y但不是x的事物”，那么表达式将是

$$x(1-y)+y(1-x);$$

记号x代表x,y代表y。然而，如果我们的意思是“要么是x的事物，或者，如果不是x，那么是y的事物”，那么表达式将是

$$x+y(1-x).$$

这个表达式假定同时既是x又是y的事物是可容许的。它可以更完整地表达为以下形式

$$xy+x(1-y)+y(1-x);$$

但这个表达式在将前两项相加之后，仅仅重现了前一个表达式。

请注意，上面给出的表达式满足对偶律的基本规律(III.16)。因此我们有

$$\begin{aligned}\{x(1-y)+y(1-x)\}^2 &= x(1-y)+y(1-x),\\ \{x+(1-x)\}^2 &= x+y(1-x).\end{aligned}$$

以后将会看到，这只不过是表达“事物的类或集合”的表达式的一般规律的一个特殊表现。

7. 这些研究的结果可以体现在以下的表达规则中。

RULE.—*Express simple names or qualities by the symbols x, y, z, &c., their contraries by $1-x$, $1-y$, $1-z$, &c.; classes of things defined by common names or qualities, by connecting the corresponding symbols as in multiplication; collections of things, consisting of portions different from each other, by connecting the expressions of those portions by the sign $+$. In particular, let the expression, "Either x's or y's," be expressed by $x(1-y)+y(1-x)$, when the classes denoted by x and y are exclusive, by $x+y(1-x)$ when they are not exclusive. Similarly let the expression, "Either x's, or y's, or z's," be expressed by $x(1-y)(1-z)+y(1-x)(1-z)+z(1-x)(1-y)$, when the classes denoted by x, y, and z, are designed to be mutually exclusive, by $x+y(1-x)+z(1-x)(1-y)$, when they are not meant to be exclusive, and so on.*

8. On this rule of expression is founded the converse rule of interpretation. Both these will be exemplified with, perhaps, sufficient fulness in the following instances. Omitting for brevity the universal subject "things," or "beings," let us assume

$$x = \text{hard}, y = \text{elastic}, z = \text{metals};$$

and we shall have the following results:

"Non-elastic metals," will be expressed by $z(1-y)$;
"Elastic substances with non-elastic metals," by $y+z(1-y)$;
"Hard substances, except metals," by $x-z$;
"Metallic substances, except those which are neither hard nor elastic," by $z-z(1-x)(1-y)$, or by $z\{1-(1-x)(1-y)\}$, *vide* (6), Chap. II.

In the last example, what we had really to express was "Metals, except not hard, not elastic, metals." Conjunctions used between *adjectives* are usually superfluous, and, therefore, must not be expressed symbolically.

Thus, "Metals hard and elastic," is equivalent to "Hard elastic metals," and expressed by xyz.

Take next the expression, "Hard substances, except those which are metallic and non-elastic, and those which are elastic and non-metallic." Here the word *those* means hard substances, so that the expression really means, *Hard substances except hard substances, metallic, non-elastic, and hard substances non-metallic, elastic*; the word *except* extending to both the classes which follow it. The complete expression is

$$\begin{aligned} x - \{xz(1-y) &+ xy(1-z)\}; \\ \text{or, } x &- xz(1-y) - xy(1-z). \end{aligned}$$

9. The preceding Proposition, with the different illustrations which have been given of it, is a necessary preliminary to the following one, which will complete the design of the present chapter.

PROPOSITION III.

规则。--用记号 、 、 、&c.表示简单的名称或性质，用 1 – x 、 1 – y 、 1 – z 、&c. 表示它们的反面；用按乘法方式连接相应记号的办法，表示由共同名称或性质所定义的事物的类；用符号 + 连接表示各个部分的表达式，来表示由彼此不同的部分所组成的事物的集合。特别地，让表达式"要么是的，要么是的"在和所表示的类互斥时，用 x(1 – y) + y(1 – x) 表示，在它们不互斥时，用 x + y(1 – x) 表示。同样，让表达式"要么是的，要么是的，要么是的"在 、 、 所表示的类被设定为互斥时，用 x(1 – y)(1 – z) + y(1 – x)(1 – z) + z(1 – x)(1 – y) 表示，在它们不被意指为互斥时，用 x + y(1 – x) + z(1 – x)(1 – y) 表示，依此类推。

8. 解释的逆向规则就建立在这条表达规则之上。这两者都将在以下实例中得到或许足够充分的例证。为简洁起见，省略全称主词"事物"或"存在物"，让我们假定

$$x = \text{hard}, y = \text{elastic}, z = \text{metals};$$

我们将得到以下结果：

"非弹性金属"将用 z(1 – y) 表示；
"带有非弹性金属的弹性物质"，用y + z(1 – y)表示；
"除金属外的硬物质"，作者x – z;
"金属物质，除了那些既不硬也无弹性的"，由 z – z(1 – x)(1 – y) 表达，或由 z { 1 –(1 – x)(1 – y) } 表达，见(6)，第二章。在最后一个例子中，我们真正要表达的是"金属，除了不硬、无弹性的金属。"用在形容词之间的连词通常是多余的，因此，不得用记号表达。

因此，"坚硬且有弹性的金属"等价于"坚硬的弹性金属"，并用xyz表示。

接下来看这个表达式："坚硬物质，除了那些是金属且非弹性的，以及那些是弹性且非金属的。"这里那些一词指的是坚硬物质，因此该表达式实际上意味着，坚硬物质，除了坚硬物质中的金属的、非弹性的，以及坚硬物质中的非金属的、弹性的;"除了"一词延伸到其后的两个类。完整的表达式是

$$\begin{aligned} x - \{xz(1-y) &+ xy(1-z)\}; \\ \text{or, } x &- xz(1-y) - xy(1-z). \end{aligned}$$

9. 前面的命题，连同已经给出的关于它的各种例证，是以下命题的必要预备，后者将完成本章的设计。

命题三。

To deduce from an examination of their possible varieties a general method for the expression of Primary or Concrete Propositions.

A primary proposition, in the most general sense, consists of two terms, between which a relation is asserted to exist. These terms are not necessarily single-worded names, but may represent any collection of objects, such as we have been engaged in considering in the previous sections. The mode of expressing those terms is, therefore, comprehended in the general precepts above given, and it only remains to discover how the relations between the terms are to be expressed. This will evidently depend upon the nature of the relation, and more particularly upon the question whether, in that relation, the terms are understood to be universal or particular, i.e. whether we speak of the whole of that collection of objects to which a term refers, or indefinitely of the whole or of a part of it, the usual signification of the prefix, "some."

Suppose that we wish to express a relation of identity between the two classes, "Fixed Stars" and "Suns," i.e. to express that "All fixed stars are suns," and "All suns are fixed stars." Here, if x stand for fixed stars, and y for suns, we shall have

$$x = y$$

for the equation required.

In the proposition, "All fixed stars are suns," the term "all fixed stars" would be called the *subject*, and " suns" the *predicate*. Suppose that we extend the meaning of the terms *subject* and *predicate* in the following manner. By subject let us mean the first term of any affirmative proposition, i. e. the term which precedes the copula *is* or *are*; and by predicate let us agree to mean the second term, i.e. the one which follows the copula; and let us admit the assumption that either of these may be universal or particular, so that, in either case, the whole class may be implied, or only a part of it. Then we shall have the following Rule for cases such as the one in the last example:–

10. RULE.—*When both Subject and Predicate of a Proposition are universal, form the separate expressions for them, and connect them by the sign =.*

This case will usually present itself in the expression of the definitions of science, or of subjects treated after the manner of pure science. Mr. Senior's definition of wealth affords a good example of this kind, viz.:

"Wealth consists of things transferable, limited in supply, and either productive of pleasure or preventive of pain."

Before proceeding to express this definition symbolically, it must be remarked that the conjunction *and* is superfluous. Wealth is really defined by its possession of three properties or qualities, not by its composition out of three classes or collections of objects. Omitting then the conjunction *and*, let us make

$$\begin{aligned} w &= \text{wealth.} \\ t &= \text{things transferable.} \\ s &= \text{limited in supply.} \\ p &= \text{productive of pleasure.} \\ r &= \text{preventive of pain.} \end{aligned}$$

从对它们可能变体的考察中，推导出一个用于表达初级命题或具体命题的一般方法。

一个主命题，在最一般的意义上，由两个项构成，二者之间的关系被断言存在。这些项不一定是单词名称，而可以代表任何对象的集合，正如我们在前面各节中所一直考虑的那样。因此，表达这些项的方式已包含在上面给出的一般规则之中，现在只须发现项之间的关系应如何表达。这显然将取决于关系的性质，尤其取决于在该关系中，这些项是被理解为全称的还是特称的，即我们所谈论的是一个项所指的那整个对象集合，还是不确定地指其整体或一部分，即前缀"some"的通常含义。

假设我们希望表达"恒星"与"太阳"这两个类之间的同一关系，即表达"所有恒星都是太阳"以及"所有太阳都是恒星"。这里，若以x表示恒星，以y表示太阳，我们便有

$$x = y$$

对于所需的方程。

在命题"All fixed stars are suns"中，项"all fixed stars"将被称为主词，而"suns"被称为谓词。假设我们按以下方式扩展主词和谓词这两个项的含义。所谓主词，我们指任何肯定命题的第一个项，即在系词*is*或*are*之前的项；所谓谓词，我们约定指第二个项，即在系词之后的项；并且我们承认这样的假定：这两者中的任何一个都可以是全称的或特称的，因此在任何一种情况下，都可以隐含整个类，或只隐含它的一部分。于是，对于像上一例那样的情形，我们将有以下规则：--

10.规则。--当一个命题的主词和谓词都是全称时，分别构成它们的表达式，并用=号连接。

这种情况通常出现在表达科学的定义，或按纯粹科学方式处理的主题的定义之时。西尼尔先生关于财富的定义就是这方面的一个好例子，即：

"财富由可转移的、供给有限的、且或者产生快乐或者防止痛苦的事物构成。"

在着手用符号表达这一定义之前，必须指出，连词和是多余的。财富实际上是由其具有三种属性或性质来定义的，而不是由三个类或对象集合组合而成来定义的。那么，略去连词和，让我们令

$$w = \text{wealth.}$$

t = 可转移的事物。

s = 供给有限。

p = 产生快乐。

r = 避免痛苦之物。

Now it is plain from the nature of the subject, that the expression, "Either productive of pleasure or preventive of pain," in the above definition, is meant to be equivalent to "Either productive of pleasure; or, if not productive of pleasure, preventive of pain." Thus the class of things which the above expression, taken alone, would define, would consist of all things productive of pleasure, together with all things not productive of pleasure, but preventive of pain, and its symbolical expression would be

$$p + (1 - p)r.$$

If then we attach to this expression placed in brackets to denote that both its terms are referred to, the symbols s and t limiting its application to things "transferable" and "limited in supply," we obtain the following symbolical equivalent for the original definition, viz.:

$$w = st\{p + r(1 - p)\}. \tag{1}$$

If the expression, "Either productive of pleasure or preventive of pain," were intended to point out merely those things which are productive of pleasure without being preventive of pain, $p(1 - r)$, or preventive of pain, without being productive of pleasure, $r(1 - p)$ (exclusion being made of those things which are both productive of pleasure and preventive of pain), the expression in symbols of the definition would be

$$w = st\{p(1 - r) + r(1 - p)\}. \tag{2}$$

All this agrees with what has before been more generally stated. The reader may be curious to inquire what effect would be produced if we literally translated the expression, "Things productive of pleasure or preventive of pain," by $p + r$, making the symbolical equation of the definition to be

$$w = st(p + r). \tag{3}$$

The answer is, that this expression would be equivalent to (2), with the additional implication that the classes of things denoted by *stp* and *str* are quite distinct, so that of things transferable and limited in supply there exist none in the universe which are at the same time both productive of pleasure and preventive of pain. How the full import of any equation may be determined will be explained hereafter. What has been said may show that before attempting to translate our data into the rigorous language of symbols, it is above all things necessary to ascertain the *intended* import of the words we are using. But this necessity cannot be regarded as an evil by those who value correctness of thought, and regard the right employment of language as both its instrument and its safeguard.

11. Let us consider next the case in which the predicate of the proposition is particular, e.g. "All men are mortal."

In this case it is clear that our meaning is, "All men are some mortal beings," and we must seek the expression of the predicate, "some mortal beings."

现在，从主题的性质可以明白，上述定义中的表达式"Either productive of pleasure or preventive of pain"意在等同于"Either productive of pleasure; or, if not productive of pleasure, preventive of pain"。因此，单就上述表达式本身而言，它所定义的那一类事物将包括一切产生快乐的事物，连同一切不产生快乐但防止痛苦的事物，而其记号表达将是

$$p + (1 - p)r.$$

那么，如果我们给置于括号中的这个表达式附上符号s和t，以表示它的两项都被指涉，并把它的应用限制于"可转移的"和"供给有限的"事物，我们就得到原定义的如下符号等价式，即：

$$w = st\{p + r(1 - p)\}. \tag{1}$$

如果"或者产生快乐或者防止痛苦"这一表达式仅仅意在指出那些产生快乐而不防止痛苦的事物，p(1 – r)，或者防止痛苦而不产生快乐的事物，r(1 – p)(排除那些既产生快乐又防止痛苦的事物)，那么该定义的符号表达式将是

$$w = st\{p(1 - r) + r(1 - p)\}. \tag{2}$$

所有这些都与前面更一般地陈述的内容相符。读者或许会好奇地追问：如果我们逐字地把"产生快乐或防止痛苦的事物"这一表达式译为p + r，使该定义的符号方程为

$$w = st(p + r). \tag{3}$$

答案是，这个表达式将等同于(2)，并附加这样的含义：由stp和str所表示的那些事物类是完全有别的，因此在可转移且供应有限的事物中，宇宙间不存在任何同时既产生快乐又防止痛苦的事物。任何方程的完整含义如何确定，将在以后解释。以上所说可以表明，在试图把我们的材料翻译成严格的记号语言之前，首先必须确定我们所使用的词的*intended*含义。但是，对于那些重视思想正确性、并把语言的正确运用既视为其工具又视为其保障的人来说，这种必要性不能被视为一种坏事。

11. 接下来让我们考虑命题的谓词是特称的情形，例如"所有人都是有死的。"

在这种情况下，显然我们的意思是"所有人都是某些有死的存在物"，并且我们必须寻求谓词"某些有死的存在物"的表达式。

Represent then by v, a class indefinite in every respect but this, viz., that some of its members are mortal beings, and let x stand for "mortal beings," then will vx represent "some mortal beings." Hence if y represent men, the equation sought will be

$$y = vx.$$

From such considerations we derive the following Rule, for expressing an affirmative universal proposition whose predicate is particular:

RULE.—*Express as before the subject and the predicate, attach to the latter the indefinite symbol v, and equate the expressions.*

It is obvious that v is a symbol of the same kind as x, y, &c., and that it is subject to the general law,

$$v^2 = v, or v(1 - v) = 0.$$

Thus, to express the proposition, "The planets are either primary or secondary," we should, according to the rule, proceed thus:

Let x represent planets (the subject);

$$\begin{aligned} y &= \text{primary bodies;} \\ z &= \text{secondary bodies;} \end{aligned}$$

then, assuming the conjunction "or" to separate absolutely the class of "primary" from that of "secondary" bodies, so far as they enter into our consideration in the proposition given, we find for the equation of the proposition

$$x = v\left\{y\left(1 - z\right) + z\left(1 - y\right)\right\}. \tag{4}$$

It may be worth while to notice, that in this case the *literal* translation of the premises into the form

$$x = v(y + z) \tag{5}$$

would be exactly equivalent, v being an indefinite class symbol. The form (4) is, however, the better, as the expression

$$y\left(1 - z\right) + z\left(1 - y\right)$$

consists of terms representing classes quite distinct from each other, and satisfies the fundamental law of duality.

If we take the proposition, "The heavenly bodies are either suns, or planets, or comets," representing these classes of things by w, x, y, z, respectively, its expression, on the supposition that none of the heavenly bodies belong at once to two of the divisions above mentioned, will be

$$w = v\left\{x\left(1 - y\right)\left(1 - z\right) + y\left(1 - x\right)\left(1 - z\right) + z\left(1 - x\right)\left(1 - y\right)\right\}$$

If, however, it were meant to be implied that the heavenly bodies were either suns, or, if not suns, planets, or, if neither, suns nor planets, fixed stars, a meaning which does not exclude the supposition of some of them belonging at

那么，用v表示一个在各方面都不确定、唯有这一点确定的类，即它的某些成员是有死的存在物，并令x表示"有死的存在物"，那么vx将表示"某些有死的存在物"。因此，若y表示人，则所求的方程将是

$$y = vx.$$

由这些考虑，我们得出以下规则，用于表达谓词为特称的肯定全称命题：

规则。--像以前一样表达主词和谓词，给后者附上不定符号，并使两表达式相等。

显然，v是与x、y等等同类的符号，并且它服从一般规律，

$$v^2 = v, or v(1 - v) = 0.$$

因此，要表达命题"行星或者是主星或者是次星"，按照规则，我们应这样进行：

设 x 表示行星(主项);

y = 初级物体；
z = 次级体；

然后，假定连词"或者"把"主星"类与"次星"体类绝对地区分开来，就它们在所给命题中进入我们考虑的范围而言，我们得到该命题的方程

$$x = v\left\{y\left(1 - z\right) + z\left(1 - y\right)\right\}. \tag{4}$$

It may be worth while to notice, that in this case the *literal* translation of the premises into the form

$$x = v(y + z) \tag{5}$$

将完全等价，v是一个不定类符号。形式
(4) 然而，较好的做法是，作为表达式

$$y\left(1 - z\right) + z\left(1 - y\right)$$

由表示彼此截然不同的类的项构成，并满足二重性的基本规律。如果我们取命题"天体或者是太阳，或者是行星，或者是彗星"，分别用w、x、y、z表示这些事物类，那么在假定没有天体同时属于上述两个 division 的情况下，它的表达式将是

$$w = v\left\{x\left(1 - y\right)\left(1 - z\right) + y\left(1 - x\right)\left(1 - z\right) + z\left(1 - x\right)\left(1 - y\right)\right\}$$

然而，如果其意在于暗示天体或者是太阳，或者若不是太阳则是行星，或者若两者都不是，则既不是太阳也不是行星，而是恒星，这一含义并不排除其中某些天体同时属于

once to two or to all three of the divisions of suns, planets, and fixed stars,—the expression required would be

$$w = v\{x + y(1-x) + z(1-x)(1-y)\}. \tag{6}$$

The above examples belong to the class of descriptions, not definitions. Indeed the predicates of propositions are usually particular. When this is not the case, either the predicate is a singular term, or we employ, instead of the copula "is" or "are," some form of connexion, which implies that the predicate is to be taken universally.

12. Consider next the case of universal negative propositions, e.g. "No men are perfect beings."

Now it is manifest that in this case we do not speak of a class termed "no men," and assert of this class that all its members are "perfect beings." But we virtually make an assertion about "*all men*" to the effect that they are "*not perfect beings*." Thus the true meaning of the proposition is this:

"All men (subject) are (copula) not perfect (predicate);" whence, if y represent "men," and x "perfect beings," we shall have

$$y = v(1-x),$$

and similarly in any other case. Thus we have the following Rule:

RULE.—*To express any proposition of the form "No x's are y's," convert it into the form "All x's are not y's," and then proceed as in the previous case.*

13. Consider, lastly, the case in which the subject of the proposition is particular, e.g. "Some men are not wise." Here, as has been remarked, the negative *not* may properly be referred, certainly, at least, for the ends of Logic, to the predicate *wise*; for we do not mean to say that it is not true that "Some men are wise," but we intend to predicate of "some men" a want of wisdom. The requisite form of the given proposition is, therefore, "Some men are not-wise." Putting, then, y for "men," x for "wise," i. e. "wise beings," and introducing v as the symbol of a class indefinite in all respects but this, that it contains some individuals of the class to whose expression it is prefixed, we have

$$vy = v(1-x).$$

14. We may comprise all that we have determined in the following general Rule:

GENERAL RULE FOR THE SYMBOLICAL EXPRESSION OF PRIMARY PROPOSITIONS.

1st. *If the proposition is affirmative, form the expression of the subject and that of the predicate. Should either of them be particular, attach to it the indefinite symbol v, and then equate the resulting expressions.*

2ndly. *If the proposition is negative, express first its true meaning by attaching the negative particle to the predicate, then proceed as above.*

One or two additional examples may suffice for illustration.

太阳、行星和恒星这三个 division 中的两个或全部三个的假设，--那么所需的表达式将是

$$w = v\{x + y(1-x) + z(1-x)(1-y)\}. \tag{6}$$

上述例子属于描述，而非定义。事实上，命题的谓词通常是特称的。当情况并非如此时，要么谓词是一个单独词项，要么我们不用系词"是"或"是"的某种形式，而用某种连接形式，它意味着谓词应作全称理解。

12. 接下来考虑全称否定命题的情形，例如"没有人是完美的存在物。"

现在显而易见，在这种情况下，我们并不是谈论一个称为"没有人"的类，并对这个类断言其所有成员都是"完美的存在物"。而是我们实际上对"所有人"作出断言，大意是他们"不是完美的存在物"。因此，该命题的真正含义是这样的：

"所有人(主词)是(系词)不完美的(谓词)"；因此，若y表示"人",x表示"完美的存在物"，我们便有

$$y = v(1-x),$$

and similarly in any other case. Thus we have the following Rule:

RULE.—*To express any proposition of the form "No x's are y's," convert it into the form "All x's are not y's," and then proceed as in the previous case.*

13. 最后，考虑命题的主词是特称的情形，例如"Some men are not wise"。在这里，正如已经指出的，否定词*not*可以恰当地归到谓词*wise*上，至少就逻辑的目的而言肯定可以如此；因为我们并不是要说"Some men are wise"不成立，而是要对"some men"断言其缺乏智慧。因此，所给命题的必需形式是"Some men are not-wise"。于是，以y表示"men"，以x表示"wise"，即"wise beings"，并引入v作为这样一个类的记号：它在一切方面都不确定，唯独这一点除外，即它包含其表达式前所加的那个类的某些个体，我们便有

$$vy = v(1-x).$$

14. 我们可以将已确定的一切概括为以下一般规则：

主项符号表达式的一般规则

命题。

第一，如果命题是肯定命题，就构成主词的表达式和谓词的表达式。如果其中任一个是特称的，就在它上面附加不定记号v，然后使所得的表达式相等。第二，如果命题是否定命题，就先把否定助词附加于谓词以表达其真正含义，然后按上述步骤进行。

一两个额外的例子或许足以说明问题。

Ex.—"No men are placed in exalted stations, and free from envious regards."

Let y represent "men," x, "placed in exalted stations," z, "free from envious regards."

Now the expression of the class described as "placed in exalted station," and "free from envious regards," is xz. Hence the contrary class, i.e. they to whom this description does not apply, will be represented by $1 - xz$, and to this class all men are referred. Hence we have

$$y = v\,(1 - xz)\,.$$

If the proposition thus expressed had been placed in the equiva- lent form, "Men in exalted stations are not free from envious regards," its expression would have been

$$yx = v\,(1 - z)\,.$$

It will hereafter appear that this expression is really equivalent to the previous one, on the particular hypothesis involved, viz., that v is an indefinite class symbol.

Ex.—"No men are heroes but those who unite self-denial to courage."

Let $x =$ "men," $y =$ "heroes," $z =$ "those who practise self-denial," w, "those who possess courage."

The assertion really is, that "men who do not possess courage and practise self-denial are not heroes."

Hence we have

$$x\,(1 - zw) = v\,(1 - y)$$

for the equation required.

15. In closing this Chapter it may be interesting to compare together the great leading types of propositions symbolically expressed. If we agree to represent by X and Y the symbolical expressions of the "terms," or things related, those types will be

$$\begin{aligned} X &= vY, \\ X &= Y, \\ vX &= vY. \end{aligned}$$

In the first, the predicate only is particular; in the second, both terms are universal; in the third, both are particular. Some minor forms are really included under these. Thus, if $Y = 0$, the second form becomes

$$X = 0;$$

and if $Y = 1$ it becomes

$$X = 1;$$

both which forms admit of interpretation. It is further to be noticed, that the expressions X and Y, if founded upon a sufficiently careful analysis of the

例--"没有人处于显赫地位而又免于嫉妒之心。"令y表示"人",x表示"处于显赫地位",z表示"免于嫉妒
关于。"

现在，被描述为"处于显赫地位"且"免于嫉妒之心"的类的表达式是xz。因此，相反的类，即这一描述不适用于的那些人，将由1 – xz表示，而所有人都被归入这个类。因此我们有

$$y = v\,(1 - xz)\,.$$

如果这样表达的命题曾被置于等价形式"处于显赫地位的人不免于嫉妒之心"，它的表达式本会是

$$yx = v\,(1 - z)\,.$$

后面将会看到，在所涉及的特定假说下，即v是一个不定类符号的情况下，这个表达式实际上等价于前一个表达式。

例--"没有人是英雄，除了那些将克己与勇气结合起来的人。"设 x = "人",y = "英雄",z = "实行克己的人",w,"那些
"谁拥有勇气。"

该断言实际上是："不具备勇气且不践行
"自我否定并非英雄。"

因此我们有

$$x\,(1 - zw) = v\,(1 - y)$$

对于所需的方程。

15. 在结束本章时，将各种主要类型的命题用符号表达出来加以比较，或许是有趣的。如果我们同意用 X 和 Y 来表示"项"或相关事物的符号表达式，那么这些类型将是

$$\begin{aligned} X &= vY, \\ X &= Y, \\ vX &= vY. \end{aligned}$$

在第一种中，只有谓词是特称的；在第二种中，两个项都是全称的；在第三种中，两者都是特称的。一些次要形式实际上包含在这些形式之下。因此，如果 Y = 0，第二种形式就变成

$$X = 0;$$

而若 Y = 1，则变为

$$X = 1;$$

这两种形式都容许解释。还应注意到，表达式 X 和 Y，如果建立在对

meaning of the "terms" of the proposition, will satisfy the fundamental law of duality which requires that we have

$$X^2 = X \quad \text{or} \quad X\,(1-X) = 0,$$
$$Y^2 = Y \quad \text{or} \quad Y\,(1-Y) = 0.$$

命题的"项"的含义进行足够仔细的分析的基础上，就将满足对偶性的基本定律，该定律要求我们有

$$X^2 = X \quad \text{or} \quad X\,(1-X) = 0,$$
$$Y^2 = Y \quad \text{or} \quad Y\,(1-Y) = 0.$$

Chapter V

OF THE FUNDAMENTAL PRINCIPLES OF SYMBOLICAL REASONING, AND OF THE EXPANSION OR DEVELOPMENT OF EXPRESSIONS INVOLVING LOGICAL SYMBOLS.

1. The previous chapters of this work have been devoted to the investigation of the fundamental laws of the operations of the mind in reasoning; of their development in the laws of the symbols of Logic; and of the principles of expression, by which that species of propositions called primary may be represented in the language of symbols. These inquiries have been in the strictest sense preliminary. They form an indispensable introduction to one of the chief objects of this treatise—the construction of a system or method of Logic upon the basis of an exact summary of the fundamental laws of thought. There are certain considerations touching the nature of this end, and the means of its attainment, to which I deem it necessary here to direct attention.

2. I would remark in the first place that the generality of a method in Logic must very much depend upon the generality of its elementary processes and laws. We have, for instance, in the previous sections of this work investigated, among other things, the laws of that logical process of *addition* which is symbolized by the sign $+$. Now those laws have been determined from the study of instances, in all of which it has been a necessary condition, that the classes or things added together in thought should be mutually exclusive. The expression $x + y$ seems indeed uninterpretable, unless it be assumed that the things represented by x and the things represented by y are entirely separate; that they embrace no individuals in common. And conditions analogous to this have been involved in those acts of conception from the study of which the laws of the other symbolical operations have been ascertained. The question then arises, whether it is necessary to restrict the application of these symbolical laws and processes by the same conditions of interpretability under which the knowledge of them was obtained. If such restriction is necessary, it is manifest that no such thing as a general method in Logic is possible. On the other hand, if such restriction is unnecessary, in what light are we to contemplate processes which appear to be uninterpretable in that sphere of thought which they are designed to aid? These questions do not belong to the science of Logic alone. They are equally

第五章

论符号推理的基本原理，以及

涉及逻辑符号的表达式的展开或发展。

1. 本著作的前几章致力于研究心灵活动在推理中的基本规律；这些规律在逻辑符号的规律中的展开式；以及表达的原则，通过该原则，那种被称为特称的命题可以在符号的语言中被表示出来。这些探究在最严格的意义上是预备性的。它们构成了本论著的主要目标之一--在精确总结思维基本规律的基础上构建一个逻辑的系统或方法--不可或缺的导论。关于这一目标的性质以及实现它的手段，有一些考虑我认为有必要在此提请注意。

2. 我首先要指出，逻辑中一个方法的一般性必然在很大程度上取决于其基本过程和规律的一般性。例如，在本著作前面的各节中，我们研究了以符号+为标记的那种逻辑的加法过程的规律，以及其他事项。现在，那些规律是从对实例的研究中确定下来的，在所有这些实例中，一个必要的条件是，在思想中加在一起的类或事物应当是互斥的。表达式 x + y 似乎确实不可解释，除非假定由 x 所表示的事物和由 y 所表示的事物是完全分离的；它们不包含任何共同的个体。而与此类似的条件也涉及那些概念活动，通过对这些活动的研究，其他符号运算的规律得以确定。于是问题产生了：是否有必要将这些符号规律和过程的应用限制在获得它们知识时所依据的同样的可解释性条件之下。如果这种限制是必要的，那么显然，逻辑中一般方法这样的东西就是不可能的。另一方面，如果这种限制是不必要的，那么我们应当以何种眼光来看待那些在它们被设计来辅助的思想领域中似乎不可解释的过程呢?这些问题不仅仅属于逻辑科学。它们同样

pertinent to every developed form of human reasoning which is based upon the employment of a symbolical language.

3. I would observe in the second place, that this apparent failure of correspondency between process and interpretation does not manifest itself in the *ordinary* applications of human reason. For no operations are there performed of which the meaning and the application are not seen; and to most minds it does not suffice that merely formal reasoning should connect their premises and their conclusions; but every step of the connecting train, every mediate result which is established in the course of demonstration, must be intelligible also. And without doubt, this is both an actual condition and an important safeguard, in the reasonings and discourses of common life.

There are perhaps many who would be disposed to extend the same principle to the general use of symbolical language as an instrument of reasoning. It might be argued, that as the laws or axioms which govern the use of symbols are established upon an investigation of those cases only in which interpretation is possible, we have no right to extend their application to other cases in which interpretation is impossible or doubtful, even though (as should be admitted) such application is employed in the intermediate steps of demonstration only. Were this objection conclusive, it must be acknowledged that slight advantage would accrue from the use of a symbolical method in Logic. Perhaps that advantage would be confined to the mechanical gain of employing short and convenient symbols in the place of more cumbrous ones. But the objection itself is fallacious. Whatever our *à priori* anticipations might be, it is an unquestionable fact that the validity of a conclusion arrived at by any symbolical process of reasoning, does not depend upon our ability to interpret the formal results which have presented themselves in the different stages of the investigation. There exist, in fact, certain general principles relating to the use of symbolical methods, which, as pertaining to the particular subject of Logic, I shall first state, and I shall then offer some remarks upon the nature and upon the grounds of their claim to acceptance.

4. The conditions of valid reasoning, by the aid of symbols, are—

1st, That a fixed interpretation be assigned to the symbols employed in the expression of the data; and that the laws of the combination of those symbols be correctly determined from that interpretation.

2nd, That the formal processes of solution or demonstration be conducted throughout in obedience to all the laws determined as above, without regard to the question of the interpretability of the particular results obtained.

3rd, That the final result be interpretable in form, and that it be actually interpreted in accordance with that system of interpretation which has been employed in the expression of the data. Concerning these principles, the following observations may be made.

5. The necessity of a fixed interpretation of the symbols has already been sufficiently dwelt upon (II. 3). The necessity that the fixed result should be in such a form as to admit of that interpretation being applied, is founded on the obvious principle, that the use of symbols is a means towards an end, that end being the knowledge of some intelligible fact or truth. And that this end may

与每一种基于符号语言之运用而发展起来的人类推理形式相关。

3. 我其次要指出，这种过程与解释之间表面上的对应关系的缺失，在人类理性的普通应用中并不会显现出来。因为在那些应用中，没有哪种运算的意义和应用是未被看到的；而对于大多数心灵来说，仅仅形式推理连接其前提和结论是不够的；连接序列的每一步，在论证过程中所建立的每一个中介结果，也必须是可以理解的。毫无疑问，在日常生活和言谈的推理中，这既是一个实际条件，也是一项重要的保障。

也许有许多人倾向于将同样的原则扩展到符号语言作为推理工具的一般使用上。可以论证说，既然支配符号使用的规律或公理是仅通过对那些解释成为可能的实例的研究而建立的，我们就无权将它们应用于解释不可能或可疑的其他实例，即使(应当承认)这种应用仅用于论证的中间步骤。如果这一反对意见是决定性的，那就必须承认，在逻辑中使用符号方法所能获得的益处微乎其微。也许那种益处将仅限于以简短方便的符号代替更为笨重的符号这种机械性的收获。但这一反对意见本身是谬误的。无论我们的`*a priori*`预期可能是什么，一个不容置疑的事实是：通过任何符号推理过程所得出的结论的有效性，并不取决于我们解释在探究的不同阶段所呈现的形式结果的能力。事实上，存在某些关于符号方法使用的一般原则，它们作为与逻辑这一特定主题相关的内容，我将首先陈述，然后对其性质和其被接受的理由作一些评论。

4. 借助符号，有效推理的条件是--

第一，对于在表达已知数据时所使用的记号，必须赋予固定的解释；并且必须根据该解释正确地确定这些记号组合的规律。第二，求解或证明的形式过程必须自始至终遵循如上确定的一切规律，而不考虑所获得的特定结果是否可解释的问题。第三，最终结果必须在形式上可解释，并且必须按照在表达已知数据时所采用的那套解释体系来实际地加以解释。关于这些原则，可以作出以下观察。

5. 对记号作固定解释的必要性已经充分论述过了(II.3)。固定结果必须采取能够应用那种解释的形式，这一必要性建立在一个显而易见的原则之上：记号的使用是达到目的的手段，而这个目的就是认识某种可理解的事实或真理。并且，为了使这个目的

be attained, the final result which expresses the symbolical conclusion must be in an interpretable form. It is, however, in connexion with the second of the above general principles or conditions (V. 4), that the greatest difficulty is likely to be felt, and upon this point a few additional words are necessary.

I would then remark, that the principle in question may be considered as resting upon a general law of the mind, the knowledge of which is not given to us *à priori*, i.e. antecedently to experience, but is derived, like the knowledge of the other laws of the mind, from the clear manifestation of the general principle in the particular instance. A single example of reasoning, in which symbols are employed in obedience to laws founded upon their interpretation, but without any sustained reference to that interpretation, the chain of demonstration conducting us through intermediate steps which are not interpretable, to a final result which is interpretable, seems not only to establish the validity of the particular application, but to make known to us the general law manifested therein. No accumulation of instances can properly add weight to such evidence. It may furnish us with clearer conceptions of that common element of truth upon which the application of the principle depends, and so prepare the way for its reception. It may, where the immediate force of the evidence is not felt, serve as a verification, *à posteriori*, of the practical validity of the principle in question. But this does not affect the position affirmed, viz., that the general principle must be seen in the particular instance,—seen to be general in application as well as true in the special example. The employment of the uninterpretable symbol $\sqrt{-1}$, in the intermediate processes of trigonometry, furnishes an illustration of what has been said. I apprehend that there is no mode of explaining that application which does not covertly assume the very principle in question. But that principle, though not, as I conceive, warranted by formal reasoning based upon other grounds, seems to deserve a place among those axiomatic truths which constitute, in some sense, the foundation of the possibility of general knowledge, and which may properly be regarded as expressions of the mind's own laws and constitution.

6. The following is the mode in which the principle above stated will be applied in the present work. It has been seen, that any system of propositions may be expressed by equations involving symbols x, y, z, which, whenever interpretation is possible, are subject to laws identical in form with the laws of a system of quantitative symbols, susceptible only of the values 0 and 1 (II. 15). But as the formal processes of reasoning depend only upon the laws of the symbols, and not upon the nature of their interpretation, we are permitted to treat the above symbols, x, y, z, as if they were quantitative symbols of the kind above described. *We may in fact lay aside the logical interpretation of the symbols in the given equation; convert them into quantitative symbols, susceptible only of the values* 0 *and* 1*; perform upon them as such all the requisite processes of solution; and finally restore to them their logical interpretation.* And this is the mode of procedure which will actually be adopted, though it will be deemed unnecessary to restate in every instance the nature of the transformation employed. The processes to which the symbols x, y, z, regarded as quantitative and of the species above described, are subject, are not limited

得以达到，表达符号结论的最终结果必须采取可解释的形式。然而，很可能正是在与上述一般原则或条件中的第二条(V.4)相关的地方，会感到最大的困难，关于这一点还需要补充几句话。

那么我要指出，所讨论的这个原理可以被视为建立在心灵的一条普遍规律之上，关于这条规律的知识并不是`*a priori*地、即在经验之先给予我们的，而是像关于心灵其他规律的知识一样，从普遍原理在特殊实例中的清晰显现中得来的。推理的一个单一例子，其中记号依照建立在其解释之上的规律而被使用，但并未持续参照那种解释，论证的链条引导我们经过不可解释的中间步骤而达到一个可解释的最终结果，这似乎不仅确立了该特殊应用的有效性，而且使我们认识到其中所显现的普遍规律。任何实例的积累都不能恰当地为这样的证据增添分量。它可以为我们提供关于那条真理的共同元素的更清晰概念，而原理的应用正依赖于那条真理，从而为其接受铺平道路。在证据的直接力量未被感受到的地方，它可以作为对所述原理实践有效性的`*a posteriori*验证。但这并不影响所肯定的立场，即：普遍原理必须在特殊实例中被看到--被看到在其应用中是一般的，正如在特殊例子中是真的。在三角学的中间过程中使用不可解释的记号$\sqrt{-1}$，为上述说法提供了一个例证。我认为，没有任何解释那种应用的方式不暗中假定所讨论的这个原理本身。但是，这个原理，虽然在我看来并非由基于其他根据的形式推理所保证，却似乎值得在那些公理真理中占有一席之地，这些公理真理在某种意义上构成了普遍知识可能性的基础，并且可以恰当地被视为心灵自身规律和构造的表达。

6. 以下是上述原理在本书中将如何被应用的方式。已经看到，任何命题系统都可以通过涉及记号x、y、z的方程来表达，这些记号只要解释是可能的，就服从与一个量记号系统的规律在形式上相同的规律，只可取0和1两个值(II.15)。但由于推理的形式过程只依赖于记号的规律，而不依赖于其解释的性质，我们被允许把上述记号x、y、z当作上述那种量记号来处理。我们事实上可以搁置给定方程中记号的逻辑解释；把它们转换为量记号，只可取 0 和 1 ；对它们作为这样的记号执行一切必要的求解过程；最后恢复它们的逻辑解释。而这正是实际将采用的程序方式，尽管在每一实例中重述所采用的变换的性质将被视为不必要。记号x、y、z作为上述种类的量记号所服从的过程，并不限于

by those conditions of thought to which they would, if performed upon purely logical symbols, be subject, and a freedom of operation is given to us in the use of them, without which, the inquiry after a general method in Logic would be a hopeless quest.

Now the above system of processes would conduct us to no intelligible result, unless the final equations resulting therefrom were in a form which should render their interpretation, after restoring to the symbols their logical significance, possible. There exists, however, a general method of reducing equations to such a form, and the remainder of this chapter will be devoted to its consideration. I shall say little concerning the way in which the method renders interpretation possible,—this point being reserved for the next chapter,—but shall chiefly confine myself here to the mere process employed, which may be characterized as a process of "development." As introductory to the nature of this process, it may be proper first to make a few observations.

7. Suppose that we are considering any class of things with reference to this question, viz., the relation in which its members stand as to the possession or the want of a certain property x. As every individual in the proposed class either possesses or does not possess the property in question, we may divide the class into two portions, the former consisting of those individuals which possess, the latter of those which do not possess, the property. This possibility of dividing in thought the whole class into two constituent portions, is antecedent to all knowledge of the constitution of the class derived from any other source; of which knowledge the effect can only be to inform us, more or less precisely, to what further conditions the portions of the class which possess and which do not possess the given property are subject. Suppose, then, such knowledge is to the following effect, viz., that the members of that portion which possess the property x, possess also a certain property u, and that these conditions united are a sufficient definition of them. We may then represent that portion of the original class by the expression ux (II. 6). If, further, we obtain information that the members of the original class which do not possess the property x, are subject to a condition v, and are thus defined, it is clear, that those members will be represented by the expression $v\,(1-x)$. Hence the class in its totality will be represented by

$$ux + v\,(1-x)\,;$$

which may be considered as a general developed form for the expression of any class of objects considered with reference to the possession or the want of a given property x.

The general form thus established upon purely logical grounds may also be deduced from distinct considerations of formal law, applicable to the symbols x, y, z, equally in their logical and in their quantitative interpretation already referred to (V. 6).

8. *Definition.*—Any algebraic expression involving a symbol x is termed a function of x, and may be represented under the abbreviated general form $f\,(x)$. Any expression involving two symbols, x and y, is similarly termed a function of x and y, and may be represented under the general form $f\,(x, y)$, and so on

受那些思维条件的约束--如果这些运算施于纯粹逻辑记号之上，它们本应服从这些条件--并且我们在使用它们时获得了操作的自由，没有这种自由，对逻辑中一般方法的探求就会成为无望的追寻。

现在，上述过程系统将不会引导我们达到任何可理解的结果，除非由此产生的最终方程具有一种形式，使得在把逻辑意义恢复给记号之后，对它们的解释成为可能。然而，存在一种把方程化为这种形式的一般方法，本章的剩余部分将致力于对它的考察。关于这种方法如何使解释成为可能，我将很少谈及--这一点留待下一章--而在此将主要限于所采用的单纯过程，这个过程可以被称为"展开式"过程。作为对这一过程性质的引导，或许宜先作几点观察。

7. 假定我们正就以下问题考察任何一类事物，即：其成员在具有或不具有某种性质x方面的关系。由于所提出的类中的每一个体或者具有或者不具有所讨论的性质，我们可以把该类分为两部分，前者由具有该性质的个体组成，后者由不具有该性质的个体组成。这种在思想中把整个类分为两个组成部分的可能性，先于从任何其他来源获得的关于该类构造的一切知识；那种知识的效果只能是或多或少精确地告知我们，具有和不具有给定性质的类的各部分还服从哪些进一步的条件。那么，假定这种知识如下，即：具有性质x的那部分的成员，也具有某种性质u，并且这些联合的条件是它们的一个充分定义。那么我们便可以用表达式ux(II.6)表示原类的那个部分。如果进一步我们获得信息说，原类中不具有性质x的成员服从一个条件v，并由此得到定义，那么很清楚，那些成员将由表达式v(1 – x)表示。因此该类在总体上将由

$$ux + v\,(1-x)\,;$$

这可以被视为表达任何一类对象的一般展开形式，这些对象是就具有或缺乏某一给定性质 x 而言被考虑的。

这样纯粹基于逻辑理由建立起来的一般形式，也可以从形式定律的不同考虑中推导出来，这些考虑同样适用于记号 x、y、z，既适用于已经提到的它们的逻辑解释，也适用于它们的量的解释(V.6)。

8. 定义。--任何含有记号 x 的代数表达式都称为 x 的函数，并可以用缩写的一般形式 f(x) 来表示。任何含有两个记号 x 和 y 的表达式同样称为 x 和 y 的函数，并可以用一般形式 f(x, y) 来表示，依此类推

for any other case.

Thus the form $f(x)$ would indifferently represent any of the following functions, viz., x, $1-x$, $\frac{1+x}{1-x}$, &c.; and $f(x,y)$ would equally represent any of the forms $x+y$, $x-2y$, $\frac{x+y}{x-2y}$, &c.

On the same principles of notation, if in any function $f(x)$ we change x into 1, the result will be expressed by the form $f(1)$; if in the same function we change x into 0, the result will be expressed by the form $f(0)$. Thus, if $f(x)$ represent the function $\frac{a+x}{a-2x}$, $f(1)$ will represent $\frac{a+1}{a-2}$, and $f(0)$ will represent $\frac{a}{a}$.

9. *Definition.*—Any function $f(x)$, in which x is a logical symbol, or a symbol of quantity susceptible only of the values 0 and 1, is said to be developed, when it is reduced to the form $ax+b(1-x)$, a and b being so determined as to make the result equivalent to the function from which it was derived.

This definition assumes, that it is possible to represent any function $f(x)$ in the form supposed. The assumption is vindicated in the following Proposition.

PROPOSITION I.

10. *To develop any function $f(x)$ in which x is a logical symbol.*

By the principle which has been asserted in this chapter, it is lawful to treat x as a quantitative symbol, susceptible only of the values 0 and 1.

Assume then,

$$f(x) = ax + b(1-x),$$

and making $x=1$, we have

$$f(1) = a.$$

Again, in the same equation making $x=0$, we have

$$f(0) = b.$$

Hence the values of a and b are determined, and substituting them in the first equation, we have

$$f(x) = f(1)\,x + f(0)\,(1-x); \tag{1}$$

as the development sought.[1] The second member of the equation adequately represents the function $f(x)$, whatever the form of that function may be. For

[1]To some it may be interesting to remark, that the development of $f(x)$ obtained in this chapter, strictly holds, in the logical system, the place of the expansion of $f(x)$ in ascending powers of x in the system of ordinary algebra. Thus it may be obtained by introducing into the expression of Taylor's well-known theorem, viz.:

$$f(x) = f(0) + f'(0)\,x + f''(0)\,\frac{x^2}{1\cdot 2} + f'''(0)\,\frac{x^3}{1\cdot 2\cdot 3},\ \text{&c.} \tag{1}$$

the condition $x(1-x)=0$, whence we find $x^2=x$, $x^3=x$, &c., and

$$f(x) = f(0) + \left\{f'(0) + \frac{f''(0)}{1\cdot 2} + \frac{f'''(0)}{1\cdot 2\cdot 3} + \text{&c.}\right\}x. \tag{2}$$

But making in (1), $x=1$, we get

$$f(1) = f(0) + f'(0) + \frac{f''(0)}{1\cdot 2} + \frac{f'''(0)}{1\cdot 2\cdot 3} + \text{&c.}$$

for any other case.

Thus the form $f(x)$ would indifferently represent any of the following functions, viz., x, $1-x$, $\frac{1+x}{1-x}$, &c.; and $f(x,y)$ would equally represent any of the forms $x+y$, $x-2y$, $\frac{x+y}{x-2y}$, &c.

按照同样的记号原则，如果在任何函数 f(x) 中我们把 x 变成 1，结果将用形式 f(1) 来表示；如果在同一函数中我们把 x 变成 0，结果将用形式 f(0) 来表示。因此，如果 f(x) 表示函数 a + x

$\frac{a+x}{a-2x}$, $f(1)$ will represent $\frac{a+1}{a-2}$, and $f(0)$ will represent $\frac{a}{a}$.

9. 定义。--任何函数 f(x)，其中 x 是一个逻辑记号，或一个只能取 0 和 1 这两个值的量的记号，当它被化为形式 ax + b(1 – x) 时，就说它被展开了，其中 a 和 b 被确定为使结果等价于它所由导出的那个函数。

这个定义假定，有可能把任何函数 f(x) 表示为所设的形式。这一假定在下面的命题中得到证明。

命题一。

10. *To develop any function $f(x)$ in which x is a logical symbol.*

By the principle which has been asserted in this chapter, it is lawful to treat x as a quantitative symbol, susceptible only of the values 0 and 1.

Assume then,

$$f(x) = ax + b(1-x),$$

and making $x=1$, we have

$$f(1) = a.$$

同样，在令x = 0 的同一方程中，我们有

$$f(0) = b.$$

Hence the values of a and b are determined, and substituting them in the first equation, we have

$$f(x) = f(1)\,x + f(0)\,(1-x); \tag{1}$$

as the development sought.[1] The second member of the equation adequately represents the function $f(x)$, whatever the form of that function may be. For

1 对某些人来说，指出以下一点或许是有趣的：本章所得到的 f(x) 的展开式，在逻辑体系中严格占据着 f(x) 按 x 的升幂展开在普通代数体系中所占的位置。因此，它可以借助泰勒著名定理的表达式引入而得到，即：

$$f(x) = f(0) + f'(0)\,x + f''(0)\,\frac{x^2}{1\cdot 2} + f'''(0)\,\frac{x^3}{1\cdot 2\cdot 3},\ \text{&c.} \tag{1}$$

the condition $x(1-x)=0$, whence we find $x^2=x$, $x^3=x$, &c., and

$$f(x) = f(0) + \left\{f'(0) + \frac{f''(0)}{1\cdot 2} + \frac{f'''(0)}{1\cdot 2\cdot 3} + \text{&c.}\right\}x. \tag{2}$$

But making in (1), $x=1$, we get

$$f(1) = f(0) + f'(0) + \frac{f''(0)}{1\cdot 2} + \frac{f'''(0)}{1\cdot 2\cdot 3} + \text{&c.}$$

x regarded as a quantitative symbol admits only of the values 0 and 1, and for each of these values the development

$$f(1)\,x + f(0)\,(1-x)\,,$$

assumes the same value as the function $f(x)$.

As an illustration, let it be required to develop the function $\frac{1+x}{1+2x}$. Here, when $x=1$, we find $f(1)=\frac{2}{3}$, and when $x=0$, we find $f(0)=\frac{1}{1}$, or 1. Hence the expression required is

$$\frac{1+x}{1+2x} = \frac{2}{3}x + 1 - x\;;$$

and this equation is satisfied for each of the values of which the symbol x is susceptible.

Proposition II.

To expand or develop a function involving any number of logical symbols.

Let us begin with the case in which there are two symbols, x and y, and let us represent the function to be developed by $f(x,y)$.

First, considering $f(x,y)$ as a function of x alone, and expanding it by the general theorem (1), we have

$$f(x,y) = f(1,y)\,x + f(0,y)\,(1-x)\,; \tag{2}$$

wherein $f(1,y)$ represents what the proposed function becomes, when in it for x; we write 1, and $f(0,y)$ what the said function becomes, when in it for x we write 0.

Now, taking the coefficient $f(1,y)$, and regarding it as a function of y, and expanding it accordingly, we have

$$f(1,y) = f(1,1)\,y + f(1,0)\,(1-y)\,, \tag{3}$$

wherein $f(1,1)$ represents what $f(1,y)$ becomes when y is made equal to 1, and $f(1,0)$ what $f(1,y)$ becomes when y is made equal to 0.

In like manner, the coefficient $f(0,y)$ gives by expansion,

$$f(0,y) = f(0,1)\,y + f(0,0)\,(1-y)\,. \tag{4}$$

---

whence

$$f'(0) + \frac{f''(0)}{1\cdot 2} + \text{ \&c.} = f(1) - f(0),$$

and (2) becomes, on substitution,

$$\begin{aligned} f(x) &= f(0) + \{f(1) - f(0)\}x, \\ &= f(1)x + f(0)(1-x), \end{aligned}$$

the form in question. This demonstration in supposing $f(x)$ to be developable in a series of ascending powers of x is less general than the one in the text.

x 作为量的记号只容许取 0 和 1 这两个值，而对于这两个值中的每一个，展开式

$$f(1)\,x + f(0)\,(1-x)\,,$$

与函数f(x)取相同的值。

作为例证，设需展开函数$^{1+}$ x $^{1+2}$ x。此处，当 x = 1 时，我们求得 f(1) = 23,而当x=0 时，我们求得f(0)=11,或1。因此所求的表达式为

$$\frac{1+x}{1+2x} = \frac{2}{3}x + 1 - x\;;$$

并且这个方程对于记号 x 所能取的每一个值都得到满足。

命题二。

展开或发展一个含有任意数目逻辑记号的函数。让我们从有两个记号 x 和 y 的情形开始，并让我们用 f(x, y) 表示要展开的函数。

首先，把 f(x, y) 看作仅 x 的函数，并用一般定理(1)展开它，我们有

$$f(x,y) = f(1,y)\,x + f(0,y)\,(1-x)\,; \tag{2}$$

其中 f(1 , y) 表示所设函数在把其中的 x 写作 1 时变成什么，而 f(0 , y) 表示上述函数在把其中的 x 写作 0 时变成什么。

$$f(1,y) = f(1,1)\,y + f(1,0)\,(1-y)\,, \tag{3}$$

其中 f(1 , 1) 表示 f(1 , y) 在使 y 等于 1 时变成什么，而 f(1 , 0) 表示 f(1 , y) 在使 y 等于 0 时变成什么。

同样地，系数 f(0 , y) 通过展开给出，

$$f(0,y) = f(0,1)\,y + f(0,0)\,(1-y)\,. \tag{4}$$

---

whence

$$f'(0) + \frac{f''(0)}{1\cdot 2} + \text{ \&c.} = f(1) - f(0),$$

and (2) becomes, on substitution,

$$\begin{aligned} f(x) &= f(0) + \{f(1) - f(0)\}x, \\ &= f(1)x + f(0)(1-x), \end{aligned}$$

the form in question. This demonstration in supposing $f(x)$ to be developable in a series of ascending powers of x is less general than the one in the text.

Substitute in (2) for $f(1,y)$, $f(0,y)$, their values given in (3) and (4), and we have

$$f(x,y) = f(1,1)\,xy + f(1,0)\,x(1-y) + f(0,1)\,(1-x)\,y \quad (5)$$
$$+ f(0,0)\,(1-x)\,(1-y)\,, \quad (6)$$

for the expansion required. Here $f(1,1)$ represents what $f(x,y)$ becomes when we make therein $x=1$, $y=1$; $f(1,0)$ represents what $f(x,y)$ becomes when we make therein $x=1$, $y=0$, and so on for the rest.

Thus, if $f(x,y)$ represent the function $\frac{1-x}{1-y}$, we find

$$f(1,1) = \frac{0}{0}, \qquad f(1,0) = \frac{0}{1}, \qquad f(0,1) = \frac{1}{0}, \qquad f(0,0) = 1$$

whence the expansion of the given function is

$$\frac{0}{0}xy + 0x(1-y) + \frac{1}{0}(1-x)\,y + (1-x)(1-y)\,.$$

It will in the next chapter be seen that the forms $\frac{0}{0}$ and $\frac{1}{0}$, the former of which is known to mathematicians as the symbol of indeterminate quantity, admit, in such expressions as the above, of a very important logical interpretation.

Suppose, in the next place, that we have three symbols in the function to be expanded, which we may represent under the general form $f(x,y,z)$. Proceeding as before, we get

$$\begin{aligned} f(x,y,z) &= f(1,1,1)\,xyz + f(1,1,0)\,xy(1-z) + f(1,0,1)\,x(1-y)\,z \\ &+ f(1,0,0)\,x(1-y)(1-z) + f(0,1,1)\,(1-x)\,yz \\ &+ f(0,1,0)\,(1-x)\,y(1-z) + f(0,0,1)\,(1-x)(1-y)\,z \\ &+ f(0,0,0)\,(1-x)(1-y)(1-z) \end{aligned}$$

in which $f(1,1,1)$ represents what the function $f(x,y,z)$ becomes when we make therein $x=1$, $y=1$, $z=1$, and so on for the rest.

11. It is now easy to see the general law which determines the expansion of any proposed function, and to reduce the method of effecting the expansion to a rule. But before proceeding to the expression of such a rule, it will be convenient to premise the following observations:—

Each form of expansion that we have obtained consists of certain terms, into which the symbols x, y, &c. enter, multiplied by coefficients, into which those symbols do not enter. Thus the expansion of $f(x)$ consists of two terms, x and $1-x$, multiplied by the coefficients $f(1)$ and $f(0)$ respectively. And the expansion of $f(x,y)$ consists of the four terms xy, $x(1-y)$, $(1-x)\,y$, and $(1-x)$, $(1-y)$, multiplied by the coefficients $f(1,1)$, $f(1,0)$, $f(0,1)$, $f(0,0)$, respectively. The terms x, $1-x$, in the former case, and the terms xy, $x(1-y)$, &c., in the latter, we shall call the *constituents* of the expansion. It is evident that they are in form independent of the form of the function to be expanded. Of the constituent xy, x and y are termed the *factors*.

把(3)和(4)中给出的 f(1 , y)、f(0 , y) 的值代入(2)，我们便得到

$$f(x,y) = f(1,1)\,xy + f(1,0)\,x(1-y) + f(0,1)\,(1-x)\,y \quad (5)$$
$$+ f(0,0)\,(1-x)\,(1-y)\,, \quad (6)$$

这就是所需的展开式。这里 f(1 , 1) 表示 f(x, y) 在我们在其中令 x = 1、y = 1 时变成什么；f(1 , 0) 表示 f(x, y) 在我们在其中令 x = 1、y = 0 时变成什么，其余依此类推。

Thus, if $f(x,y)$ represent the function $\frac{1-x}{1-y}$, we find

$$f(1,1) = \frac{0}{0}, \qquad f(1,0) = \frac{0}{1}, \qquad f(0,1) = \frac{1}{0}, \qquad f(0,0) = 1$$

由此得到给定函数的展开式

$$\frac{0}{0}xy + 0x(1-y) + \frac{1}{0}(1-x)\,y + (1-x)(1-y)\,.$$

在下一章中将可看到，形式00和10,其中前者被数学家称为不定量的记号，在像上面这样的表达式中容许一种非常重要的逻辑解释。

其次，假定要展开的函数中有三个记号，我们可以用一般形式 f(x, y, z) 来表示它。像以前一样进行，我们得到

$$\begin{aligned} f(x,y,z) &= f(1,1,1)\,xyz + f(1,1,0)\,xy(1-z) + f(1,0,1)\,x(1-y)\,z \\ &+ f(1,0,0)\,x(1-y)(1-z) + f(0,1,1)\,(1-x)\,yz \\ &+ f(0,1,0)\,(1-x)\,y(1-z) + f(0,0,1)\,(1-x)(1-y)\,z \\ &+ f(0,0,0)\,(1-x)(1-y)(1-z) \end{aligned}$$

其中 f(1 , 1 , 1) 表示函数 f(x, y, z) 在我们在其中令 x = 1、y = 1、z = 1 时变成什么，其余依此类推。

11. 现在很容易看出决定任何所设函数之展开的一般定律，并把实现展开的方法归结为一条规则。但在着手表述这样一条规则之前，先提出以下观察将是方便的：--

我们所得到的每一种展开式都由某些项组成，记号 x、y 等进入这些项，并乘以系数，而那些记号不进入这些系数。因此，f(x) 的展开式由两项组成，即 x 和 1 – x，分别乘以系数 f(1) 和 f(0)。而 f(x, y) 的展开式由四项组成，即 xy、x(1 – y)、(1 – x) y 和 (1 – x)、(1 – y)，分别乘以系数 f(1 , 1)、f(1 , 0)、f(0 , 1)、f(0 , 0)。在前一种情形中的项 x、1 – x，以及在后一种情形中的项 xy、x(1 – y) 等，我们将称之为展开式的构成成分。显然，它们在形式上独立于所要展开的函数的形式。在构成成分 xy 中，x 和 y 被称为因子。

The general rule of development will therefore consist of two parts, the first of which will relate to the formation of the *constituents* of the expansion, the second to the determination of their respective coefficients. It is as follows:

1st. *To expand any function of the symbols x, y, z.*—Form a series of constituents in the following manner: Let the first constituent be the product of the symbols; change in this product any symbol z into $1-z$, for the second constituent. Then in both these change any other symbol y into $1-y$, for two more constituents. Then in the four constituents thus obtained change any other symbol x into $1-x$, for four new constituents, and so on until the number of possible changes is exhausted.

2ndly. *To find the coefficient of any constituent.*—If that constituent involves x as a factor, change in the original function x into 1; but if it involves $1-x$ as a factor, change in the original function x into 0. Apply the same rule with reference to the symbols y, z, &c.: the final calculated value of the function thus transformed will be the coefficient sought.

The sum of the constituents, multiplied each by its respective coefficient, will be the expansion required.

12. It is worthy of observation, that a function may be developed with reference to symbols which it does not explicitly contain. Thus if, proceeding according to the rule, we seek to develop the function $1-x$, with reference to the symbols x and y, we have,

When	x = 1	and	y = 1	the given function	= 0.
	x = 1	”	y = 0	” ”	= 0.
	x = 0	”	y = 1	” ”	= 1.
	x = 0	”	y = 0	” ”	= 1.

Whence the development is

$$1-x=0xy+0x\,(1-y)+(1-x)\,y+(1-x)\,(1-y)\,;$$

and this is a true development. The addition of the terms $(1-x)\,y$ and $(1-x)\,(1-y)$ produces the function $1-x$.

The symbol 1 thus developed according to the rule, with respect to the symbol x, gives

$$x+1-x.$$

Developed with respect to x and y, it gives

$$xy+x\,(1-y)+(1-x)\,y+(1-x)\,(1-y)\,.$$

Similarly developed with respect to any set of symbols, it produces a series consisting of all possible constituents of those symbols.

13. A few additional remarks concerning the nature of the general expansions may with propriety be added. Let us take, for illustration, the general theorem (5), which presents the type of development for functions of two logical symbols.

In the first place, that theorem is perfectly true and intelligible when x and y are quantitative symbols of the species considered in this chapter, whatever algebraic form may be assigned to the function $f\,(x,y)$, and it may therefore be

因此，展开的一般规则将由两部分组成，第一部分涉及展开的成分的形成，第二部分涉及它们各自系数的确定。规则如下：

第一。展开记号 、 、 的任一函数 z。- 按以下方式形成一系列构成成分：令第一个构成成分为这些记号的乘积；在这个乘积中，将任一记号 z 变为 1 – z，作为第二个构成成分。然后在这两者中，将任何另一个记号 y 变为 1 – y，作为另外两个构成成分。然后在由此得到的四个构成成分中，将任何另一个记号 x 变为 1 – x，作为四个新的构成成分，如此继续，直到可能的变化数目用尽为止。

其次。求任一成分的系数。--若该成分含有x作为因子，则将原函数中的x变为1；但若该成分含有1 – x作为因子，则将原函数中的x变为0。对记号y、z等应用同一规则：如此变换后的函数最终计算所得的值即为所求的系数。

各成分分别乘以其相应系数后的总和，即为所求的展开式。

12.值得注意的是，一个函数可以相对于它并未明确含有的记号而展开。因此，若按照规则，我们试图将函数1 – x相对于记号x和y展开，我们便得到，

当	x = 1	且	y = 1	给定的函数	= 0.
	x = 1	?	y = 0	? ?	= 0.
	x = 0	?	y = 1	? ?	= 1.
	x = 0	?	y = 0	? ?	= 1.

由此发展而来

$$1-x=0xy+0x\,(1-y)+(1-x)\,y+(1-x)\,(1-y)\,;$$

and this is a true development. The addition of the terms $(1-x)\,y$ and $(1-x)\,(1-y)$ produces the function $1-x$.

The symbol 1 thus developed according to the rule, with respect to the symbol x, gives

$$x+1-x.$$

Developed with respect to x and y, it gives

$$xy+x\,(1-y)+(1-x)\,y+(1-x)\,(1-y)\,.$$

类似地，相对于任何一组记号展开，它产生一个由这些记号的所有可能成分组成的级数。

13. 关于一般展开式的性质，还可以适当地补充几点意见。让我们以一般定理(5)为例，它呈现了两个逻辑符号的函数的展开式的类型。首先，当 x 和 y 是本章所考虑的那类量的记号时，无论赋予函数 f(x, y) 什么代数的形式，该定理都是完全真实且可理解的，因此它可以

intelligibly employed in any stage of the process of analysis intermediate between the change of interpretation of the symbols from the logical to the quantitative system above referred to, and the final restoration of the logical interpretation.

Secondly. The theorem is perfectly true and intelligible when x and y are logical symbols, provided that the form of the function $f(x, y)$ is such as to represent a *class or collection of things,* in which case the second member is always logically interpretable. For instance, if $f(x, y)$ represent the function $1 - x + xy$, we obtain on applying the theorem

$$\begin{aligned} 1 - x + xy &= xy + 0x(1 - y) + (1 - x)y + (1 - x)(1 - y), \\ &= xy + (1 - x)y + (1 - x)(1 - y), \end{aligned}$$

and this result is intelligible and true.

Thus we may regard the theorem as true and intelligible for quantitative symbols of the species above described, *always*; for logical symbols, *always when interpretable.* Whensoever therefore it is employed in this work it must be understood that the symbols x, y are quantitative and of the particular species referred to, if the expansion obtained is not interpretable.

But though the expansion is not always immediately interpretable, it always conducts us at once to results which are interpretable. Thus the expression $x - y$ gives on development the form

$$x(1 - y) - y(1 - x),$$

which is not generally interpretable. We cannot take, in thought, from the class of things which are x's and not y's, the class of things which are y's and not x's, because the latter class is not contained in the former. But if the form $x - y$ presented itself as the first member of an equation, of which the second member was 0, we should have on development

$$x(1 - y) - y(1 - x) = 0.$$

Now it will be shown in the next chapter that the above equation, x and y being regarded as quantitative and of the species described, is resolvable at once into the two equations

$$x(1 - y) = 0, \quad y(1 - x) = 0,$$

and these equations are directly interpretable in Logic when logical interpretations are assigned to the symbols x and y. And it may be remarked, that though *functions* do not necessarily become interpretable upon development, yet *equations* are always reducible by this process to interpretable forms.

14. The following Proposition establishes some important properties of constituents. In its enunciation the symbol t is employed to represent indifferently any constituent of an expansion. Thus if the expansion is that of a function of two symbols x and y, t represents any of the four forms xy, $x(1 - y)$, $(1 - x)y$, and $(1 - x)(1 - y)$. Where it is necessary to represent the constituents of an expansion by single symbols, and yet to distinguish them from each other, the distinction will be marked by suffixes. Thus t_1 might be employed to represent xy, t_2 to represent $x(1 - y)$, and so on.

可以在分析过程的任何阶段被可理解地运用，该阶段介于上述记号解释从逻辑系统到定量系统的改变与逻辑解释的最终恢复之间。

其次。当x和y是逻辑记号时，只要函数f(x, y)的形式能够表示一个类或事物的集合，该定理便完全真实且可理解，在此情形下第二项总是逻辑上可解释的。例如，若f(x, y)表示函数1 – x + xy，则应用该定理我们得到

$$\begin{aligned} 1 - x + xy &= xy + 0x(1 - y) + (1 - x)y + (1 - x)(1 - y), \\ &= xy + (1 - x)y + (1 - x)(1 - y), \end{aligned}$$

and this result is intelligible and true.

因此我们可以认为，对于上述种类的定量记号，该定理是真实且可理解的，总是如此；对于逻辑记号，当可解释时总是如此。因此，无论何时在本著作中运用该定理，都必须理解为：若所得的展开式不可解释，则记号x、y是定量的且属于所提及的特定种类。

然而，尽管展开式并不总是直接可解释的，它却总能立即引导我们得到可解释的结果。因此，表达式x – y展开后给出形式

$$x(1 - y) - y(1 - x),$$

该形式一般不可解释。我们不能在思想中从那些是x而非y的事物的类中，取出那些是y而非x的事物的类，因为后一类并不包含在前一类之中。但若形式x – y作为某一方程的第一项出现，而该方程的第二项为0，则展开后我们便得到

$$x(1 - y) - y(1 - x) = 0.$$

现在，下一章将表明，上述方程，在x和y被视为定量的且属于所述种类时，可立即分解为两个方程

$$x(1 - y) = 0, \quad y(1 - x) = 0,$$

而当记号x和y被赋予逻辑解释时，这些方程在逻辑中直接可解释。并且可以指出，尽管函数在展开后并不必然变得可解释，然而方程总能通过这一过程化为可解释的形式。

14. 下述命题确立了构成成分的一些重要性质。在其表述中，记号 t 被用来随意表示展开式的任一构成成分。因此，如果展开式是两个记号 x 和 y 的函数的展开式，则 t 表示四种形式 xy、x(1 – y)、(1 – x) y 和 (1 – x)(1 – y) 中的任何一种。在需要用单个记号表示展开式的构成成分、同时又要将它们彼此区分开来的地方，这种区分将用下标来标记。因此，t_1可以用来表示 xy,t_2用来表示 x(1 – y)，如此等等。

PROPOSITION III.

Any single constituent t *of an expansion satisfies the law of duality whose expression is*

$$t\,(1-t)=0.$$

The product of any two distinct constituents of an expansion is equal to 0, *and the sum of all the constituents is equal to* 1.

1st. Consider the particular constituent xy. We have

$$xy\times xy=x^2y^2.$$

But $x^2=x$, $y^2=y$, by the fundamental law of class symbols; hence

$$xy\times xy=xy.$$

Or representing xy by t,

$$t\times t=t,$$

or

$$t\,(1-t)=0.$$

Similarly the constituent $x\,(1-y)$ satisfies the same law. For we have

$$x^2=x,(1-y)^2=1-y,$$
$$\therefore\{x\,(1-y)\}^2=x\,(1-y)\,,\text{ or } t\,(1-t)=0.$$

Now every factor of every constituent is either of the form x or of the form $1-x$. Hence the square of each factor is equal to that factor, and therefore the square of the product of the factors, i.e. of the constituent, is equal to the constituent; wherefore t representing any constituent, we have

$$t^2=t,\text{ or } t\,(1-t)=0.$$

2ndly. The product of any two constituents is 0. This is evident from the general law of the symbols expressed by the equation $x\,(1-x)=0$; for whatever constituents in the same expansion we take, there will be at least one factor x in the one, to which will correspond a factor $1-x$ in the other.

3rdly. The sum of all the constituents of an expansion is unity. This is evident from addition of the two constituents x and $1-x$, or of the four constituents, xy, $x(1-y)$, $(1-x)y$, $(1-x)(1-y)$. But it is also, and more generally, proved by expanding 1 in terms of any set of symbols (V. 12). The constituents in this case are formed as usual, and all the coefficients are unity.

15. With the above Proposition we may connect the following.

PROPOSITION IV.

命题三。

Any single constituent t *of an expansion satisfies the law of duality whose expression is*

$$t\,(1-t)=0.$$

The product of any two distinct constituents of an expansion is equal to 0, *and the sum of all the constituents is equal to* 1.

1st. Consider the particular constituent xy. We have

$$xy\times xy=x^2y^2.$$

但根据类记号的基本规律，x 2 = x,y 2 = y；因此

$$xy\times xy=xy.$$

Or representing xy by t,

$$t\times t=t,$$

or

$$t\,(1-t)=0.$$

Similarly the constituent $x\,(1-y)$ satisfies the same law. For we have

$$x^2=x,(1-y)^2=1-y,$$
$$\therefore\{x\,(1-y)\}^2=x\,(1-y)\,,\text{ or } t\,(1-t)=0.$$

现在，每一成分的每一因子要么具有x的形式，要么具有1 – x的形式。因此每一因子的平方等于该因子，从而各因子之积(即该成分)的平方等于该成分；因此，以t表示任一成分，我们有

$$t^2=t,\text{ or } t\,(1-t)=0.$$

其次。任意两个成分之积为0。这从方程x(1 – x) = 0所表达的记号的一般规律即可明显看出；因为无论我们在同一展开式中取哪些成分，其中一个中至少会有一个因子x，与之相对应，另一个中会有一个因子1 – x。

第三。展开式的所有构成成分之和为 1。这从两个构成成分 x 和 1 – x 的相加，或四个构成成分 xy、x(1 – y)、(1 – x) y、(1 – x)(1 – y) 的相加，是显然的。但它也可以、并且更一般地，通过按任意一组记号展开 1 来证明(V.12)。在这种情形中，构成成分按通常方式形成，且所有系数均为 1。

15. 与上述命题相联系，我们可引出以下内容。

命题四。

If V represent the sum of any series of constituents, the separate coefficients of which are 1*, then is the condition satisfied,*

$$V\,(1-V)=0$$

Let t_1, t_2 $\ldots t_n$ be the constituents in question, then

$$V=t_1+t_2\cdots+t_n.$$

Squaring both sides, and observing that $t_1^2=t_1$, $t_1t_2,=0$, &c., we have

$$V^2=t_1+t_2\cdots+t_n;$$

whence

$$V=V^2.$$

Therefore

$$V\,(1-V)=0.$$

如果表示任一 *constituent* 级数之和，其各自的系数为 1 ，那么条件得到满足，

$$V\,(1-V)=0$$

设 t 1、t 2.....t n 为所讨论的 constituents，则

$$V=t_1+t_2\cdots+t_n.$$

两边平方，并注意到 t 21=t1,t1t2,=0,&c.,我们有

$$V^{\text{“}}=t_1+t_2\cdots+t_n;$$

由此

$$V=V^2.$$

因此 erefore

$$V\,(1-V)=0.$$

Chapter VI

OF THE GENERAL INTERPRETATION OF LOGICAL EQUATIONS, AND THE RESULTING ANALYSIS OF PROPOSITIONS. ALSO, OF THE CONDITION OF INTERPRETABILITY OF LOGICAL FUNCTIONS.

1. It has been observed that the complete expansion of any function by the general rule demonstrated in the last chapter, involves two distinct sets of elements, viz., the constituents of the expansion, and their coefficients. I propose in the present chapter to inquire, first, into the interpretation of constituents, and afterwards into the mode in which that interpretation is modified by the coefficients with which they are connected.

The terms "logical equation," "logical function," &c., will be employed generally to denote any equation or function involving the symbols x, y, &c., which may present itself either in the expression of a system of premises, or in the train of symbolical results which intervenes between the premises and the conclusion. If that function or equation is in a form not immediately interpretable in Logic, the symbols x, y, &c., must be regarded as quantitative symbols of the species described in previous chapters (II. 15), (V. 6), as satisfying the law,

$$x\,(1-x)=0.$$

By the problem, then, of the interpretation of any such logical function or equation, is meant the reduction of it to a form in which, when logical values are assigned to the symbols x, y, &c., it shall become interpretable, together with the resulting interpretation. These conventional definitions are in accordance with the general principles for the conducting of the method of this treatise, laid down in the previous chapter.

Proposition I.

2. The constituents of the expansion of any function of the logical symbols x, y, &c., are interpretable, and represent the several exclusive divisions of the universe of discourse, formed by the predication and denial in every possible way of the qualities denoted by the symbols x, y, &c.

第六章

论逻辑方程的一般解释，以及由此产生的命题分析。兼论逻辑函数的可解释性条件。

1. 已经注意到，按照上一章所证明的一般规则，任何函数的完全展开都涉及两组不同的元素，即展开式的 constituents 及其系数。本章拟先探究 constituents 的解释，然后再探究该解释受与之相联系的系数所修饰的方式。

"逻辑方程"、"逻辑函数"等术语将一般地用来指称任何涉及记号x、y等的方程或函数，这些方程或函数可能出现在一个前提系统的表达式中，或出现在介于前提与结论之间的符号结果的序列中。如果该函数或方程的形式不能立即在逻辑中被解释，则记号x、y等必须被视为前几章(II.15)、(V.6)中所描述的那类定量记号，满足定律，

$$x\,(1-x)=0.$$

因此，所谓任何这类逻辑函数或方程的解释问题，就是指把它化归为这样一种形式：当把逻辑值赋予记号 x、y 等时，它将成为可解释的，并连同由此产生的解释。这些约定性定义符合上一章所奠定的、进行本论著方法的一般原则。

命题一。

2. 任何逻辑记号 、 等的函数的展开式的 *constituents* 是可解释的，并且表示论域的若干互斥划分，这些划分是通过以一切可能的方式肯定和否定记号 、 等所表示的性质而形成的。

For greater distinctness of conception, let it be supposed that the function expanded involves two symbols x and y, with reference to which the expansion has been effected. We have then the following constituents, viz.:

$$xy,\ x\,(1-y)\,,\ (1-x)\,y,\ (1-x)\ (1-y)\,.$$

Of these it is evident, that the first xy represents that class of objects which at the same time possess both the elementary qualities expressed by x and y, and that the second $x\,(1-y)$ represents the class possessing the property x, but not the property y. In like manner the third constituent represents the class of objects which possess the property represented by y, but not that represented by x; and the fourth constituent $(1-x)\,(1-y)$, represents that class of objects, the members of which possess neither of the qualities in question.

Thus the constituents in the case just considered represent all the four classes of objects which can be described by affirmation and denial of the properties expressed by x and y. Those classes are distinct from each other. No member of one is a member of another, for each class possesses some property or quality contrary to a property or quality possessed by any other class. Again, these classes together make up the universe, for there is no object which may not be described by the presence or the absence of a proposed quality, and thus each individual thing in the universe may be referred to some one or other of the four classes made by the possible combination of the two given classes x and y, and their contraries.

The remarks which have here been made with reference to the constituents of $f\,(x,y)$ are perfectly general in character. The constituents of any expansion represent classes—those classes are mutually distinct, through the possession of contrary qualities, and they together make up the universe of discourse.

3. These properties of constituents have their expression in the theorems demonstrated in the conclusion of the last chapter, and might thence have been deduced. From the fact that every constituent satisfies the fundamental law of the individual symbols, it might have been conjectured that each constituent would represent a class. From the fact that the product of any two constituents of an expansion vanishes, it might have been concluded that the classes they represent are mutually exclusive. Lastly, from the fact that the sum of the constituents of an expansion is unity, it might have been inferred, that the classes which they represent, together make up the universe.

4. Upon the laws of constituents and the mode of their interpretation above determined, are founded the analysis and the interpretation of logical equations. That all such equations admit of interpretation by the theorem of development has already been stated. I propose here to investigate the forms of possible solution which thus present themselves in the conclusion of a train of reasoning, and to show how those forms arise. Although, properly speaking, they are but manifestations of a single fundamental type or principle of expression, it will conduce to clearness of apprehension if the minor varieties which they exhibit are presented separately to the mind.

The forms, which are three in number, are as follows:

为使概念更为清晰，设所展开的函数涉及两个记号 x 和 y，展开是相对于它们进行的。于是我们有以下 constituents，即：

$$xy,\ x\,(1-y)\,,\ (1-x)\,y,\ (1-x)\ (1-y)\,.$$

在这些项中，显然第一个xy代表同时具有由x和y所表达的基本性质的那一类对象，而第二个x(1 – y)代表具有性质x但不具有性质y的那一类。同样地，第三个成分代表具有由y所表示的性质但不具有由x所表示的性质的那一类对象；而第四个成分(1 – x)(1 – y)代表这样一类对象，其成员既不具有所讨论的性质中的任何一个。

因此，刚才所考虑的情形中的成分代表了可以由对x和y所表达的性质的肯定和否定来描述的全部四类对象。这些类彼此不同。一个类的成员不是另一个类的成员，因为每一类都具有某种与任何其他类所具有的性质或性质相反的性质或性质。再者，这些类合在一起构成论域，因为没有哪个对象不能由某一给定性质的有无来描述，因此论域中的每一个个别事物都可以被归入由两个给定类x和y及其相反者的可能组合所形成的四个类中的某一个。

这里就 f(x, y) 的 constituents 所作的评论在性质上完全是普遍的。任何展开式的 constituents 都表示类--这些类由于具有相反的性质而彼此区别，并且它们共同构成论域。

3. 成分的这些性质在上一章结论中所证明的定理中得到了表达，因而本可以从那里推导出来。从每个成分都满足个体记号的基本定律这一事实，本可以猜想每个成分都代表一个类。从展开式的任意两个成分的乘积为零这一事实，本可以得出结论说它们所代表的类互相排斥。最后，从展开式的各成分之和为一这一事实，本可以推断出它们所代表的类合在一起构成论域。

4. 逻辑方程的分析和解释是建立在上述所确定的成分的定律及其解释方式之上的。一切这样的方程都可以通过展开定理而得到解释，这一点已经陈述过了。我在此打算考察在一系列推理的结论中如此呈现出来的可能解的形式，并表明这些形式是如何产生的。尽管严格说来它们只不过是一个单一的基本类型或表达原则的表现，但把它们所呈现的次要变化分别呈现于心灵之前，将有助于理解的清晰。

这些形式共有三种，具体如下：

FORM I.

5. The form we shall first consider arises when any logical equation $V = 0$ is developed, and the result, after resolution into its component equations, is to be interpreted. The function is supposed to involve the logical symbols x, y, &c., in combinations which are not fractional. Fractional combinations indeed only arise in the class of problems which will be considered when we come to speak of the third of the forms of solution above referred to.

PROPOSITION II.

To interpret the logical equation $V = 0$.

For simplicity let us suppose that V involves but two symbols, x and y, and let us represent the development of the given equation by

$$axy + bx\,(1-y) + c\,(1-x)\,y + d\,(1-x)\,(1-y) = 0; \qquad (1)$$

a, b, c, and d being definite numerical constants.

Now, suppose that any coefficient, as a, does not vanish. Then multiplying each side of the equation by the constituent xy, to which that coefficient is attached, we have

$$axy = 0,$$

whence, as a does not vanish,

$$xy = 0,$$

and this result is quite independent of the nature of the other coefficients of the expansion. Its interpretation, on assigning to x and y their logical significance, is "No individuals belonging at once to the class represented by x, and the class represented by y, exist."

But if the coefficient a *does* vanish, the term axy does not appear in the development (1), and, therefore, the equation $xy = 0$ cannot thence be deduced.

In like manner, if the coefficient b does not vanish, we have

$$x\,(1-y) = 0,$$

which admits of the interpretation, "There are no individuals which at the same time belong to the class x, and do not belong to the class y."

Either of the above interpretations may, however, as will subsequently be shown, be exhibited in a different form.

The sum of the distinct interpretations thus obtained from the several terms of the expansion whose coefficients do not vanish, will constitute the complete interpretation of the equation $V = 0$. The analysis is essentially independent of the number of logical symbols involved in the function V, and the object of the proposition will, therefore, in all instances, be attained by the following Rule: –

RULE.—*Develop the function* V*, and equate to* 0 *every constituent whose coefficient does not vanish. The interpretation of these results collectively will constitute the interpretation of the given equation.*

式 i。

5. 我们将首先考虑的形式，是在任何逻辑方程 V = 0 被展开，并且其结果在分解为各分量方程之后加以解释时出现的。该函数被假定涉及逻辑记号 x、y 等，其组合不是分数形式的。事实上，分数组合只出现在我们谈到上述第三种解的形式时将考虑的那类问题中。

命题二。

解释逻辑方程 V = 0 。

为简单起见，设 V 只涉及两个记号 x 和 y，并设给定方程的展开式表示为

$$axy + bx\,(1-y) + c\,(1-x)\,y + d\,(1-x)\,(1-y) = 0; \qquad (1)$$

a、b、c 和 d 为确定的数值常数。

现在，设任一系数，如 a，不为零。那么，将方程两边乘以该系数所附的 constituent xy，我们得到

$$axy = 0,$$

因此，既然 a 不为零，

$$xy = 0,$$

而这个结果与展开式其他系数的性质完全无关。当把逻辑意义赋予 x 和 y 时，它的解释是："不存在同时属于 x 所表示的类和 y 所表示的类的个体。"

但如果系数 a 确实为零，则项 axy 不出现在展开式(1)中，因此不能由此推出方程 xy = 0。

同样地，如果系数 b 不为零，我们有

$$x\,(1-y) = 0,$$

这可以解释为："不存在同时属于 x 类而不属于 y 类的个体。"

然而，如上两种解释中的任何一种，都可以如下文将要表明的那样，以不同的形式展示出来。

这样从展开式中系数不为零的各项所得到的各个不同解释之和，将构成方程 V = 0 的完整解释。这一分析与函数 V 所涉及的逻辑记号的数目本质上无关，因此，本命题的目的在一切情况下都将通过以下规则达到：--

规则。--展开函数 ，并将每个系数不为零的 *constituent* 等于 0 。这些结果的解释合在一起将构成给定方程的解释。

6. Let us take as an example the definition of "clean beasts," laid down in the Jewish law, viz., "Clean beasts are those which both divide the hoof and chew the cud," and let us assume

x = clean beasts;
y = beasts dividing the hoof; Then the given proposition will be represented
z = beasts chewing the cud;

by the equation

$$x = yz$$

which we shall reduce to the form

$$x - yz = 0,$$

and seek that form of interpretation to which the present method leads. Fully developing the first member, we have

$$0xyz + xy\,(1-z) + x\,(1-y)\,z + x\,(1-y)\,(1-z)$$
$$-\,(1-x)\,yz + 0\,(1-x)\,y\,(1-z) + 0\,(1-x)\,(1-y)\,z + 0\,(1-x)\,(1-y)\,(1-z)\,.$$

Whence the terms, whose coefficients do not vanish, give

$$xy\,(1-z) = 0,\ xz\,(1-y) = 0,\ x\,(1-y)\,(1-z) = 0,\ (1-x)\,yz = 0.$$

These equations express a denial of the existence of certain classes of objects, viz.:

1st. Of beasts which are clean, and divide the hoof, but do not chew the cud.

2nd. Of beasts which are clean, and chew the cud, but do not divide the hoof.

3rd. Of beasts which are clean, and neither divide the hoof nor chew the cud.

4th. Of beasts which divide the hoof, and chew the cud, and are not clean.

Now all these several denials are really involved in the original proposition. And conversely, if these denials be granted, the original proposition will follow as a necessary consequence. They are, in fact, the separate elements of that proposition. Every primary proposition can thus be resolved into a series of denials of the existence of certain defined classes of things, and may, from that system of denials, be itself reconstructed. It might here be asked, how it is possible to make an assertive proposition out of a series of denials or negations? From what source is the positive element derived? I answer, that the mind assumes the existence of a universe not *à priori* as a fact independent of experience, but either *à posteriori* as a deduction from experience, or *hypothetically* as a foundation of the possibility of assertive reasoning. Thus from the Proposition, "There are no men who are not fallible," which is a negation or denial of the existence of "infallible men," it may be inferred either hypothetically, "All men (if men exist) are fallible," or absolutely, (experience having assured us of the existence of the race), "All men are fallible."

The form in which conclusions are exhibited by the method of this Proposition may be termed the form of "Single or Conjoint Denial."

6. 让我们以犹太律法中关于"洁净兽"的定义为例，即"洁净兽是既分蹄又反刍的兽"，并设

x = 洁净兽；y = 分蹄的兽；z = 反刍的兽；

那么给定的命题将被表示为

由方程

$$x = yz$$

我们将把它化为以下形式

$$x - yz = 0,$$

and seek that form of interpretation to which the present method leads. Fully developing the first member, we have

$$0xyz + xy\,(1-z) + x\,(1-y)\,z + x\,(1-y)\,(1-z)$$
$$-\,(1-x)\,yz + 0\,(1-x)\,y\,(1-z) + 0\,(1-x)\,(1-y)\,z + 0\,(1-x)\,(1-y)\,(1-z)\,.$$

Whence the terms, whose coefficients do not vanish, give

$$xy\,(1-z) = 0,\ xz\,(1-y) = 0,\ x\,(1-y)\,(1-z) = 0,\ (1-x)\,yz = 0.$$

These equations express a denial of the existence of certain classes of objects, viz.:

1st. Of beasts which are clean, and divide the hoof, but do not chew the cud.

2nd. Of beasts which are clean, and chew the cud, but do not divide the hoof.

3rd. Of beasts which are clean, and neither divide the hoof nor chew the cud.

第四，关于分蹄、反刍而不洁净的兽。所有这些个别的否定实际上都包含在原命题之中。反过来，如果承认这些否定，原命题便作为必然结论随之成立。事实上，它们就是该命题的各个独立元素。因此，每一个初始命题都可以被分解为对某些确定的事物类之存在的一系列否定，并且可以从那个否定系统本身重新构造出来。这里或许可以问：如何可能从一系列否定或否认中构造出一个肯定命题?那个肯定的要素又从何而来?我回答说：心灵假定有一个宇宙存在，这既不是`先验地作为独立于经验的事实，也不是`后验地作为从经验得出的演绎，就是假设地作为肯定推理之可能性的基础。于是，从命题"没有不会犯错误的人"出发--这是对"不会犯错误的人"之存在的否定或否认--可以或者假设地推出"所有人(如果人存在)都会犯错误"，或者绝对地推出(经验已向我们保证这一族类的存在)"所有人都会犯错误"。

本命题的方法所展示的结论形式，可以称为"单一或联合否定"的形式。

FORM II.

7. As the previous form was derived from the development and interpretation of an equation whose second member is 0, the present form, which is supplementary to it, will be derived from the development and interpretation of an equation whose second member is 1. It is, however, readily suggested by the analysis of the previous Proposition.

Thus in the example last discussed we deduced from the equation

$$x - yz = 0$$

the conjoint denial of the existence of the classes represented by the constituents

$$xy\,(1-z)\,, \quad xz\,(1-y)\,, \quad x\,(1-y)\,(1-z)\,, \quad (1-x)\,yz,$$

whose coefficients were not equal to 0. It follows hence that the remaining constituents represent classes which make up the universe. Hence we shall have

$$xyz + (1-x)\,y\,(1-z) + (1-x)\,(1-y)\,z + (1-x)\,(1-y)\,(1-z) = 1.$$

This is equivalent to the affirmation that all existing things belong to some one or other of the following classes, viz.:

1st. Clean beasts both dividing the hoof and chewing the cud.
2nd. Unclean beasts dividing the hoof, but not chewing the cud.
3rd. Unclean beasts chewing the cud, but not dividing the hoof.
4th. Things which are neither clean beasts, nor chewers of the cud, nor dividers of the hoof.

This form of conclusion may be termed the form of "Single or Disjunctive Affirmation,"—single when but one constituent appears in the final equation; disjunctive when, as above, more constituents than one are there found.

Any equation, $V = 0$, wherein V satisfies the law of duality, may also be made to yield this form of interpretation by reducing it to the form $1 - V = 1$, and developing the first member. The case, however, is really included in the next general form. Both the previous forms are of slight importance compared with the following one.

FORM III.

8. In the two preceding cases the functions to be developed were equated to 0 and to 1 respectively. In the present case I shall suppose the corresponding function equated to any logical symbol w. We are then to endeavour to interpret the equation $V = w$, V being a function of the logical symbols x, y, z, &c. In the first place, however, I deem it necessary to show how the equation $V = w$, or, as it will usually present itself, $w = V$, arises.

Let us resume the definition of "clean beasts," employed in the previous examples, viz., "Clean beasts are those which both divide the hoof and chew the cud," and suppose it required to determine the relation in which "beasts

式 ii。

7. 正如前一形式是从第二项为 0 的方程的展开和解释中得出的，当前这一作为其补充的形式，将从第二项为 1 的方程的展开和解释中得出。不过，它很容易从前一命题的分析中得到提示。

因此在最后讨论的例子中，我们从方程推导出

$$x - yz = 0$$

对 constituents 所表示的类的存在的联合否定

$$xy\,(1-z)\,, \quad xz\,(1-y)\,, \quad x\,(1-y)\,(1-z)\,, \quad (1-x)\,yz,$$

其系数不等于 0。由此可知，其余 constituents 所表示的类构成论域。因此我们将有

xyz +(1 – x) y(1 – z) +(1 – x)(1 – y) z +(1 – x)(1 – y)(1 – z) = 1 .这等价于肯定一切存在物都属于下列各类中的某一类，即：

第一，分蹄且反刍的洁净牲畜。
第二，分蹄但不反刍的走兽是不洁净的。
第三，反刍而不分蹄的不洁之兽。
第四，既不是洁净的兽，也不是反刍的，也不是分蹄的。

这种结论形式可以称为"单一或析取肯定"的形式--当最终方程中只出现一个 constituent 时为单一；当如上文那样出现多于一个 constituent 时为析取。

任何方程，V = 0，其中V满足对偶律，也可以通过将其化为形式1 – V = 1并展开第一项，使之产生这种形式的解释。然而，这种情况实际上包含在下一个一般形式中。前两种形式与以下形式相比都稍显次要。

形式 iii。

8. 在前两个例子中，需要展开的函数被等同于
分别对应0和1。在当前情况下，我将假定相应的函数等于任意逻辑记号w。然后我们要努力解释方程V = w，其中V是逻辑记号x、y、z等的函数。然而，首先我认为有必要说明方程V = w，或者通常表现为w = V，是如何产生的。

让我们重新采用前面例子中使用的"洁净兽"的定义，即"洁净兽是那些既分蹄又反刍的兽"，并假定需要确定"反刍的兽"

chewing the cud" stand to "clean beasts" and "beasts dividing the hoof." The equation expressing the given proposition is

$$x = yz,$$

and our object will be accomplished if we can determine z as an interpretable function of x and y.

Now treating x, y, z as symbols of quantity subject to a peculiar law, we may deduce from the above equation, by solution,

$$z = \frac{x}{y}.$$

But this equation is not at present in an interpretable form. If we can reduce it to such a form it will furnish the relation required.

On developing the second member of the above equation, we have

$$z = xy + \frac{1}{0}x\,(1-y) + 0\,(1-x)\,y + \frac{0}{0}\,(1-x)\,(1-y)\,,$$

and it will be shown hereafter (Prop. 3) that this admits of the following interpretation:

"Beasts which chew the cud consist of all clean beasts (which also divide the hoof), together with an indefinite remainder (some, none, or all) of unclean beasts which do not divide the hoof."

9. Now the above is a particular example of a problem of the utmost generality in Logic, and which may thus be stated:—"Given any logical equation connecting the symbols x, y, z, w, required an interpretable expression for the relation of the class represented by w to the classes represented by the other symbols x, y, z, &c."

The solution of this problem consists in all cases in determining, from the equation *given*, the expression of the above symbol w, in terms of the other symbols, and rendering that expression interpretable by development. Now the equation given is always of the first degree with respect to each of the symbols involved. The required expression for w can therefore always be found. In fact, if we develop the given equation, whatever its form may be with respect to w, we obtain an equation of the form

$$Ew + E'\,(1-w) = 0, \tag{1}$$

E and E' being functions of the remaining symbols. From the above we have

$$E' = (E' - E)\,w.$$

Therefore

$$w = \frac{E'}{E' - E} \tag{2}$$

and expanding the second member by the rule of development, it will only remain to interpret the result in logic by the next proposition.

与"洁净兽"和"分蹄的兽"之间的关系。表达给定命题的方程是

$$x = yz,$$

如果我们能将z确定为x和y的可解释函数，我们的目标就将实现。

现在将x、y、z视为服从特殊规律的量的记号，我们可以通过求解从上述方程中推出

$$z = \frac{x}{y}.$$

但这个方程目前不是可解释的形式。如果我们能将其化为这种形式，它就能提供所需的关系。

展开上述方程的第二项，我们得到

$$z = xy + \frac{1}{0}x\,(1-y) + 0\,(1-x)\,y + \frac{0}{0}\,(1-x)\,(1-y)\,,$$

后文(命题3)将表明，这允许以下解释：

"反刍的兽由所有洁净兽(它们也分蹄)以及不洁净兽中不分蹄的不定剩余部分(一些、没有或全部)组成。"

9. 上述是逻辑中一个具有最大普遍性的问题的特殊例子，该问题可以这样表述："给定连接记号x、y、z、w的任何逻辑方程，要求给出一个可解释的表达式，表示由w所代表的类与其他记号x、y、z等所代表的类之间的关系。"

这个问题的解决在所有情况下都在于：从给定的方程出发，确定上述记号w用其他记号表示的表达式，并通过展开式使该表达式成为可解释的。现在，给定的方程相对于其中所涉及的每一个记号而言总是一次的。因此，w所要求的表达式总是可以找到的。事实上，如果我们展开给定的方程，无论它相对于w的形式如何，我们都会得到一个如下形式的方程

$$Ew + E'\,(1-w) = 0, \tag{1}$$

E和E′是其余记号的函数。由上述我们得到

$$E' = (E' - E)\,w.$$

Therefore

$$w = \frac{E'}{E' - E} \tag{2}$$

and expanding the second member by the rule of development, it will only remain to interpret the result in logic by the next proposition.

If the fraction $\frac{E'}{E'-E}$ has common factors in its numerator and denominator, we are not permitted to reject them, unless they are mere numerical constants. For the symbols x, y, &c., regarded as quantitative, may admit of such values 0 and 1 as to cause the common factors to become equal to 0, in which case the algebraic rule of reduction fails. This is the case contemplated in our remarks on the failure of the algebraic axiom of division (II. 14). To *express* the solution in the form (2), and without attempting to perform any unauthorized reductions, to interpret the result by the theorem of development, is a course strictly in accordance with the general principles of this treatise.

If the relation of the class expressed by $1 - w$ to the other classes, x, y, &c. is required, we deduce from (1), in like manner as above,

$$1 - w = \frac{E}{E - E'},$$

to the interpretation of which also the method of the following Proposition is applicable:

Proposition III.

10. *To determine the interpretation of any logical equation of the form $w = V$, in which w is a class symbol, and V a function of other class symbols quite unlimited in its form.*

Let the second member of the above equation be fully expanded. Each coefficient of the result will belong to some one of the four classes, which, with their respective interpretations, we proceed to discuss.

1st. Let the coefficient be 1. As this is the symbol of the universe, and as the product of any two class symbols represents those individuals which are found in both classes, any constituent which has unity for its coefficient must be interpreted without limitation, i.e. the whole of the class which it represents is implied.

2nd. Let the coefficient be 0. As in Logic, equally with Arithmetic, this is the symbol of Nothing, no part of the class represented by the constituent to which it is prefixed must be taken.

3rd. Let the coefficient be of the form $\frac{0}{0}$. Now, as in Arithmetic, the symbol $\frac{0}{0}$ represents an *indefinite number*, except when otherwise determined by some special circumstance, analogy would suggest that in the system of this work the same symbol should represent an *indefinite class*. That this is its true meaning will be made clear from the following example:

Let us take the Proposition, "Men not mortal do not exist;" represent this Proposition by symbols; and seek, in obedience to the laws to which those symbols have been proved to be subject, a reverse definition of "mortal beings," in terms of "men."

Now if we represent "men" by y, and "mortal beings" by x, the Proposition, "Men who are not mortals do not exist," will be expressed by the equation

$$y\,(1 - x) = 0,$$

如果分数 E′ E′−E 在其分子和分母中有公

我们不被允许舍弃它们，除非它们仅仅是数值常数。因为记号x、y等，若被视为量的，可能容许这样的值0和1，以致公因子变为等于0，在这种情况下代数的约简规则失效。这正是我们在关于代数除法公理失效的评论(II.14)中所考虑的情形。将解表达为形式(2)，并且不试图进行任何未经授权的约简，而是通过展开式定理来解释结果，这一做法严格符合本论著的一般原则。

如果需要由1 – w所表达的类与其他类x、y等的关系，我们如上所述从(1)推出，

$$1 - w = \frac{E}{E - E'},$$

对其解释也适用以下命题的方法：

命题三。

10. 确定任何形式为 w = V 的逻辑方程的解释，其中是一个类记号，而是一个形式完全不受限制的其他类记号的函数。

设上述方程的第二项完全展开。结果的每个系数将属于四个类中的某一个，我们接着讨论这些类及其各自的解释。

第一，设系数为1。由于这是论域的记号，并且任意两个类记号的乘积代表同时存在于两个类中的个体，任何以1为系数的构成项都必须无限制地解释，即它所代表的整个类都被蕴含。

第二，设系数为0。正如在逻辑中与算术中一样，这是"无"的记号，凡以它为前缀的构成项所代表的类，其任何部分都不得取用。

第三。设系数具有形式00.现在，正如在算术中，该记号00代表一个不定数,除非当另有确定由某些特殊情形时，类比会提示在本著作的体系中同一记号应代表一个不定类。这确实是其真正含义，将从以下例子中清楚看出：

让我们取命题"不死的人不存在"；用记号表示这个命题；并按照已证明这些记号所服从的规律，寻求用"人"来反向定义"会死的存在者"。

现在，如果我们用y表示"人"，用x表示"会死的存在者"，则命题"不是会死的人不存在"将用方程表达为

$$y\,(1 - x) = 0,$$

from which we are to seek the value of x. Now the above equation gives

$$y - yx = 0, \text{ or } yx = y.$$

Were this an ordinary algebraic equation, we should, in the next place, divide both sides of it by y. But it has been remarked in Chap. 11. that the operation of division cannot be *performed* with the symbols with which we are now engaged. Our resource, then, is to *express* the operation, and develop the result by the method of the preceding chapter. We have, then, first,

$$x = \frac{y}{y},$$

and, expanding the second member as directed,

$$x = y + \frac{0}{0}(1 - y).$$

This implies that mortals (x) consist of all men (y), together with such a remainder of beings which are not men ($1 - y$), as be indicated by the coefficient $\frac{0}{0}$. Now let us inquire what remainder of "not men" is implied by the premiss. It might happen that the remainder included all the beings who are not men, or it might include only some of them, and not others, or it might include none, and any one of these assumptions would be in perfect accordance with our premiss. In other words, whether those beings which are not men are *all*, or *some*, or *none*, of them *mortal*, the truth of the premiss which virtually asserts that all men are mortal, will be equally unaffected, and therefore the expression $\frac{0}{0}$ here indicates that *all*, *some*, or *none* of the class to whose expression it is affixed must be taken.

Although the above determination of the significance of the symbol $\frac{0}{0}$ is founded only upon the examination of a particular case, yet the principle involved in the demonstration is general, and there are no circumstances under which the symbol can present itself to which the same mode of analysis is inapplicable. We may properly term $\frac{0}{0}$ an *indefinite class symbol*, and may, if convenience should require, replace it by an uncompounded symbol v, subject to the fundamental law, $v(1 - v) = 0$.

4th. It may happen that the coefficient of a constituent in an expansion does not belong to any of the previous cases. To ascertain its true interpretation when this happens, it will be necessary to premise the following theorem:

11. THEOREM.—*If a function V, intended to represent any class or collection of objects, w, be expanded, and if the numerical coefficient, a, of any constituent in its development, do not satisfy the law.*

$$a(1 - a) = 0,$$

then the constituent in question must be made equal to 0.

To prove the theorem generally, let us represent the expansion given, under the form

$$w = a_1 t_1 + a_2 t_2 + a_3 t_3 + \&c., \tag{1}$$

我们要从中求 x 的值。现在上述方程给出

$$y - yx = 0, \text{ or } yx = y.$$

如果这是一个普通的代数方程，我们接下来应该用它除以y。但第11章已经指出，除法运算不能用我们现在所处理的记号来进行。那么，我们的办法是表达该运算，并用前一章的方法展开结果。于是，我们首先有，

$$x = \frac{y}{y},$$

并且，按指示展开第二项，

$$x = y + \frac{0}{0}(1 - y).$$

这意味着有死者(x)由全部人(y)连同那些不是人(1 – y)的存在者中由系数00所指示的剩余部分构成。现在让我们来探究什么剩余部分的"不是人"是由前提所蕴含的。可能会发生的是，这个剩余部分包括所有不是人的存在者，或者它可能只包括其中一些而不包括另一些，或者它可能一个也不包括，而这些假定中的任何一个都会与我们的前提完全一致。换言之，无论那些不是人的存在者是它们中的全部、还是有些、还是没有是有死的，那个实际上断言所有人都是有死的前提之真都将同样不受影响，因此表达式00在这里表示：其表达式被附加上该系数的那个类，必须取全部、有些或没有。

尽管上述对记号00is意义的确定仅建立在对一个特例的考察之上，然而论证中所涉及的原理是普遍的，不存在该记号可能呈现而同一分析方式不适用的情形。我们可以恰当地称00an不定类记号,并且可以，如果方便性要求，用一个非复合的记号 v 来替换它，服从基本规律，v(1 – v) = 0。

第四，展开式中某一成分的系数可能不属于上述任何一种情形。当此情形发生时，为确定其真正的解释，须先设定以下定理：

11. 定理。--如果一个函数，意在表示任何类或对象的集合，，被展开，并且如果其展开式中任何构成项的数值系数，不满足该规律。

$$a(1 - a) = 0,$$

then the constituent in question must be made equal to 0.

To prove the theorem generally, let us represent the expansion given, under the form

$$w = a_1 t_1 + a_2 t_2 + a_3 t_3 + \&c., \tag{1}$$

in which t_1, t_2, t_3, &c. represent the constituents, and a_1, a_2, a_3, &c. the coefficients; let us also suppose that a_1 and a_2 do not satisfy the law

$$a_1 (1 - a_1) = 0, \quad a_2 (1 - a_2) = 0;$$

but that the other coefficients are subject to the law in question, so that we have

$$a_3{}^2 = a_3, \text{ &c.}$$

Now multiply each side of the equation (1) by itself. The result will be

$$w = a_1{}^2 t_1 + a_2{}^2 t_2 + \text{ &c.} \tag{2}$$

This is evident from the fact that it must represent the development of the equation

$$w = V^2,$$

but it may also be proved by actually squaring (1), and observing that we have

$$t_1{}^2 = t_1, \quad t_2{}^2 = t_2, \quad t_1 t_2 = 0, \text{ &c.}$$

by the properties of constituents. Now subtracting (2) from (1), we have

$$\left(a_1 - a_1{}^2\right) t_1 + \left(a_2 - a_2{}^2\right) t_2 = 0.$$

Or,

$$a_1 (1 - a_1) t_1 + a_2 (1 - a_2) t_2 = 0.$$

Multiply the last equation by t_1; then since $t_1 t_2 = 0$, we have

$$a_1 (1 - a_1) t_1 = 0, \text{ whence } t_2 = 0.$$

In like manner multiplying the same equation by t_2, we have

$$a_2 (1 - a_2) t_2 = 0, \text{ whence } t_2 = 0.$$

Thus it may be shown generally that any constituent whose coefficient is not subject to the same fundamental law as the symbols themselves must be separately equated to 0. The usual form under which such coefficients occur is $\frac{1}{0}$. This is the algebraic symbol of infinity. Now the nearer any number approaches to infinity (allowing such an expression), the more does it depart from the condition of satisfying the fundamental law above referred to.

The symbol $\frac{0}{0}$, whose interpretation was previously discussed, does not necessarily disobey the law we are here considering, for it admits of the numerical values 0 and 1 indifferently. Its actual interpretation, however, as an indefinite class symbol, cannot, I conceive, except upon the ground of analogy, be deduced from its arithmetical properties, but must be established experimentally.

12. We may now collect the results to which we have been led, into the following summary:

其中t ^{1}、t ^{2}、t ^{3}等代表构成项，a ^{1}、a ^{2}、a ^{3}等代表系数；我们还假定a ^{1}和a ^{2}不满足该规律

$$a_1 (1 - a_1) = 0, \quad a_2 (1 - a_2) = 0;$$

但其他系数受所讨论的规律支配，因此我们有

$$a_3{}^2 = a_3, \text{ &c.}$$

现在将方程(1)的两边各自相乘。结果将是

$$w = a_1{}^2 t_1 + a_2{}^2 t_2 + \text{ &c.} \tag{2}$$

This is evident from the fact that it must represent the development of the equation

$$w = V^2,$$

但也可以通过实际对(1)式平方，并观察到我们有

$$t_1{}^2 = t_1, \quad t_2{}^2 = t_2, \quad t_1 t_2 = 0, \text{ &c.}$$

由成分的性质。现在从(1)中减去(2)，我们有

$$\left(a_1 - a_1{}^2\right) t_1 + \left(a_2 - a_2{}^2\right) t_2 = 0.$$

Or,

$$a_1 (1 - a_1) t_1 + a_2 (1 - a_2) t_2 = 0.$$

Multiply the last equation by t_1; then since $t_1 t_2 = 0$, we have

$$a_1 (1 - a_1) t_1 = 0, \text{ whence } t_2 = 0.$$

以同样方式将同一方程乘以t ^{2}，我们得到

$$a_2 (1 - a_2) t_2 = 0, \text{ whence } t_2 = 0.$$

因此可以一般地表明，任何系数不服从与记号本身相同的基本规律的构成项，都必须单独等于0。这种系数出现的通常形式是$\frac{1}{0}$。这是无穷大的代数记号。现在，任何数越接近无穷大(如果允许这种表达)，它就越偏离满足上述基本规律的条件。

记号$\frac{0}{0}$,其解释曾先前讨论过，并不必然违背我们在此考虑的规律，因为它同等地容许数值0和1。然而，其作为不定类记号的真正解释，我认为，除非依据类比，不能从其算术性质推出，而必须由经验确立。

12. 我们现在可以将所得的结果汇集为以下摘要：

1st. The symbol 1, as the coefficient of a term in a development, indicates that the whole of the class which that constituent represents, is to be taken.

2nd. The coefficient 0 indicates that none of the class are to be taken.

3rd. The symbol $\frac{0}{0}$ indicates that a perfectly *indefinite* portion of the class, i.e. *some*, *none*, or *all* of its members are to be taken.

4th. Any other symbol as a coefficient indicates that the constituent to which it is prefixed must be equated to 0.

It follows hence that if the solution of a problem, obtained by development, be of the form

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D,$$

that solution may be resolved into the two following equations, viz.,

$$w = A + vC, \tag{3}$$
$$D = O, \tag{4}$$

v being an indefinite class symbol. The interpretation of (3) shows what elements enter, or may enter, into the composition of w, the class of things whose definition is required; and the interpretation of (4) shows what relations exist among the elements of the original problem, in perfect independence of w.

Such are the canons of interpretation. It may be added, that they are universal in their application, and that their use is always unembarrassed by exception or failure.

13. *Corollary.*–If V be an independently interpretable logical function, it will satisfy the symbolical law, $V(1 - V) = 0$.

By an independently interpretable logical function, I mean one which is interpretable, without presupposing any relation among the things represented by the symbols which it involves. Thus $x(1 - y)$ is independently interpretable, but $x - y$ is not so. The latter function presupposes, as a condition of its interpretation, that the class represented by y is wholly contained in the class represented by x; the former function does not imply any such requirement.

Now if V be independently interpretable, and if w represent the collection of individuals which it contains, the equation $w = V$ will hold true without entailing as a consequence the vanishing of any of the constituents in the development of V; since such vanishing of constituents would imply relations among the classes of things denoted by the symbols in V. Hence the development of V will be of the form

$$a_1 t_1 + a_2 t_2 + \&c.$$

the coefficients a_1, a_2, &c. all satisfying the condition

$$a_1(1 - a_1) = 0, a_2(1 - a_2) = 0, \&c.$$

Hence by the reasoning of Prop. 4, Chap. v. the function V will be subject to the law

$$V(1 - V) = 0.$$

第一，记号1作为展开式中某一项的系数，表示该成分所代表的整个类都应被采取。

第二，系数0表示该类中没有一个应被选取。

第三。记号00表示该类完全不确定的部分,即该类的成员中，取某些、没有或全部。第四。任何其他记号作为系数，表示以它作前缀的构成部分必须等于 0。

It follows hence that if the solution of a problem, obtained by development, be of the form

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D,$$

该解可分解为以下两个方程，即，

$$w = A + vC, \tag{3}$$
$$D = O, \tag{4}$$

v是一个不确定的类记号。(3)的解释表明哪些元素进入或可能进入w的构成，即需要定义其定义的那个事物类；而(4)的解释表明原问题的元素之间存在哪些关系，完全独立于w。

这些就是解释的准则。可以补充说，它们在应用中是普遍的，并且它们的使用总是不会因例外或失败而受到妨碍。

13. 系。--若V是一个可独立解释的逻辑函数，它将满足符号规律，V(1 – V) = 0。所谓可独立解释的逻辑函数，我指的是无需预设其所含记号所代表的事物之间的任何关系即可解释的函数。因此x(1 – y)是可独立解释的，但x – y则不是。后一函数作为其解释的条件预设了：由y所代表的类完全包含于由x所代表的类之中；前一函数不蕴含任何此类要求。

现在，如果V可独立解释，并且如果w表示它所包含的个体的集合，那么方程w = V将成立，而不会导致V的展开式中任何成分消失；因为这种成分的消失将意味着V中记号所表示的事物类之间的关系。因此，V的展开式将具有以下形式

$$a_1 t_1 + a_2 t_2 + \&c.$$

系数a ^{1}、a ^{2}等等，全都满足条件

$$a_1(1 - a_1) = 0, a_2(1 - a_2) = 0, \&c.$$

Hence by the reasoning of Prop. 4, Chap. v. the function V will be subject to the law

$$V(1 - V) = 0.$$

This result, though evident *à priori* from the fact that V is supposed to represent a class or collection of things, is thus seen to follow also from the properties of the constituents of which it is composed. The condition $V(1-V) = 0$ may be termed "the condition of interpretability of logical functions."

14. The general form of solutions, or logical conclusions developed in the last Proposition, may be designated as a "Relation between terms." I use, as before, the word "terms" to denote the parts of a proposition, whether simple or complex, which are connected by the copula "is" or "are." The classes of things represented by the individual symbols may be called the elements of the proposition.

15. Ex. 1.–Resuming the definition of "clean beasts," (VI.6), required a description of "unclean beasts."

Here, as before, x standing for " clean beasts," y for "beasts dividing the hoof," z for "beasts chewing the cud," we have

$$x = yz; \tag{5}$$

whence

$$1 - x = 1 - yz;$$

and developing the second member,

$$1 - x = y(1 - z) + z(1 - y) + (1 - y)(1 - z);$$

which is interpretable into the following Proposition: *Unclean beasts are all which divide the hoof without chewing the cud, all which chew the cud without dividing the hoof, and all which neither divide the hoof nor chew the cud.*

Ex. 2.–The same definition being given, required a description of beasts which do not divide the hoof.

From the equation $x = yz$ we have

$$y = \frac{x}{z};$$

therefore,

$$1 - y = \frac{z - x}{z};$$

and developing the second member,

$$1 - y = 0\,xz + \frac{-1}{0}x(1 - z) + (1 - x)z + \frac{0}{0}(1 - x)(1 - z).$$

s Here, according to the Rule, the term whose coefficients is $\frac{-1}{0}$, must be separately equated to 0, whence we have

$$1 - y = (1 - x)z + v(1 - x)(1 - z),$$
$$x(1 - z) = 0;$$

whereof the first equation gives by interpretation the Proposition: *Beasts which do not divide the hoof consist of all unclean beasts which chew the cud, and an*

这个结果虽然从V被假定为表示一个类或事物的集合这一事实来看是`先验地明显的，但由此也可见它同样是从构成它的成分的性质中得出的。条件V(1 – V) = 0可以被称为"逻辑函数的可解释性条件"。

14. 在上一命题中展开的解或逻辑结论的一般形式，可以被称为"项之间的关系"。我像以前一样，使用"项"这个词来表示命题中由系词"是"或"是"连接的部分，无论是简单的还是复杂的。由各个记号所表示的事物类可以称为命题的元素。

15. 例1--重新采用"洁净兽"(VI.6)的定义，要求描述"不洁净兽"。

这里，如前所述，x代表"洁净兽",y代表"分蹄的兽",z代表"反刍的兽"，我们有

$$x = yz; \tag{5}$$

whence

$$1 - x = 1 - yz;$$

and developing the second member,

$$1 - x = y(1 - z) + z(1 - y) + (1 - y)(1 - z);$$

这可以解释为以下命题：不洁净兽是所有分蹄而不反刍的兽、所有反刍而不分蹄的兽，以及所有既不分蹄也不反刍的兽。

例2--给出同样的定义，要求描述不分蹄的兽。

由方程 x = yz 可得

$$y = \frac{x}{z};$$

therefore,

$$1 - y = \frac{z - x}{z};$$

and developing the second member,

$$1 - y = 0\,xz + \frac{-1}{0}x(1 - z) + (1 - x)z + \frac{0}{0}(1 - x)(1 - z).$$

这里，根据规则，系数为– 10的项，必须被分别等于0，由此我们有 —

$$1 - y = (1 - x)z + v(1 - x)(1 - z),$$
$$x(1 - z) = 0;$$

其中第一个方程通过解释给出命题：不分蹄的兽由所有反刍的不洁净兽，以及一个

indefinite remainder (some, none, or all) of unclean beasts which do not chew the cud.

The second equation gives the Proposition: *There are no clean beasts which do not chew the cud.* This is one of the independent relations above referred to. We sought the direct relation of "Beasts not dividing the hoof," to "Clean beasts and beasts which chew the cud." It happens, however, that independently of any relation to beasts not dividing the hoof, there exists, in virtue of the premiss, a separate relation between clean beasts and beasts which chew the cud. This relation is also necessarily given by the process.

Ex. 3.–Let us take the following definition, viz.: "Responsible beings are all rational beings who are either free to act, or have voluntarily sacrificed their freedom," and apply to it the preceding analysis.

Let	x	stand for	responsible beings.
	y	"	rational beings.
	z	"	those who are free to act,
	w	"	those who have voluntarily sacrificed their freedom of action.

In the expression of this definition I shall assume, that the two alternatives which it presents, viz.: "Rational beings free to act," and "Rational beings whose freedom of action has been voluntarily sacrificed," are mutually exclusive, so that no individuals are found at once in both these divisions. This will permit us to interpret the proposition literally into the language of symbols, as follows:

$$x = yz + yw. \tag{6}$$

Let us first determine hence the relation of "rational beings" to responsible beings, beings free to act, and beings whose freedom of action has been voluntarily abjured. Perhaps this object will be better stated by saying, that we desire to express the relation among the elements of the premiss in such a form as will enable us to determine how far rationality may be inferred from responsibility, freedom of action, a voluntary sacrifice of freedom, and their contraries.

From (6) we have

$$y = \frac{x}{(z+w)},$$

and developing the second member, but rejecting terms whose coefficients are 0,

$$y = \frac{1}{2}xzw + xz(1-w) + x(1-z)w + \frac{1}{0}x(1-z)(1-w) + \frac{0}{0}(1-x)(1-z)(1-w),$$

whence, equating to 0 the terms whose coefficients are $\frac{1}{2}$ and $\frac{1}{0}$, we have

不确定的剩余部分(某些、没有或全部)不反刍的不洁净兽组成。

第二个方程给出命题：没有不反刍的洁净兽。这是上文所提到的独立关系之一。我们寻求的是"不分开蹄的兽"与"洁净兽和反刍的兽"之间的直接关系。然而，恰好独立于与不分开蹄的兽的任何关系，凭借前提，在洁净兽与反刍的兽之间存在着一种单独的关系。这一关系也必然由该过程给出。

例3--让我们采用以下定义，即："负责任的存在者是所有要么有行动自由、要么自愿牺牲其自由的有理性的存在者"，并对它应用前面的分析。

设代表有责任的存在者。

y	?	有理性的存在者。
z	?	那些有行动自由的人，那些自愿牺牲其行动自由的
w	?	人。

在这个定义的表述中，我将假定它所呈现的两个选项，即："有行动自由的有理性的存在者"和"其行动自由已被自愿牺牲的有理性的存在者"，是相互排斥的，因此没有个体同时属于这两个部分。这将允许我们把这个命题逐字地解释为记号的语言，如下：

$$x = yz + yw. \tag{6}$$

让我们首先由此确定"有理性的存在者"与负责任的存在者、有行动自由的存在者以及其行动自由已被自愿放弃的存在者之间的关系。也许更好的说法是，我们希望以这样一种形式表达前提各元素之间的关系，使我们能够确定从负责任、行动自由、自愿牺牲自由及其反面可以推出多少理性。

由(6)我们得到

$$y = \frac{x}{(z+w)},$$

并展开第二项，但舍弃那些系数为为0，

$$y = \frac{1}{2}xzw + xz(1-w) + x(1-z)w + \frac{1}{0}x(1-z)(1-w) + \frac{0}{0}(1-x)(1-z)(1-w),$$

由此，令系数为12和10的项等于0,我们有

$$y = xz(1-w) + xw(1-z) + v(1-x)(1-z)(1-w); \tag{7}$$

$$xzw = 0; \tag{8}$$

$$x(1-z)(1-w) = 0; \tag{9}$$

whence by interpretation—

DIRECT CONCLUSION.—*Rational beings are all responsible beings who are either free to act, not having voluntarily sacrificed their freedom, or not free to act, having voluntarily sacrificed their freedom, together with an indefinite remainder (some, none, or all) of beings not responsible, not free, and not having voluntarily sacrificed their freedom.*

FIRST INDEPENDENT RELATION.—*No responsible beings are at the same time free to act, and in the condition of having voluntarily sacrificed their freedom.*

SECOND.–*No responsible beings are not free to act, and at the same time in the condition of not having sacrificed their freedom.*

The independent relations above determined may, however, be put in another and more convenient form. Thus (8) gives

$$xw = \frac{0}{z} = 0z + \frac{0}{0}(1-z), \text{ on development;}$$

or,

$$xw = v(1-z); \tag{10}$$

and in like manner (9) gives

$$x(1-w) = \frac{0}{1-z} = \frac{0}{0}z + 0(1-z);$$

or,

$$x(1-w) = vz; \tag{11}$$

and (10) and (11) interpreted give the following Propositions:

1st. *Responsible beings who have voluntarily sacrificed their freedom are not free.*

2nd. *Responsible beings who have not voluntarily sacrificed their freedom are free.*

These, however, are merely different forms of the relations before determined.

16. In examining, these results, the reader must bear in mind, that the sole province of a method of inference or analysis, is to determine those relations which are necessitated by the *connexion* of the terms in the original proposition. Accordingly, in estimating the completeness with which this object is effected, we have nothing whatever to do with those other relations which may be suggested to our minds by the *meaning* of the terms employed, as distinct from their expressed connexion. Thus it seems obvious to remark, that "They who

$$y = xz(1-w) + xw(1-z) + v(1-x)(1-z)(1-w); \tag{7}$$

$$xzw = 0; \tag{8}$$

$$x(1-z)(1-w) = 0; \tag{9}$$

由此通过解释--

直接结论--有理性的存在者是所有要么有行动自由而未自愿牺牲其自由、要么没有行动自由而自愿牺牲了其自由的负责任的存在者，连同不负责任、没有自由且未自愿牺牲其自由的存在者的一个不确定的剩余部分(某些、没有或全部)。

第一个独立关系--没有负责任的存在者同时有行动自由并处于自愿牺牲其自由的状态。

第二个--没有负责任的存在者没有行动自由并同时处于然而，上面确定的独立关系可以用另一种

未曾牺牲其自由的状态。

而更为便利的形式。因此(8)给出

$$xw = \frac{0}{z} = 0z + \frac{0}{0}(1-z), \text{ on development;}$$

or,

$$xw = v(1-z); \tag{10}$$

并以同样的方式(9)给出

$$x(1-w) = \frac{0}{1-z} = \frac{0}{0}z + 0(1-z);$$

or,

$$x(1-w) = vz; \tag{11}$$

and (10) and (11) interpreted give the following Propositions:

1st. *Responsible beings who have voluntarily sacrificed their freedom are not free.*

2nd. *Responsible beings who have not voluntarily sacrificed their freedom are free.*

然而，这些仅仅是先前所确定的关系的不同形式。16. 在考察这些结果时，读者必须牢记，推论或分析方法的唯一职责，是确定那些由原命题中各项的联系所必然导致的关系。因此，在评估这一目标实现得是否完备时，我们与那些可能由所用各项的含义--有别于其表达出的联系--在我们心灵中提示的其他关系毫无干系。于是，似乎显而易见地可以指出，"那些

have voluntarily sacrificed their freedom are not free," this being a relation implied in the very meaning of the terms. And hence it might appear, that the first of the two independent relations assigned by the method is on the one hand needlessly limited, and on the other hand superfluous. However, if regard be had merely to the connexion of the terms in the original premiss, it will be seen that the relation in question is not liable to either of these charges. The solution, as expressed in the direct conclusion and the independent relations, conjointly, is perfectly complete, without being in any way superfluous.

If we wish to take into account the implicit relation above referred to, viz., "They who have voluntarily sacrificed their freedom are not free," we can do so by making this a distinct proposition, the proper expression of which would be

$$w = v(1 - z).$$

This equation we should have to employ together with that expressive of the original premiss. The mode in which such an examination must be conducted will appear when we enter upon the theory of systems of propositions in a future chapter. The sole difference of result to which the analysis leads is, that the first of the independent relations deduced above is superseded.

17. Ex. 4. – Assuming the same definition as in Example 2, let it be required to obtain a description of irrational persons.

We have

$$\begin{aligned}
1 - y &= 1 - \frac{x}{z + w} \\
&= \frac{z + w - x}{z + w} \\
&= \frac{1}{2}xzw + 0xz(1 - w) + 0x(1 - z)w - \frac{1}{0}x(1 - z)(1 - w) \\
&+ (1 - x)zw + (1 - x)z(1 - w) + (1 - x)(1 - z)w + \frac{0}{0}(1 - x)(1 - z)(1 - w) \\
&= (1 - x)zw + (1 - x)z(1 - w) + (1 - x)(1 - z)w + v(1 - x)(1 - z)(1 - w) \\
&= (1 - x)z + (1 - x)(1 - z)w + v(1 - x)(1 - z)(1 - w),
\end{aligned}$$

with $xzw = 0, \quad x(1 - z)(1 - w) = 0.$

The independent relations here given are the same as we before arrived at, as they evidently ought to be, since whatever relations prevail independently of the existence of a given class of objects y, prevail independently also of the existence of the contrary class $1 - y$.

The direct solution afforded by the first equation is:–*Irrational persons consist of all irresponsible beings who are either free to act, or have voluntarily sacrificed their liberty, and are not free to act; together with an indefinite remainder of irresponsible beings who have not sacrificed their liberty, and are not free to act.*

18. The propositions analyzed in this chapter have been of that species called definitions. I have discussed none of which the second or predicate term

自愿牺牲其自由的人并不自由"，这是项的含义本身所蕴含的一种关系。因此，可能看来，该方法所指定的两个独立关系中的第一个，一方面是不必要地受限的，另一方面又是多余的。然而，若仅顾及原前提中各项的联系，就会看出所论及的关系并不应受这两种指责中的任何一种。该解答，如直接结论与独立关系合起来所表述的，是完全完备的，而绝非多余。

如果我们希望考虑上面提到的隐含关系，即"那些自愿牺牲其自由的人没有自由"，我们可以通过把这作为一个独立的命题来做到，其恰当表述将是

$$w = v(1 - z).$$

我们将不得不把这个方程与表达原始前提的方程一起使用。这种考察必须进行的方式将在未来一章中我们进入命题系统理论时出现。分析所得结果的唯一区别是，上面推导出的第一个独立关系被取代。

17. Ex. 4. – Assuming the same definition as in Example 2, let it be required to obtain a description of irrational persons.

We have

$$\begin{aligned}
1 - y &= 1 - \frac{x}{z + w} \\
&= \frac{z + w - x}{z + w} \\
&= \frac{1}{2}xzw + 0xz(1 - w) + 0x(1 - z)w - \frac{1}{0}x(1 - z)(1 - w)
\end{aligned}$$

+(1 – x) zw +(1 – x) z(1 – w) +(1 – x)(1 – z) w + $\frac{0}{0}$ $^{(1-x)(1=z)(1-w)}$ =(1 – x) zw +(1 – x) z(1 – w) +(1 – x)(1 – z) w + v(1 – x)(1 – z)(1 – w) =(1 – x) z +(1 – x)(1 – z) w + v(1 – x)(1 – z)(1 – w) ， 其中 xzw = 0 , x(1 – z)(1 – w) = 0。此处给出的独立关系与我们先前所得相同，正如它们显然应当相同，因为凡独立于某一给定对象类 y 的存在而成立的关系，也独立于相反类 1 – y 的存在而成立。

第一个方程所提供的直接解是：--非理性的人由所有要么有行动自由、要么自愿牺牲其自由而没有行动自由的不负责任的存在者，连同未牺牲其自由且没有行动自由的不负责任的存在者的一个不确定的剩余部分构成。

18. 本章所分析的命题属于被称为定义的那一种。我没有讨论任何一个其第二项或谓词项

is particular, and of which the general type is $Y = vX$, Y and X being functions of the logical symbols x, y, z, &c., and v an indefinite class symbol. The analysis of such propositions is greatly facilitated (though the step is not an essential one) by the elimination of the symbol v, and this process depends upon the method of the next chapter. I postpone also the consideration of another important problem necessary to complete the theory of single propositions, but of which the analysis really depends upon the method of the reduction of systems of propositions to be developed in a future page of this work.

是特称的，其一般形式为 Y = vX，其中 Y 与 X 是逻辑记号 x、y、z 等的函数，而 v 是一个不定类记号。此类命题的分析因消去记号 v 而大为便利(尽管这一步并非本质性的)，而这一过程依赖于下一章的方法。我还推迟考虑另一个重要问题，该问题对于完成单一命题的理论是必要的，但其分析实际上依赖于将命题系统化归的方法，而这一方法将在本书后面的篇幅中展开。

Chapter VII

ON ELIMINATION.

1. In the examples discussed in the last chapter, all the elements of the original premiss re-appeared in the conclusion, only in a different order, and with a different connexion. But it more usually happens in common reasoning, and especially when we have more than one premiss, that some of the elements are required not to appear in the conclusion. Such elements, or, as they are commonly called, "middle terms," may be considered as introduced into the original propositions only for the sake of that connexion which they assist to establish among the other elements, which are alone designed to enter into the expression of the conclusion.

2. Respecting such intermediate elements, or middle terms, some erroneous notions prevail. It is a general opinion, to which, however, the examples contained in the last chapter furnish a contradiction, that inference consists peculiarly in the elimination of such terms, and that the elementary type of this process is exhibited in the elimination of one middle term from two premises, so as to produce a single resulting conclusion into which that term does not enter. Hence it is commonly held, that *syllogism* is the basis, or else the common type, of all inference, which may thus, however complex its form and structure, be resolved into a series of syllogisms. The propriety of this view will be considered in a subsequent chapter. At present I wish to direct attention to an important, but hitherto unnoticed, point of difference between the system of Logic, as expressed by symbols, and that of common algebra, with reference to the subject of elimination. In the algebraic system we are able to eliminate one symbol from two equations, two symbols from three equations, and generally $n-1$ symbols from n equations. There thus exists a definite connexion between the number of independent equations given, and the number of symbols of quantity which it is possible to eliminate from them. But it is otherwise with the system of Logic. No fixed connexion there prevails between the number of equations given representing propositions or premises, and the number of typical symbols of which the elimination can be effected. From a single equation an indefinite number of such symbols may be eliminated. On the other hand, from an indefinite number of equations, a single class symbol only may be eliminated. We may affirm, that in this peculiar system, the problem

第七章

论消元法。

1. 在上一章讨论的例子中，原前提的所有元素都在结论中重新出现，只是顺序不同，连接方式也不同。但在通常的推理中，尤其是当我们有不止一个前提时，更常见的情况是某些元素被要求不出现在结论中。这样的元素，或通常所谓的"中项"，可以被视为仅仅为了帮助在其他元素之间建立那种连接而被引入原命题的，而那些其他元素才是被设计来进入结论的表达的。

2. 关于这样的中间元素或中项，流行着一些错误的观念。一种普遍的观点是--然而上一章所包含的例子却提供了反证--推论特别在于消去这样的项，并且这一过程的基本类型表现在从两个前提中消去一个中项，从而产生一个该中项不进入的单一结论。因此通常认为，三段论是一切推论的基础，或者是其共同类型，而推论无论其形式和结构多么复杂，都可以分解为一系列三段论。这一观点的恰当性将在后续一章中考虑。目前我希望提请注意逻辑体系(如用记号所表达的)与普通代数体系之间在消元问题上一个重要的、但迄今未被注意的差异点。在代数体系中，我们能够从两个方程中消去一个记号，从三个方程中消去两个记号，一般地，从n个方程中消去n – 1个记号。因此，在给定的独立方程数目与可能从中消去的量的记号的数目之间存在着确定的联系。但逻辑体系则不然。在表示命题或前提的给定方程的数目与能够被消去的典型记号的数目之间，不存在固定的联系。从一个单一方程中可以消去不定数目的这样的记号。另一方面，从不定数目的方程中，可能只消去一个类记号。我们可以断言，在这个独特的体系中，问题

of elimination is resolvable under all circumstances alike. This is a consequence of that remarkable law of duality to which the symbols of Logic are subject. To the equations furnished by the premises given, there is added another equation or system of equations drawn from the fundamental laws of thought itself, and supplying the necessary means for the solution of the problem in question. Of the many consequences which flow from the law of duality, this is perhaps the most deserving of attention.

3. As in Algebra it often happens, that the elimination of symbols from a given system of equations conducts to a mere identity in the form $0 = 0$, no independent relations connecting the symbols which remain; so in the system of Logic, a like result, admitting of a similar interpretation, may present itself. Such a circumstance does not detract from the generality of the principle before stated. The object of the method upon which we are about to enter is to eliminate any number of symbols from any number of logical equations, and to exhibit in the result the actual relations which remain. Now it may be, that no such residual relations exist. In such a case the truth of the method is shown by its leading us to a merely identical proposition.

4. The notation adopted in the following Propositions is similar to that of the last chapter. By $f(x)$ is meant any expression involving the logical symbol x, with or without other logical symbols. By $f(1)$ is meant what $f(x)$ becomes when x is therein changed into 1; by $f(0)$ what the same function becomes when x is changed into 0.

PROPOSITION I.

5. If $f(x) = 0$ be any logical equation involving the class symbol x, with or without other class symbols, then will the equation

$$f(1)f(0) = 0$$

be true, independently of the interpretation of x; and it will be the complete result of the elimination of x from the above equation.

In other words, the elimination of x from any given equation, $f(x) = 0$, will be effected by successively changing in that equation x into 1, and x into 0, and multiplying the two resulting equations together.

Similarly the complete result of the elimination of any class symbols, x, y, etc.,from any equation of the form $V = 0$, will be obtained by completely expanding the first member of that equation in constituents of the given symbols, and multiplying together all the coefficients of those constituents, and equating the product to 0.

Developing the first member of the equation $f(x) = 0$, we have (V. 10),

$$\begin{aligned} f(1)x + f(0)(1-x) &= 0; \\ \text{or, } [f(1) - f(0)]x + f(0) &= 0. \therefore x = \frac{f(0)}{f(0) - f(1)}; \\ \text{and } 1 - x &= -\frac{f(1)}{f(0) - f(1)}. \end{aligned}$$

的消元在所有情况下都同样可解。这是逻辑记号所服从的那种显著的对偶律的一个推论。在由给定前提所提供的方程之外，还添加了另一个方程或方程组，它来自思维规律本身，并为所讨论问题的解决提供了必要的手段。在对偶律所产生的许多推论中，这一点或许最值得注意。

3. 正如在代数中经常发生的那样，从给定的方程组中消去记号会导致仅仅形如0 = 0的恒等式，而不再有连接剩余记号的独立关系；同样，在逻辑体系中，也可能出现类似的结果，容许类似的解释。这种情况并不减损前述原则的普遍性。我们即将进入的方法，其目标是从任意数目的逻辑方程中消去任意数目的记号，并在结果中展示实际剩余的关系。然而，可能不存在这样的剩余关系。在这种情况下，方法的真理性通过引导我们得到一个仅仅同一的命题而显示出来。

4. 以下各命题中采用的记号与上一章类似。以f(x)表示任何含有逻辑记号x的表达式，无论是否含有其他逻辑记号。以f(1)表示当f(x)中的x变为1时该式所成为者；以f(0)表示同一函数当x变为0时所成为者。

命题一。

5. 若f(x)=0是任何含有类记号x的逻辑方程，无论是否含有其他类记号，则方程

$$f(1)f(0) = 0$$

为真，独立于x的解释；并且这将是上述方程中消去x的完整结果。换言之，从任意给定方程f(x) = 0中消去x，将通过在该方程中依次将x变为1、将x变为0，并将所得两个方程相乘来实现。类似地，从任意形如V = 0的方程中消去任意类记号x、y等的完整结果，将通过将该方程的第一项完全展开为给定记号的成分，并将所有这些成分的系数相乘，令乘积等于0来获得。

展开方程f(x)=0的第一端，我们有(V.10),

$$\begin{aligned} f(1)x + f(0)(1-x) &= 0; \\ \text{or, } [f(1) - f(0)]x + f(0) &= 0. \therefore x = \frac{f(0)}{f(0) - f(1)}; \\ \text{and } 1 - x &= -\frac{f(1)}{f(0) - f(1)}. \end{aligned}$$

Substitute these expressions for x and $1-x$ in the fundamental equation

$$x(1-x)=0,$$

and there results

$$-\frac{f(0)f(1)}{[f(0)-f(1)]^2}=0;$$
$$\text{or, } f(1)f(0)=0, \tag{1}$$

the form required.

6. It is seen in this process, that the elimination is really effected between the given equation $f(x)=0$ and the universally true equation $x(1-x)=0$, expressing the fundamental law of logical symbols, *qua* logical. There exists, therefore, no need of more than one premiss or equation, in order to render possible the elimination of a term, the necessary law of thought virtually supplying the other premiss or equation. And though the demonstration of this conclusion may be exhibited in other forms, yet the same element furnished by the mind itself will still be virtually present. Thus we might proceed as follows:

Multiply (1) by x, and we have

$$f(1)x=0, \tag{2}$$

and let us seek by the forms of ordinary algebra to eliminate x from this equation and (1).

Now if we have two algebraic equations of the form

$$ax+b=0,$$

$$a'x+b'=0;$$

it is well known that the result of the elimination of x is

$$ab'-a'b=0 \tag{3}$$

But comparing the above pair of equations with (1) and (3) respectively, we find

$$a=f(1)-f(0),\quad b=f(0);$$
$$a'=f(1)\quad b'=0;$$

which, substituted in (4), give

$$f(1)f(0)=0,$$

as before. In this form of the demonstration, the fundamental equation $x(1-x)=0$, makes its appearance in the derivation of (3) from (1).

7. I shall add yet another form of the demonstration, partaking of a half logical character, and which may set the demonstration of this important theorem in a clearer light.

将这些表达式代入基本方程中的x和1–x,

$$x(1-x)=0,$$

并且由此得出结果

$$-\frac{f(0)f(1)}{[f(0)-f(1)]^2}=0;$$
$$\text{or, } f(1)f(0)=0, \tag{1}$$

所需的形式。

6. 在这一过程中可以看出，消元实际上是在给定方程 f(x) = 0 与全称真方程 x(1 – x) = 0 之间进行的，后者表达了逻辑记号作为逻辑记号的基本规律。因此，为了使一个项的消元成为可能，并不需要多于一个前提或方程，因为必要的思维规律实际上提供了另一个前提或方程。而且，尽管这一结论的证明可以用其他形式展示，但由心灵本身所提供的同一元素仍然实际上在场。于是我们可以如下进行：

将(1)乘以x，我们得到

$$f(1)x=0, \tag{2}$$

并让我们借助普通代数的形式，从该方程与(1)中消去x。

现在，如果我们有两个形式如下的代数方程

$$ax+b=0,$$

$$a'x+b'=0;$$

众所周知，消去x的结果是

$$ab'-a'b=0 \tag{3}$$

But comparing the above pair of equations with (1) and (3) respectively, we find

$$a=f(1)-f(0),\quad b=f(0);$$
$$a'=f(1)\quad b'=0;$$

将其代入(4)中，得到

$$f(1)f(0)=0,$$

如前。在这种论证形式中，基本方程x(1–x) = 0，出现在由(1)推导(3)的过程中。

7. 我将再补充另一种论证形式，它带有半逻辑的性质，或可使这一重要定理的论证更为清晰。

We have as before

$$f(1)x + f(0)(1-x) = 0.$$

Multiply this equation first by x, and secondly by $1-x$, we get

$$f(1)x = 0 \quad f(0)(1-x) = 0.$$

From these we have by solution and development,

$$f(1) = \frac{0}{x} = \frac{0}{0}(1-x), \text{ on development,}$$
$$f(0) = \frac{0}{1-x} = \frac{0}{0}x.$$

The direct interpretation of these equations is–

1st. Whatever individuals are included in the class represented by $f(1)$, are not x's.

2nd. Whatever individuals are included in the class represented by $f(0)$, are x's.

Whence by common logic, there are no individuals at once in the class $f(1)$ and in the class $f(0)$, i.e. there are no individuals in the class $f(1)f(0)$. Hence,

$$f(1)f(0) = 0. \tag{4}$$

Or it would suffice to multiply together the developed equations, whence the result would immediately follow.

8. The theorem (5) furnishes us with the following Rule :

TO ELIMINATE ANY SYMBOL FROM A PROPOSED EQUATION.

RULE.–*The terms of the equation having been brought, by transposition if necessary, to the first side, give to the symbol successively the values 1 and 0, and multiply the resulting equations together.*

The first part of the Proposition is now proved.

9. Consider in the next place the general equation

$$f(x, y) = 0;$$

the first member of which represents any function of x, y, and other symbols.

By what has been shown, the result of the elimination of y from this equation will be

$$f(x, 1)f(x, 0) = 0;$$

for such is the form to which we are conducted by successively changing in the given equation y into 1, and y into 0, and multiplying the results together.

Again, if in the result obtained we change successively x into 1, and x into 0, and multiply the results together, we have

$$f(1, 1)f(1, 0)f(0, 1)f(0, 0) = 0; \tag{5}$$

We have as before

$$f(1)x + f(0)(1-x) = 0.$$

Multiply this equation first by x, and secondly by $1-x$, we get

$$f(1)x = 0 \quad f(0)(1-x) = 0.$$

由这些，我们通过求解与展开，

$$f(1) = \frac{0}{x} = \frac{0}{0}(1-x), \text{ on development,}$$
$$f(0) = \frac{0}{1-x} = \frac{0}{0}x.$$

这些方程的直接解释是--

第一，凡包含于f(1)所表示的类中的个体，都不是x。

第二，凡包含于f(0)所表示的类中的个体，都是x。

因此，按普通逻辑，没有任何个体同时属于类f(1)和类f(0)，即没有任何个体属于类f(1)f(0)。因此，

$$f(1)f(0) = 0. \tag{4}$$

或者，只需将展开后的各方程相乘，结果便会立即得出。

8. 定理(5)为我们提供了以下规则：

从一给定方程中消去任何记号。规则--将方程的各项(必要时通过移项)移至第一端，依次给该记号以值*1*和*0*，并将所得各方程相乘。

该命题的第一部分现已得证。

9. 其次考虑一般方程

$$f(x, y) = 0;$$

the first member of which represents any function of x, y, and other symbols.

By what has been shown, the result of the elimination of y from this equation will be

$$f(x, 1)f(x, 0) = 0;$$

因为这就是我们通过在给定方程中依次将y变为1、将y变为0并将结果相乘所得到的形式。

再者，若在所得结果中依次将x变为1、将x变为0并将结果相乘，我们便得到

$$f(1, 1)f(1, 0)f(0, 1)f(0, 0) = 0; \tag{5}$$

as the final result of elimination. But the four factors of the first member of this equation are the four coefficients of the complete expansion of $f(x, y)$, the first member of the original equation; whence the second part of the Proposition is manifest.

EXAMPLES.

10. Ex. 1. – Having given the Proposition, "All men are mortal," and its symbolical expression, in the equation,

$$y = vx,$$

in which y represents "men," and x "mortals," it is required to eliminate the indefinite class symbol v, and to interpret the result.

Here bringing the terms to the first side, we have

$$y - vx = 0.$$

When $v = 1$ this becomes

$$y - x = 0;$$

and when $v = 0$ it becomes

$$y = 0;$$

and these two equations multiplied together, give

$$y - yx = 0,$$

or $y(1 - x) = 0$,
it being observed that $y^2 = y$.

The above equation is the required result of elimination, and its interpretation is, *Men who are not mortal do not exist*, – an obvious conclusion.

If from the equation last obtained we seek a description of beings who are not mortal, we have

$$x = \frac{y}{y},$$

$$\therefore 1 - x = \frac{0}{y}.$$

Whence, by expansion, $1 - x = \frac{0}{0}(1 - y)$, which interpreted gives, *They who are not mortal are not men.* This is an example of what in the common logic is called conversion by contraposition, or negative conversion. [1]

Ex. 2.–Taking the Proposition, "No men are perfect," as represented by the equation

$$y = v(1 - x),$$

[1] Whately's Logic, Book II. chap. II. sec. 4.

作为消元的最终结果。但该方程第一端的四个因子，就是原方程第一端f(x,y)的完全展开式的四个系数；由此本命题的第二部分便显而易见了。

示例。

10. 例1--已给命题"所有人都是有死的"及其在方程中的记号表达式，

$$y = vx,$$

其中y表示"人",x表示"有死的"，要求消去不定类记号v，并解释结果。

将各项移至第一边，我们得到

$$y - vx = 0.$$

当 $v = 1$ 时，这变为

$$y - x = 0;$$

而当 $v = 0$ 时，它变为

$$y = 0;$$

将这两个方程相乘，得到

$$y - yx = 0,$$

or $y(1 - x) = 0$,
it being observed that $y^2 = y$.

The above equation is the required result of elimination, and its interpretation is, *Men who are not mortal do not exist*, – an obvious conclusion.

If from the equation last obtained we seek a description of beings who are not mortal, we have

$$x = \frac{y}{y},$$

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Ex. 2.–Taking the Proposition, "No men are perfect," as represented by the equation

$$y = v(1 - x),$$

[1] Whately's Logic, Book II. chap. II. sec. 4.

wherein y represents "men," and x "perfect beings," it is required to eliminate v, and find from the result a description both of *perfect beings* and of *imperfect beings*. We have

$$y - v(1 - x) = 0.$$

Whence, by the rule of elimination,

$$\{y - (1 - x)\} \times y = 0,$$

or

$$y - y(1 - x) = 0,$$

or

$$yx = 0;$$

which is interpreted by the Proposition, *Perfect men do not exist.* From the above equation we have

$$x = \frac{0}{y} = \frac{0}{0}(1 - y) \text{ by development;}$$

whence, by interpretation, *No perfect beings are men.* Similarly,

$$1 - x = 1 - \frac{0}{y} = \frac{y}{y} = y + \frac{0}{0}(1 - y),$$

which, on interpretation, gives, *Imperfect beings are all men with an indefinite remainder of beings, which are not men.*

11. It will generally be the most convenient course, in the treatment of propositions, to eliminate first the indefinite class symbol v, wherever it occurs in the corresponding equations. This will only modify their form, without impairing their significance. Let us apply this process to one of the examples of Chap. IV. For the Proposition, "No men are placed in exalted stations and free from envious regards," we found the expression

$$y = v(1 - xz),$$

and for the equivalent Proposition, "Men in exalted stations are not free from envious regards," the expression

$$yx = v(1 - z);$$

and it was observed that these equations, v being an indefinite class symbol, were themselves equivalent. To prove this, it is only necessary to eliminate from each the symbol v. The first equation is

$$y - v(1 - xz) = 0,$$

whence, first making $v = 1$, and then $v = 0$, and multiplying the results, we have

其中y表示"人",x表示"完善的存在者"，要求消去v，并从结果中得出对完善的存在者以及对不完善的存在者的描述。我们有

$$y - v(1 - x) = 0.$$

Whence, by the rule of elimination,

$$\{y - (1 - x)\} \times y = 0,$$

或

$$y - y(1 - x) = 0,$$

或

$$yx = 0;$$

它被解释为命题：完善的人不存在。由上述方程我们有

$$x = \frac{0}{y} = \frac{0}{0}(1 - y) \text{ by development;}$$

由此，通过解释，没有完美的人是凡人。同样，

$$1 - x = 1 - \frac{v}{y} = \frac{y}{y} = y + \frac{v}{0}(1 - y),$$

对其加以解释，便得出：不完善的存在者就是全部的人，外加一个不定剩余的非人的存在者。

11. 在处理命题时，通常最方便的做法是，首先消去不定类记号v，无论它出现在相应方程的何处。这只会改变方程的形式，而不损害其意义。让我们将此过程应用于第四章的一个例子。对于命题"没有既处于显赫地位又免于嫉妒之念的人"，我们得到表达式

$$y = v(1 - xz),$$

而对于等价的命题"处于显赫地位的人不免于嫉妒之念"，得到表达式

$$yx = v(1 - z);$$

并且已经指出，由于v是不定类记号，这些方程本身是等价的。为证明这一点，只需从每个方程中消去记号v。第一个方程是

$$y - v(1 - xz) = 0,$$

因此，先令v=1，再令v=0，并将结果相乘，我们便得到

$$(y - 1 + xz)y = 0,$$
$$\text{or } yxz = 0.$$

Now the second of the given equations becomes on transposition

$$yx - v(1 - z) - 0;$$
$$\text{whence } (x - 1 + z)yx = 0,$$
$$\text{or } yxz = 0,$$

as before. The reader will easily interpret the result.

12. Ex. 3.–As a subject for the general method of this chapter, we will resume Mr. Senior's definition of wealth, viz.: "Wealth consists of things transferable, limited in supply, and either productive of pleasure or preventive of pain." We shall consider this definition, agreeably to a former remark, as including all things which possess at once both the qualities expressed in the last part of the definition, upon which assumption we have, as our representative equation,

$$w = st\{pr + p(1 - r) + r(1 - p)\},$$
$$\text{or } w = st\{p + r(1 - p)\},$$

wherein

w	stands for	wealth.
s	"	things limited in supply.
t	"	things transferable.
p	"	things productive of pleasure.
r	"	things preventive of pain.

From the above equation we can eliminate any symbols that we do not desire to take into account, and express the result by solution and development, according to any proposed arrangement of subject and predicate.

Let us first consider what the expression for w, wealth, would be if the element r, referring to prevention of pain, were eliminated. Now bringing the terms of the equation to the first side, we get

$$w - st(p + r - rp) = 0.$$

Making $r = 1$, the first member becomes $w - st$, and making $r = 0$ it becomes $w - stp$; whence we have by the Rule,

$$(w - st)(w - stp) = 0 \tag{6}$$

or

$$(y - 1 + xz)y = 0,$$
$$\text{or } yxz = 0.$$

现在，给定方程中的第二个在移项后变为

$$yx - v(1 - z) - 0;$$
$$\text{whence } (x - 1 + z)yx = 0,$$
$$\text{or } yxz = 0,$$

和以前一样。读者将很容易解释这个结果。

12. 例3--作为本章一般方法的主题，我们将重提西尼尔先生对财富的定义，即："财富由可转移的、供给有限的、并且或者产生快乐或者防止痛苦的事物所构成。"依照先前的评述，我们将这一定义视为包含一切同时具有该定义最后一部分所表达的两项性质的事物，在此假定下，我们得到作为其代表方程的

$$w = st\{pr + p(1 - r) + r(1 - p)\},$$
$$\text{or } w = st\{p + r(1 - p)\},$$

其中

w代表财富。s"供给有限的事物。t"可转移的事物。
p"产生快乐的事物。r"防止痛苦的事物。

由上述方程，我们可以消去任何我们不想考虑的记号，并按照任何给定的主词与谓词的安排，通过求解与展开来表达结果。

让我们首先考虑，如果消去涉及防止痛苦的元素r,w即财富的表达式会是什么。现在将方程的各项移至第一端，我们得到

$$w - st(p + r - rp) = 0.$$

Making $r = 1$, the first member becomes $w - st$, and making $r = 0$ it becomes $w - stp$; whence we have by the Rule,

$$(w - st)(w - stp) = 0 \tag{6}$$

或

$$w - wstp - wst + stp = 0; \tag{7}$$

whence

$$w = \frac{stp}{st + stp - 1};$$

the development of the second member of which equation gives

$$w = stp + \frac{0}{0}st(1-p). \tag{8}$$

Whence we have the conclusion,–*Wealth consists of all things limited in supply, transferable, and productive of pleasure, and an indefinite remainder of things limited in supply, transferable, and not productive of pleasure.* This is sufficiently obvious.

Let it be remarked that it is not necessary to perform the multiplication indicated in (7), and reduce that equation to the form (8), in order to determine the expression of w in terms of the other symbols. The process of development may in all cases be made to supersede that of multiplication. Thus if we develop (7) in terms of w, we find

$$(1 - sf)(1 - stp)w + stp(1 - w) = 0,$$

whence

$$w = \frac{stp}{stp - (1 - st)(1 - stp)};$$

and this equation developed will give, as before,

$$w = stp + \frac{0}{0}st(1-p).$$

13. Suppose next that we seek a description of things limited in supply, as dependent upon their relation to wealth, transferableness, and tendency to produce pleasure, omitting all reference to the prevention of pain.

From equation (8), which is the result of the elimination of r from the original equation, we have

$$w - s\,(wt + wtp - tp) = 0;$$

whence

$$\begin{aligned} s = \frac{w}{wt + wtp - tp} \\ = wtp + wt\,(1-p) + \frac{1}{0}w\,(1-t)\,p + \frac{1}{0}w\,(1-t)\,(1-p) \\ +0\,(1-w)\,tp + \frac{0}{0}\,(1-w)\,t\,(1-p) + \frac{0}{0}\,(1-w)\,(1-t)\,p \\ +\frac{0}{0}\,(1-w)\,(1-t)\,(1-p)\,. \end{aligned}$$

$$w - wstp - wst + stp = 0; \tag{7}$$

由此

$$w = \frac{stp}{st + stp - 1};$$

the development of the second member of which equation gives

$$w = stp + \frac{\breve{0}}{0}st(1-p). \tag{8}$$

由此我们得出如下结论--财富由一切供给有限的、可转移的、产生快乐的事物，外加一个不定剩余的供给有限的、可转移的、不产生快乐的事物所构成。这是足够明显的。

需要指出的是，为了确定 w 用其他记号表示的表达式，不必执行(7)中所示的乘法并将该方程化为(8)的形式。展开的过程在所有情况下都可以取代乘法的过程。因此，如果我们按 w 展开(7)，我们得到

$$(1 - sf)(1 - stp)w + stp(1 - w) = 0,$$

由此

$$w = \frac{stp}{stp - (1 - st)(1 - stp)};$$

而所推导出的这一方程将如前所述给出，

$$w = stp + \frac{\breve{0}}{0}st(1-p).$$

13. 其次，假设我们寻求对供应有限之物的描述，将其视为依赖于它们与财富、可转移性以及产生快乐之倾向的关系，而省略一切关于防止痛苦的提及。

From equation (8), which is the result of the elimination of r from the original equation, we have

$$w - s\,(wt + wtp - tp) = 0;$$

whence

$$\begin{aligned} s = \frac{w}{wt + wtp - tp} \\ = wtp + wt\,(1-p) + \frac{1}{0}w\,(1-t)\,p + \frac{1}{0}w\,(1-t)\,(1-p) \\ +0\,(1-w)\,tp + \frac{0}{0}\,(1-w)\,t\,(1-p) + \frac{0}{0}\,(1-w)\,(1-t)\,p \\ +\frac{0}{0}\,(1-w)\,(1-t)\,(1-p)\,. \end{aligned}$$

We will first give the direct interpretation of the above solution, term by term; afterwards we shall offer some general remarks which it suggests; and, finally, show how the expression of the conclusion may be somewhat abbreviated.

First, then, the direct interpretation is, Things limited in supply consist of *All wealth transferable and productive of pleasure–all wealth transferable, and not productive of pleasure,–an indefinite amount of what is not wealth, but is either transferable, and not productive of pleasure, or intransferable and productive of pleasure, or neither transferable nor productive of pleasure.*

To which the terms whose coefficients are $\frac{1}{0}$ permit us to add the following independent relations, viz.:

1st. *Wealth that is intransferable, and productive of pleasure, does not exist.*

2ndly. *Wealth that is intransferable, and not productive of pleasure, does not exist.*

14. Respecting this solution I suppose the following remarks are likely to be made.

First, it may be said, that in the expression above obtained for "things limited in supply," the term "All wealth transferable," &c., is in part redundant; since all wealth is (as implied in the original proposition, and directly asserted in the *independent relations*) necessarily transferable.

I answer, that although in ordinary speech we should not deem it necessary to add to "wealth" the epithet "transferable," if another part of our reasoning had led us to express the conclusion, that there is no wealth which is not transferable, yet it pertains to the perfection of this method that it in all cases fully defines the objects represented by each term of the conclusion, by stating the relation they bear to each quality or element of distinction that we have chosen to employ. This is necessary in order to keep the different parts of the solution really distinct and independent, and actually prevents redundancy. Suppose that the pair of terms we have been considering had not contained the word "transferable," and had unitedly been "All wealth," we could then logically resolve the single term "All wealth" into the two terms "All wealth transferable," and "All wealth intransferable." But the latter term is shown to disappear by the "independent relations." Hence it forms no part of the description required, and is therefore redundant. The remaining term agrees with the conclusion actually obtained.

Solutions in which there cannot, by logical divisions, be produced any superfluous or redundant terms, may be termed *pure solutions*. Such are all the solutions obtained by the method of development and elimination above explained. It is proper to notice, that if the common algebraic method of elimination were adopted in the cases in which that method is possible in the present system, we should not be able to depend upon the purity of the solutions obtained. Its want of generality would not be its only defect.

15. In the second place, it will be remarked, that the conclusion contains two terms, the aggregate significance of which would be more conveniently expressed by a single term. Instead of "All wealth productive of pleasure, and transferable," and "All wealth not productive of pleasure, and transferable," we might simply say, "All wealth transferable." This remark is quite just. But it must be noticed that whenever any such simplifications are possible, they are

我们将首先逐项给出上述解的直接解释；之后我们将提出它所暗示的一些一般性评论；最后，展示结论的表达式如何可以稍作简略。

那么，首先，直接解释是：供应有限之物由以下构成：全部可转移且产生快乐的财富--全部可转移但不产生快乐的财富--以及不定量的既非财富、但或者可转移而不产生快乐、或者不可转移而产生快乐、或者既不可转移也不产生快乐的东西。

对此，系数为$\frac{1}{0}$的项允许我们添加以下独立的关系，即：

1. 不可转让且能产生快乐的财富不存在。
2. 不可转让且不能产生快乐的财富不存在。

14. 关于这个解，我想以下评论很可能会被提出。

首先，可能会说，在上面得到的"供应有限之物"的表达式中，"全部可转移的财富"等项部分是多余的；因为一切财富(如原命题所隐含、并在独立关系中直接断定的那样)必然是可转移的。

我回答说，尽管在通常的言说中，如果我们的推理的另一部分已经引导我们表达出"没有不可转让的财富"这一结论，我们就不认为有必要在"财富"上添加"可转让的"这一修饰语，然而，这种方法之所以完善，正在于它在一切情况下都通过陈述每个项与我们选择采用的每一种区别性质或元素所保持的关系，来充分界定结论中每个项所表示的对象。这对于保持解答的不同部分真正彼此区分和独立是必要的，并且实际上防止了冗余。假设我们一直在考虑的那对项不包含"可转让的"一词，而合起来是"一切财富"，那么我们逻辑上就可以把单一的一项"一切财富"分解为两项："一切可转让的财富"和"一切不可转让的财富"。但后一项被"独立关系"表明为消失。因此它不构成所需描述的一部分，因而就是冗余的。剩下的项与实际得到的结论一致。

通过逻辑划分不可能产生任何多余或冗余项的解答，可以称为纯解答。通过上述展开式和消元方法得到的一切解答都是如此。应当注意，如果在目前这一体系中该方法可能适用的情况下采用通常的代数的消元方法，我们就不能依赖所得到的解答的纯粹性。缺乏普遍性并不是它唯一的缺陷。

15. 其次，将会指出，结论包含两个项，它们的合计意义用一个单项来表达会更为方便。与其说"全部产生快乐且可转移的财富"和"全部不产生快乐且可转移的财富"，我们不如简单地说"全部可转移的财富"。这一评论完全公正。但必须注意，凡此类简化是可能的，它们就是

immediately suggested by the form of the equation we have to interpret; and if that equation be reduced to its simplest form, then the interpretation to which it conducts will be in its simplest form also. Thus in the original solution the terms wtp and $wt(1-p)$, which have unity for their coefficient, give, on addition, wt; the terms $w\,(1-t)\,p$ and $w\,(1-t)\,(1-p)$, which have $\frac{1}{0}$ for their coefficient give $w\,(1-t)$; and the terms $(1-w)\,(1-t)\,p$ and $(1-w)\,(1-t)\,(1-p)$, which have $\frac{0}{0}$ for their coefficient, give $(1-w)\,(1-t)$. Whence the complete solution is

$$s = wt + \frac{0}{0}\,(1-w)\,(1-t) + \frac{0}{0}\,(1-w)\,t\,(1-p)\,,$$

with the independent relation,

$$w\,(1-t) = 0,\ \text{or}\ w = \frac{0}{0}t.$$

The interpretation would now stand thus:–

1st. *Things limited in supply consist of all wealth transferable, with an indefinite remainder of what is not wealth and not transferable, and of transferable articles which are not wealth, and are not productive of pleasure.*

2nd. *All wealth is transferable.*

This is the simplest form under which the general conclusion, with its attendant condition, can be put.

16. When it is required to eliminate two or more symbols from a proposed equation we can either employ (6) Prop. I., or eliminate them in succession, the order of the process being indifferent. From the equation

$$w = st\,(p + r - pr)\,,$$

we have eliminated r, and found the result,

$$w - wst - wstp + stp = 0.$$

Suppose that it had been required to eliminate both r and t, then taking the above as the first step of the process, it remains to eliminate from the last equation t. Now when $t = 1$ the first member of that equation becomes

$$w - ws - wsp + sp,$$

and when $t = 0$ the same member becomes w. Whence we have

$$w\,(w - ws - wsp + sp) = 0,$$

or

$$w - ws = 0,$$

for the required result of elimination.

If from the last result we determine w, we have

立即由我们所要解释的方程的形式所提示；而如果该方程被化为其最简形式，那么它所导出的解释也将同样处于其最简形式。因此，在原解中，系数为1的那些项 wtp 与 wt(1 – p)，相加后给出 wt；系数为10fortheircoefficient的那些项 w(1 – t) p 与 w(1 – t)(1 – p)，给出 w(1 – t)；而系数为00fortheircoefficient,give(1–w)(1–t).Whencethecompletesolution的那些项(1 – w)(1 – t) p 与(1 – w)(1 – t)(1 – p),

是

$$s = wt + \frac{0}{0}\,(1-w)\,(1-t) + \frac{0}{0}\,(1-w)\,t\,(1-p)\,,$$

与独立关系，

$$w\,(1-t) = 0,\ \text{or}\ w = \frac{0}{0}t.$$

现在的解释将如下所示：--1. 供给有限的物品由一切可转让的财富，连同非财富且不可转让之物、以及非财富且不能产生快乐的可转让物品的不定剩余组成。

第二，一切财富皆可转让。

这是该一般结论连同其附带条件所能采取的最简形式。

16. 当需要从一给定方程中消去两个或更多记号时，我们可以采用(6)命题I，或者依次消去它们，过程的次序无关紧要。从方程

$$w = st\,(p + r - pr)\,,$$

我们已经消去了 r，并求得结果，

$$w - wst - wstp + stp = 0.$$

假设曾需要同时消去 r 和 t，那么以上述作为过程的第一步，尚需从最后的方程中消去 t。现在当 t = 1 时，该方程的第一项变为

$$w - ws - wsp + sp,$$

而当 t = 0 时，同一成员变为 w。由此我们得到

$$w\,(w - ws - wsp + sp) = 0,$$

or

$$w - ws = 0,$$

for the required result of elimination.

If from the last result we determine w, we have

$$w = \frac{0}{1-s} = \frac{0}{0}s,$$

whence "*All wealth is limited in supply.*" As p does not enter into the equation, it is evident that the above is true, irrespectively of any relation which the elements of the conclusion bear to the quality "productive of pleasure."

Resuming the original equation, let it be required to eliminate s and t. We have

$$w = st(p + r - pr).$$

Instead, however, of separately eliminating s and t according to the Rule, it will suffice to treat st as a single symbol, seeing that it satisfies the fundamental law of the symbols by the equation

$$st(1 - st) = 0.$$

Placing, therefore, the given equation under the form

$$w - st(p + r - pr) = 0;$$

and making st successively equal to 1 and to 0, and taking the product of the results, we have

$$(w - p - r + pr)w = 0,$$
$$\text{or } w - wp - wr + wpr = 0,$$

for the result sought.

As a particular illustration, let it be required to deduce an expression for "things productive of pleasure" (p), in terms of "wealth" (w), and "things preventive of pain" (r).

We have, on solving the equation,

$$p = \frac{w(1-r)}{w(1-r)}$$
$$= \frac{0}{0}wr + w(1-r) + \frac{0}{0}(1-w)r + \frac{0}{0}(1-w)(1-r)$$
$$= w(1-r) + \frac{0}{0}wr + \frac{0}{0}(1-w).$$

Whence the following conclusion:–*Things productive of pleasure are, all wealth not preventive of pain, an indefinite amount of wealth that is preventive of pain, and an indefinite amount of what is not wealth.*

From the same equation we get

$$1 - p = 1 - \frac{w(1-r)}{w(1-r)} = \frac{0}{w(1-r)},$$

$$w = \frac{0}{1-s} = \frac{0}{0}s,$$

由此得"一切财富都是供应有限的。"由于 p 不进入该方程，显然上述结论为真，与结论的各元素同"产生快乐"这一性质有何关系无关。

回到原方程，设需要消去 s 和 t。我们有

$$w = st(p + r - pr).$$

然而，与其按照规则分别消去 s 和 t，只需将 st 当作单一记号处理即可，因为由方程

$$st(1 - st) = 0.$$

Placing, therefore, the given equation under the form

$$w - st(p + r - pr) = 0;$$

and making st successively equal to 1 and to 0, and taking the product of the results, we have

$$(w - p - r + pr)w = 0,$$
$$\text{or } w - wp - wr + wpr = 0,$$

所寻求的结果。

作为一个具体例证，设需要推导出"产生快乐之物"(p)用"财富"(w)和"防止痛苦之物"(r)表示的表达式。

解此方程，我们得到

$$p = \frac{w(1-r)}{w(1-r)}$$
$$= \frac{0}{0}wr + w(1-r) + \frac{0}{0}(1-w)r + \frac{0}{0}(1-w)(1-r)$$
$$= w(1-r) + \frac{0}{0}wr + \frac{0}{0}(1-w).$$

由此得以下结论：--产生快乐之物是：全部不防止痛苦的财富，不定量的防止痛苦的财富，以及不定量的非财富之物。

由同一方程我们得到

$$1 - p = 1 - \frac{w(1-r)}{w(1-r)} = \frac{0}{w(1-r)},$$

which developed, gives

$$w(1-p) = \frac{0}{0}wr + \frac{0}{0}(1-w)\cdot r + \frac{0}{0}(1-w)\cdot(1-r)$$
$$= \frac{0}{0}wr + \frac{0}{0}(1-w).$$

Whence, *Things not productive of pleasure are either wealth, preventive of pain, or what is not wealth.*

Equally easy would be the discussion of any similar case.

17. In the last example of elimination, we have eliminated the compound symbol *st* from the given equation, by treating it as a single symbol. The same method is applicable to any combination of symbols which satisfies the fundamental law of individual symbols. Thus the expression $p+r-pr$ will, on being multiplied by itself, reproduce itself, so that if we represent $p+r-pr$ by a single symbol as y, we shall have the fundamental law obeyed, the equation

$$y = y^2, \text{ or } y(1-y) = 0,$$

being satisfied. For the rule of elimination for symbols is founded upon the supposition that each individual symbol is subject to that law; and hence the elimination of any function or combination of such symbols from an equation, may be effected by a single operation, whenever that law is satisfied by the function.

Though the forms of interpretation adopted in this and the previous chapter show, perhaps better than any others, the direct significance of the symbols 1 and $\frac{0}{0}$, modes of expression more agreeable to those of common discourse may, with equal truth and propriety, be employed. Thus the equation (9) may be interpreted in the following manner: *Wealth is either limited in supply, transferable, and productive of pleasure, or limited in supply, transferable, and not productive of pleasure.* And reversely, *Whatever is limited in supply, transferable, and productive of pleasure, is wealth.* Reverse interpretations, similar to the above, are always furnished when the final development introduces terms having unity as a coefficient.

18. NOTE.–The fundamental equation $f(1)f(0) = 0$, expressing the result of the elimination of the symbol x from any equation $f(x) = 0$, admits of a remarkable interpretation.

It is to be remembered, that by the equation $f(x) = 0$ is implied some proposition in which the individuals represented by the class x, suppose "men," are referred to, together, it may be, with other individuals; and it is our object to ascertain whether there is implied in the proposition any relation among the other individuals, independently of those found in the class *men*. Now the equation $f(1) = 0$ expresses what the original proposition would become if *men* made up the universe, and the equation $f(0) = 0$ expresses what that original proposition would become if men ceased to exist, wherefore the equation $f(1)f(0) = 0$ expresses what in virtue of the original proposition would be

其发展得出

$$w(1-p) = \frac{0}{0}wr + \frac{0}{0}(1-w)\cdot r + \frac{0}{0}(1-w)\cdot(1-r)$$
$$= \frac{0}{0}wr + \frac{0}{0}(1-w).$$

由此得，不产生快乐之物或者是财富、防止痛苦的，或者是非财富之物。

任何类似情形的讨论也同样容易。

17. 在消元的最后一个例子中，我们把复合记号 st 当作单一记号来处理，从而从给定方程中消去了它。同样的方法适用于任何满足个体记号基本规律的记号组合。因此，表达式 p + r – pr 在自乘时将重现自身，所以如果我们用单一记号如 y 来表示 p + r – pr，我们就将遵守基本规律，方程

$$y = y^2, \text{ or } y(1-y) = 0,$$

得到满足。因为记号的消元规则是建立在如下假定之上的：每个个别记号都服从该定律；因此，只要该函数满足该定律，从方程中消去此类记号的任何函数或组合，都可以通过一次运算完成。

尽管本章及前一章所采用的解释形式，或许比任何其他形式都更好地显示了记号 1 和 00,方式的表达更符合于那些属于日常话语的直接意义，但同样真实而恰当的方式也可以被采用。于是方程(9)可以按如下方式解释：财富要么是供给有限、可转让并产生快乐的，要么是供给有限、可转让但不产生快乐的。反过来，凡供给有限、可转让并产生快乐的东西，都是财富。当最终的展开式引入系数为 unity 的项时，类似于上述的反向解释总是会随之给出。

18. 注--基本方程 f(1) f(0) = 0，表达从任何方程 f(x) = 0 中消去记号 x 的结果，容许一种值得注意的解释。

应当记住，方程 f(x) = 0 隐含了某个命题，其中由类 x(假定为"men")所代表的个体被提及，并且可能与其他个体一起被提及；而我们的目标是确定该命题中是否隐含了其他个体之间的某种关系，独立于类*men*中所发现的那些个体。现在，方程 f(1) = 0 表达了如果*men*构成了全集，原命题会变成什么；方程 f(0) = 0 表达了如果 men 不复存在，原命题会变成什么；因此方程 f(1) f(0) = 0 表达了凭借原命题将会是

equally true on either assumption, i. e. equally true whether "men" were "all things" or "nothing." Wherefore the theorem expresses that *what is equally true, whether a given class of objects embraces the whole universe or disappears from existence, is independent of that class altogether, and vice versâ.* Herein we see another example of the interpretation of formal results, immediately deduced from the mathematical laws of thought, into general axioms of philosophy.

在任一假定下同样为真，即在"人"是"一切事物"或"无"的情况下同样为真。因此该定理表达的是：无论某一给定类别的对象是囊括整个论域还是从存在中消失都同样为真者，完全独立于该类别，反之亦然。在此我们看到又一个例子，说明如何将直接由思维的数学规律推导出的形式结果解释为哲学的一般公理。

Chapter VIII

ON THE REDUCTION OF SYSTEMS OF PROPOSITIONS.

1. In the preceding chapters we have determined sufficiently for the most essential purposes the theory of single primary propositions, or, to speak more accurately, of primary propositions expressed by a single equation. And we have established upon that theory an adequate method. We have shown how any element involved in the given system of equations may be eliminated, and the relation which connects the remaining elements deduced in any proposed form, whether of denial, of affirmation, or of the more usual relation of subject and predicate. It remains that we proceed to the consideration of systems of propositions, and institute with respect to them a similar series of investigations. We are to inquire whether it is possible from the equations by which a system of propositions is expressed to eliminate, *ad libitum*, any number of the symbols involved; to deduce by interpretation of the result the whole of the relations implied among the remaining symbols; and to determine in particular the expression of any single element, or of any interpretable combination of elements, in terms of the other elements, so as to present the conclusion in any admissible form that may be required. These questions will be answered by showing that it is possible to reduce any system of equations, or any of the equations involved in a system, to an equivalent single equation, to which the methods of the previous chapters may be immediately applied. It will be seen also, that in this reduction is involved an important extension of the theory of single propositions, which in the previous discussion of the subject we were compelled to forego. This circumstance is not peculiar in its nature. There are many special departments of science which cannot be completely surveyed from within, but require to be studied also from an external point of view, and to be regarded in connexion with other and kindred subjects, in order that their full proportions be understood.

This chapter will exhibit two distinct modes of reducing systems of equations to equivalent single equations. The first of these rests upon the employment of arbitrary constant multipliers. It is a method sufficiently simple in theory, but it has the inconvenience of rendering the subsequent processes of elimination and development, when they occur, somewhat tedious. It was, however, the method

第八章

论命题系统的化简。

1. 在前几章中，我们已经为最本质的目的充分确定了单一原始命题的理论，或者更准确地说，是由单一方程所表达的原始命题的理论。并且我们已在该理论之上建立了一种充分的方法。我们已经表明，如何消去给定方程组中所涉及的任一元素，并以任何所要求的形成--无论是否定、肯定，还是更常见的主词与谓词的关系--推出连接其余元素的关系。接下来，我们着手考察命题系统，并对它们进行一系列类似的探究。我们要问：是否可能从表达一个命题系统的那些方程中，随意地消去任意数目的所涉记号；通过对结果的解释推出其余记号之间所隐含的全部关系；并特别确定任一单个元素、或元素的任一可解释组合，用其他元素表示的表达式，从而以任何可能要求的可容许形式呈现结论。这些问题将通过表明以下一点来回答：可以将任何方程组、或一个系统中所涉及的任何方程，化为一个等价的单一方程，而前几章的方法可直接应用于该方程。还将看到，在这一化归中涉及单一命题理论的一项重要扩展，而这一扩展是我们在先前对该主题的讨论中不得不放弃的。这种情况在性质上并不特殊。科学中有许多特殊部门不能从内部得到完整的考察，而还需要从外部观点加以研究，并与其他同类主题联系起来加以看待，以便理解它们的全貌。

本章将展示两种把方程组化为等价的单一方程的不同方式。第一种方式依赖于使用任意常数乘数。它在理论上足够简单，但有一个不便之处，即使得随后出现的消元和展开过程变得相当冗长。然而，它是最早发现的

of reduction first discovered, and partly on this account, and partly on account of its simplicity, it has been thought proper to retain it. The second method does not require the introduction of arbitrary constants, and is in nearly all respects preferable to the preceding one. It will, therefore, generally be adopted in the subsequent investigations of this work.

2. We proceed to the consideration of the first method.

Proposition I.

Any system of logical equations may be reduced to a single equivalent equation, by multiplying each equation after the first by a distinct arbitrary constant quantity, and adding all the results, including the first equation, together.

By Prop. 2, Chap, VI., the interpretation of any single equation, $f(x, y..) = 0$ is obtained by equating to 0 those constituents of the development of the first member, whose coefficients do not vanish. And hence, if there be given two equations, $f(x, y..) = 0$, and $F(x, y..) = 0$, their united import will be contained in the system of results formed by equating to 0 all those constituents which thus present themselves in both, or in either, of the given equations developed according to the Rule of Chap. VI. Thus let it be supposed, that we have the two equations

$$xy - 2x = 0, \tag{1}$$

$$x - y = 0; \tag{2}$$

The development of the first gives

$$-xy - 2x(1 - y) = 0;$$

$$\text{whence, } xy = 0, x(1 - y) = 0. \tag{3}$$

The development of the second equation gives

$$x(1 - y) - y(1 - x) = 0;$$

$$\text{whence, } x(1 - y) = 0, y(1 - x) = 0. \tag{4}$$

The constituents whose coefficients do not vanish in both developments are xy, $x(1 - y)$, and $(1 - x)y$, and these would together give the system

$$xy = 0, x(1 - y) = 0, (l - x)y = 0; \tag{5}$$

which is equivalent to the two systems given by the developments separately, seeing that in those systems the equation $x(1 - y) = 0$ is repeated. Confining ourselves to the case of binary systems of equations, it remains then to determine a single equation, which on development shall yield the same constituents with

化归方法，部分出于这一原因，部分出于其简单性，人们认为保留它是适当的。第二种方法不需要引入任意常数，并且在几乎所有方面都优于前一种。因此，在本著作随后的研究中将普遍采用它。

2. 我们 proceed 到第一种方法的考虑。

命题一。

任何逻辑方程组都可以化为一个等价的单一方程，方法是将第一个方程之后的每个方程乘以一个不同的任意常数，并将所有结果连同第一个方程一起相加。根据第六章命题 2，任何单一方程 f(x, y..) = 0 的解释，是通过将第一项展开式中系数不为零的那些成分等于0而获得的。因此，如果给定两个方程 f(x, y..) = 0 和 F(x, y..) = 0，它们的联合含义将包含在由以下方式形成的结果系统中：将按第六章规则展开的给定方程中在两者或任一者中如此呈现的所有那些成分等于0。于是，假设我们有两个方程

$$xy - 2x = 0, \tag{1}$$

$$x - y = 0; \tag{2}$$

The development of the first gives

$$-xy - 2x(1 - y) = 0;$$

$$\text{whence, } xy = 0, x(1 - y) = 0. \tag{3}$$

The development of the second equation gives

$$x(1 - y) - y(1 - x) = 0;$$

$$\text{whence, } x(1 - y) = 0, y(1 - x) = 0. \tag{4}$$

The constituents whose coefficients do not vanish in both developments are xy, $x(1 - y)$, and $(1 - x)y$, and these would together give the system

$$xy = 0, x(1 - y) = 0, (l - x)y = 0; \tag{5}$$

这等价于分别由展开式给出的两个方程组，因为在这些方程组中方程 x(1 – y) = 0 被重复。将我们限制在二元方程组的情形，那么剩下的就是确定一个单一方程，它在展开时将与

coefficients which do not vanish, as the given equations produce. Now if we represent by

$$V_1 = 0, V_2 = 0,$$

the given equations, V_1 and V_2 being functions of the logical symbols x, y, z, &c.; then the single equation

$$V_1 + cV_2 = 0, \tag{6}$$

c being an arbitrary constant quantity, will accomplish the required object. For let At represent any term in the full development V, wherein t is a constituent and A its numerical coefficient, and let Bt represent the corresponding term in the full development of V_2, then will the corresponding term in the development of (6) be

$$(A + cB)t.$$

The coefficient of t vanishes if A and B both vanish, but not otherwise. For if we assume that A and B do not both vanish, and at the same time make

$$A + cB = 0, \tag{7}$$

the following cases alone can present themselves.

1st. That A vanishes and B does not vanish. In this case the above equation becomes

$$cB = 0,$$

and requires that $c = 0$. But this contradicts the hypothesis that c is an *arbitrary* constant.

2nd. That B vanishes and A does not vanish. This assumption reduces (7) to

$$A = 0,$$

by which the assumption is itself violated.

3rd. That neither A nor B vanishes. The equation (7) then gives

$$c = \frac{-A}{B}$$

which is a definite value, and, therefore, conflicts with the hypothesis that c is arbitrary.

Hence the coefficient $A + cB$ vanishes when A and B both vanish, but not otherwise. Therefore, the same constituents will appear in the development of (6), with coefficients which do not vanish, as in the equations $V_1 = 0, V_2 = 0$, singly or together. And the equation $V_1 + cV_2 = 0$, will be equivalent to the system $V_1 = 0, V_2 = 0$.

By similar reasoning it appears, that the general system of equations

$$V_1 = 0, V_2 = 0, V_3 = 0, \text{ \&c.};$$

coefficients which do not vanish, as the given equations produce. Now if we represent by

$$V_1 = 0, V_2 = 0,$$

给定的方程，V [1]和 V [2]是逻辑记号 x, y, z， 等的函数；那么单一方程

$$V_1 + cV_2 = 0, \tag{6}$$

c 为任意常数，将实现所需的目标。因为设 At 表示完整展开式 V 中的任一项，其中 t 是一个成分而 A 是其数值系数，并设 Bt 表示 V [2]的完整展开式中的对应项，那么(6)的展开式中的对应项将是

$$(A + cB)t.$$

若 A 和 B 同时为零，则 t 的系数消失，否则不然。因为如果我们假定 A 和 B 不同时为零，同时使

$$A + cB = 0, \tag{7}$$

则只有以下情形可能出现。第一。A 为零而 B 不为零。在此情形下上述方程变为

$$cB = 0,$$

并且要求 c = 0。但这与 c 是任意常数的假说相矛盾。

第二，B 消失而 A 不消失。这一假定化简为(7)到

$$A = 0,$$

由此该假定本身即被违反。

第三，A与B均不消失。此时方程(7)给出

$$c = \frac{-A}{B}$$

这是一个确定的值，因此与 c 是任意的这一假说相冲突。

因此，系数 A + cB 在 A 和 B 两者都消失时消失，但在其他情况下则不消失。因此，同样的成分将出现在(6)的展开式中，其系数不消失，正如在方程 V [1] = 0 , V [2] = 0 中那样，单独地或一起地。而方程 V [1] + cV [2] = 0，将等价于系统 V [1] = 0 , V [2] = 0。

由类似的推理可知，一般的方程组

$$V_1 = 0, V_2 = 0, V_3 = 0, \text{ \&c.};$$

may be replaced by the single equation

$$V_1 + cV_2 + c'V_3 + \ \&c. = 0\,,$$

c, c', &c., being arbitrary constants. The equation thus formed may be treated in all respects as the ordinary logical equations of the previous chapters. The arbitrary constants c_1, c_2, &c., are not *logical* symbols. They do not satisfy the law,

$$c_1(1-c_1) = 0, c_2(1-c_2) = 0\,.$$

But their introduction is justified by that general principle which has been stated in (II. 15) and (V. 6), and exemplified in nearly all our subsequent investigations, viz., that equations involving the symbols of Logic may be treated in all respects as if those symbols were symbols of quantity, subject to the special law $x(1-x) = 0$, until in the final stage of solution they assume a form interpretable in that system of thought with which Logic is conversant.

3. The following example will serve to illustrate the above method.

Ex. 1.–Suppose that an analysis of the properties of a particular class of substances has led to the following general conclusions, viz.:

1st. That wherever the properties A and B are combined, either the property C, or the property D, is present also; but they are not jointly present.

2nd. That wherever the properties B and C are combined, the properties A and D are either both present with them, or both absent.

3rd. That wherever the properties A and B are both absent, the properties C and D are both absent also; and vice versa, where the properties C and D are both absent, A and B are both absent also.

Let it then be required from the above to determine what may be concluded in any particular instance from the presence of the property A with respect to the presence or absence of the properties B and C, paying no regard to the property D.

Represent the property A by x;
" the property B by y;
" the property C by z;
" the property D by w.

Then the symbolical expression of the premises will be

$$xy - v(w(1-z)+z(1-w)); yz = v(xw+(1-x)(1-w)); (1-x)(1-y) = (1-z)(1-w).$$

From the first two of these equations, separately eliminating the indefinite class symbol v, we have

$$xy(1 - w(1-z) - z(1-w)) = 0;$$
$$yz(1 - xw - (1-x)(1-w)) = 0.$$

Now if we observe that by development

$$1 - w(1-z) - z(1-w) = wz + (1-w)(1-z),$$

可用单一方程代替

$$V_1 + cV_2 + c'V_3 + \ \&c. = 0\,,$$

c, c′ ，等等，为任意常数。如此形成的方程可以在一切方面当作前几章的普通逻辑方程来处理。任意常数 c 1 , c 2等等，不是逻辑的记号。它们不满足那条规律，

$$c_1(1-c_1) = 0, c_2(1-c_2) = 0\,.$$

但它们的引入由那条一般原则所证明，该原则已在(II.15)和(V.6)中陈述，并在我们其后几乎所有的研究中得到例示，即：涉及逻辑记号的方程可以在一切方面当作那些记号是量的记号来处理，服从特殊规律 x(1 – x) = 0，直到在求解的最后阶段它们采取一种可在逻辑所熟悉的那套思想体系中解释的形式。

3. 下例将用以说明上述方法。

例1。--假设对某一类物质的性质的分析导致了以下一般结论，即：

第一，凡性质 A 与 B 相结合之处，或者性质 C，或者性质 D，也出现；但它们不是共同出现。

第二，凡性质 B 与 C 相结合之处，性质 A 与 D 或者都与它们一同出现，或者都一同不出现。

第三，凡性质 A 与 B 都不出现之处，性质 C 与 D 也都一同不出现；反之，凡性质 C 与 D 都不出现之处，A 与 B 也都一同不出现。

于是要求从上述结论出发，就性质 A 的出现，判定在任一特殊情形下关于性质 B 与 C 的出现或不出现可以得出什么结论，而不顾及性质 D。

用x表示性质A;
" 性质 B 用 y 表示；" 性质 C 用 z
表示；" 性质 D 用 w 表示。

那么，前提的符号表达式将是

$$xy - v(w(1-z)+z(1-w)); yz = v(xw+(1-x)(1-w)); (1-x)(1-y) = (1-z)(1-w).$$

从这些方程的前两个中，分别消去不定类记号 v，我们得到

$$xy(1 - w(1-z) - z(1-w)) = 0;$$
$$yz(1 - xw - (1-x)(1-w)) = 0.$$

Now if we observe that by development

$$1 - w(1-z) - z(1-w) = wz + (1-w)(1-z),$$

and

$$1 - xw - (1-x)(1-w) = x(1-w) + w(1-x),$$

and in these expressions replace, for simplicity,

$$1 - xby\bar{x}, 1 - yby\bar{y}, \&c.,$$

we shall have from the three last equations,

$$xy(wz + \bar{w}\bar{z}) = 0; \quad (1)$$
$$yz(x\bar{w} + \bar{x}w) = 0; \quad (2)$$
$$\bar{x}\bar{y} = \bar{w}\bar{z}; \quad (3)$$

and from this system we must eliminate w.

Multiplying the second of the above equations by c, and the third by c', and adding the results to the first, we have

$$xy(wz + \bar{w}\bar{z}) + cyz(x\bar{w} + \bar{x}w) + c'(\bar{x}\bar{y} - \bar{w}\bar{z}) = 0.$$

When w is made equal to 1, and therefore $\bar{w}$ to 0, the first member of the above equation becomes

$$xyz + c\bar{x}yz + c'\bar{x}\bar{y}.$$

And when in the same member w is made 0 and $\bar{w} = 1$, it becomes

$$xy\bar{z} + cxyz + c'\bar{x}\bar{y} - c'\bar{z}.$$

Hence the result of the elimination of w may be expressed in the form

$$(xyz + c\bar{x}yz + c'\bar{x}\bar{y})(xy\bar{z} + cxyz + c'\bar{x}\bar{y} - c'\bar{z}) = 0; \quad (4)$$

and from this equation x is to be determined.

Were we now to proceed as in former instances, we should multiply together the factors in the first member of the above equation ; but it may be well to show that such a course is not at all necessary. Let us develop the first member of (4) with reference to x, the symbol whose expression is sought, we find

$$yz(y\bar{z} + cyz - c'\bar{z})x + (cyz + c'\bar{y})(c'\bar{y} - c'z)(1-x) = 0;$$
$$\text{or, } cyzx + (cyz + c'\bar{y})(c'\bar{y} - c'\bar{z})(1-x) = 0;$$

whence we find,

$$x = \frac{(cyz + c'\bar{y})(c'\bar{y} - c'\bar{z})}{(cyz + c'\bar{y})(c'\bar{y} - c'\bar{z}) - cyz};$$

and developing the second member with respect to y and z,

$$x = 0yz + \frac{0}{0}y\bar{z} + \frac{c'^2}{c'^2}\bar{y}z + \frac{0}{0}\bar{y}\bar{z};$$

且

$$1 - xw - (1-x)(1-w) = x(1-w) + w(1-x),$$

and in these expressions replace, for simplicity,

$$1 - xby\bar{x}, 1 - yby\bar{y}, \&c.,$$

we shall have from the three last equations,

$$xy(wz + \bar{w}\bar{z}) = 0; \quad (1)$$
$$yz(x\bar{w} + \bar{x}w) = 0; \quad (2)$$
$$\bar{x}\bar{y} = \bar{w}\bar{z}; \quad (3)$$

而我们必须从这一系统中消去w。

将上述方程的第二个乘以 c，第三个乘以 c′，并将结果加到第一个上，我们得到

$$xy(wz + \bar{w}\bar{z}) + cyz(x\bar{w} + \bar{x}w) + c'(\bar{x}\bar{y} - \bar{w}\bar{z}) = 0.$$

When w is made equal to 1, and therefore $\bar{w}$ to 0, the first member of the above equation becomes

$$xyz + c\bar{x}yz + c'\bar{x}\bar{y}.$$

而当在同一成员中令 w 为 0 且 w = 1 时，它变为

$$xy\bar{z} + cxyz + c'\bar{x}\bar{y} - c'\bar{z}.$$

Hence the result of the elimination of w may be expressed in the form

$$(xyz + c\bar{x}yz + c'\bar{x}\bar{y})(xy\bar{z} + cxyz + c'\bar{x}\bar{y} - c'\bar{z}) = 0; \quad (4)$$

and from this equation x is to be determined.

倘若我们现在像先前诸例中那样进行，我们就应当把上述方程第一个成员中的各因子相乘；但最好表明这样一种做法完全不必要。让我们就 x--其表达式正被寻求的那个记号--来展开(4)的第一个成员，我们得到

$$yz(y\bar{z} + cyz - c'\bar{z})x + (cyz + c'\bar{y})(c'\bar{y} - c'z)(1-x) = 0;$$
$$\text{or, } cyzx + (cyz + c'\bar{y})(c'\bar{y} - c'\bar{z})(1-x) = 0;$$

whence we find,

$$x = \frac{(cyz + c'\bar{y})(c'\bar{y} - c'\bar{z})}{(cyz + c'\bar{y})(c'\bar{y} - c'\bar{z}) - cyz};$$

and developing the second member with respect to y and z,

$$x = 0yz + \frac{0}{0}y\bar{z} + \frac{c'^2}{c'^2}\bar{y}z + \frac{0}{0}\bar{y}\bar{z};$$

or,

$$x = (1-y)\,z + \frac{0}{0}y\,(1-z) + \frac{0}{0}\,(1-y)\,(1-z)\,;$$

or,

$$x = (1-y)\,z + \frac{0}{0}\,(1-z)\,;$$

the interpretation of which is, *Wherever the property A is present, there either C is present and B absent, or C is absent.* And inversely, *Wherever the property C is present, and the property B absent, there the property A is present.*

These results may be much more readily obtained by the method next to be explained. It is, however, satisfactory to possess different modes, serving for mutual verification, of arriving at the same conclusion.

4. We proceed to the second method.

PROPOSITION II.

If any equations, $V_1 = 0$, $V_2 = 0$, &c., are such that the developments of their first members consist only of constituents with positive coefficients, those equations may be combined together into a single equivalent equation by addition.

For, as before, let At represent any term in the development of the function V_1, Bt the corresponding term in the development of V_2 and so on. Then will the corresponding term in the development of the equation

$$V_1 + V_2 + \&\text{c.} = 0, \tag{1}$$

formed by the addition of the several given equations, be

$$(A + B + \&\text{c.})\,t.$$

But as by hypothesis the coefficients A, B, &c. are none of them negative, the aggregate coefficient $A + B$, &c. in the derived equation will only vanish when the separate coefficients A, B, &c. vanish together. Hence the same constituents will appear in the development of the equation (1) as in the several equations $V_1 = 0$, $V_2 = 0$, &c. of the original system taken collectively, and therefore the interpretation of the equation (1) will be equivalent to the collective interpretations of the several equations from which it is derived.

PROPOSITION III.

5. *If $V_1 = 0, V_2 = 0, \&c.$ represent any system of equations, the terms of which have by transposition been brought to like first side, then the combined interpretation of the system will be involved in the single equation,*

$$V_1^2 + V_2^2 + \&c. = 0,$$

formed by adding together the squares of the given equations.

For let any equation of the system, as $V_1 = 0$, produce on development an equation

或者，

$$x = (1-y)\,z + \frac{0}{0}y\,(1-z) + \frac{0}{0}\,(1-y)\,(1-z)\,;$$

或者，

$$x = (1-y)\,z + \frac{0}{0}\,(1-z)\,;$$

其解释是，凡该性质出现之处，那里或者出现且不出现，或者不出现。并且反过来，凡该性质出现、而该性质不出现之处，那里该性质出现。

这些结果可以通过下面即将说明的方法更为容易得多地得到。然而，拥有不同的方式以达到同一结论、并用以相互验证，是令人满意的。

4. 我们接着讨论第二种方法。

命题二。

若任何方程， $V^1 = 0$, $V^2 = 0$, & c.，是这样的，即其第一个成员的展开式只由具有正系数的成分组成，则那些方程可以通过相加而结合成一个单一的等价方程。

因为，如前所述，令 At 表示函数 V^1的展开式中的任一项，Bt 表示 V^2的展开式中的对应项，依此类推。那么方程

$$V_1 + V_2 + \&\text{c.} = 0, \tag{1}$$

由几个给定方程相加而成的，是

$$(A + B + \&\text{c.})\,t.$$

但根据假说，系数 A、B 等均非否定，因此在导出方程中的总系数 A + B 等仅当各别系数 A、B 等一同消失时才会消失。因此，在方程(1)的展开式中出现的元素将与原系统各方程 $V^1 = 0$、$V^2 = 0$ 等合并考虑时出现的元素相同，因此方程(1)的解释将等价于其所导出的各方程之集合解释。

命题三。

5. 若 $V^1 = 0$, $V^2 = 0$, & c.表示任一方程组，其各项已经过移项而被置于相同的第一个边，则该组的联合解释将包含在单一方程中，

$$V_1^2 + V_2^2 + \&c. = 0,$$

由给定各方程的平方相加而成。

因为令该组中任一方程，如 $V^1 = 0$，在展开时产生一个方程

$$a_1t_1 + a_2t_2 + \&c. = 0$$

in which $t_1, t_2, \&c.$ are constituents, and $a_1, a_2, \&c.$ their corresponding coefficients. Then the equation $V_1^2 = 0$ will produce on development an equation

$$a_1^2t_1 + a_2^2t_2 + \&c. = 0,$$

as may be proved either from the law of the development or by squaring the function $a_1t_1 + a_2t_2, \&c.$ in subjection to the conditions

$$t_1^2 = t_1,\ t_2^2 = t_2,,\ t_1t_2 = 0$$

assigned in Prop. 3, Chap. v. Hence the constituents which appear in the expansion of the equation $V_1^2 = 0$, are the same with those which appear in the expansion of the equation $V_1 = 0$, and they have positive coefficients. And the same remark applies to the equations $V_2 = 0, \&c.$ Whence, by the last Proposition, the equation

$$V_1^2 + V_2^2 + \&c. = 0$$

will be equivalent in interpretation to the system of equations

$$V_1 = 0,\ V_2 = 0,\ \&c.$$

Corollary.–Any equation, $V = 0$, of which the first member already satisfies the condition

$$V^2 = V, \text{ or } V(1 - V) = 0,$$

does not need (as it would remain unaffected by) the process of squaring. Such equations are, indeed, immediately developable into a series of constituents, with coefficients equal to 1, Chap. v. Prop. 4.

Proposition IV.

6. *Whenever the equations of a system have by the above process of squaring, or by any other process, been reduced to a form such that all the constituents exhibited in their development have positive coefficients, any derived equations obtained by elimination will possess the same character, and may be combined with the other equations by addition.*

Suppose that we have to eliminate a symbol x from any equation $V = 0$, which is such that none of the constituents, in the full development of its first member, have negative coefficients. That expansion may be written in the form

$$V_1x + V_0(1 - x) = 0$$

V_1 and V_0 being each of the form

$$a_1t_1 + a_2t_2 ... + a_nt_n,$$

$$a_1t_1 + a_2t_2 + \&c. = 0$$

其中 t 1 , t 2 , & c.是成分，而 a 1 , a 2 , & c.是它们对应的系数。那么方程 V 21 = 0 在展开时将产生一个方程

$$a_1^2t_1 + a_2^2t_2 + \&c. = 0,$$

这可以由展开式的规律来证明，或者通过将函数 a 1 t 1 + a 2 t 2 ， 与 c.在服从这些条件的情况下平方来证明

$$t_1^2 = t_1,\ t_2^2 = t_2,,\ t_1t_2 = 0$$

已在第 v 章命题 3 中指定。因此，出现在方程 V 21=0,的展开式中的成分与出现在方程 V 1 = 0 的展开式中的那些成分相同，并且它们具有正系数。同样的评述适用于方程 V 2 = 0 ， 与 c.。因此，根据上一个命题，方程

$$V_1^2 + V_2^2 + \&c. = 0$$

在解释上等同于方程组

$$V_1 = 0,\ V_2 = 0,\ \&c.$$

系定理。--任何方程，V = 0，其第一个成员已经满足条件

$$V^2 = V, \text{ or } V(1 - V) = 0,$$

不需要(因为它将不受其影响)平方的过程。这样的方程确实可以立即展开为一系列成分，其系数等于 1，见第 v 章命题 4。

命题四。

6. 凡一个组的方程已通过上述平方过程、或通过任何其他过程，被化归为这样一种形式，使得在其展开式中显示出来的所有成分都具有正系数，则通过消元得到的任何导出方程都将具有同样的性质，并可以通过相加与其他方程结合。

假设我们必须从任一方程 V = 0 中消去一个记号 x，该方程是这样的，即在其第一个成员的完整展开式中，没有任何成分具有负系数。那个展开式可以写成如下形式

$$V_1x + V_0(1 - x) = 0$$

V 1和 V 0各自具有以下形式

$$a_1t_1 + a_2t_2 ... + a_nt_n,$$

in which $t_1t_2...t_n$ are constituents of the other symbols, and $a_1a_2...a_n$ in each case positive or vanishing quantities. The result of elimination is

$$V_1V_2 = 0;$$

and as the coefficients in V_1 and V_2, are none of them negative, there can be no negative coefficients in the product V_1V_2. Hence the equation $V_1V_2 = 0$ may be added to any other equation, the coefficients of whose constituents are positive, and the resulting equation will combine the full significance of those from which it was obtained.

Proposition V.

7. *To deduce from the previous Propositions a practical rule or method for the reduction of systems of equations expressing propositions in Logic.*

We have by the previous investigations established the following points, viz.:

1st. That any equations which are of the form $V = 0$, V satisfying the fundamental law of duality $V(1-V) = 0$, may be combined together by simple addition.

2ndly. That any other equations of the form $V = 0$ may be reduced, by the process of squaring, to a form in which the same principle of combination by mere addition is applicable.

It remains then only to determine what equations in the actual expression of propositions belong to the former, and what to the latter, class.

Now the general types of propositions have been set forth in the conclusion of Chap. IV. The division of propositions which they represent is as follows:

1st. Propositions, of which the subject is universal, and the predicate particular.

The symbolical type (IV. 15) is

$$X = vY,$$

X and Y satisfying the law of duality. Eliminating v, we have

$$X(1-Y) = 0, \tag{1}$$

and this will be found also to satisfy the same law. No further reduction by the process of squaring is needed.

2nd. Propositions of which both terms are universal, and of which the symbolical type is

$$X = Y,$$

X and Y separately satisfying the law of duality. Writing the equation in the form $X - Y = 0$, and squaring, we have

$$\begin{gathered} X - 2XY + Y = 0, \\ \text{or, } X(1-Y) + Y(1-X) = 0. \end{gathered} \tag{2}$$

其中 t 1 t 2 ...t n 是其他记号的成分，而 a 1 a 2 ...a n 在每一情形下都是正量或消失量。消元的结果是

$$V_1V_2 = 0;$$

并且由于 V 1与 V 2中的系数没有一个是负的，所以在乘积 V 1 V 2中不可能有负系数。因此方程 V 1 V 2 = 0 可以加到任何其他方程上，只要其成分的系数是正的，而所得方程将结合那些据以得到它的方程的全部意义。

命题五。

7. 从先前诸命题推导出一条实用的规则或方法，用以化归在逻辑中表达命题的方程组。

我们已通过先前的研究确立了以下几点，即：第一，任何形如 V = 0 的方程，其中 V 满足对偶性的基本规律 V (1 –V) = 0，都可以通过简单相加而结合在一起。第二，任何其他形如 V = 0 的方程，都可以通过平方的过程化为一种形式，在其中同样的仅凭相加而结合的原则是适用的。那么，剩下的就只是确定在命题的实际表达中，哪些方程属于前一类，哪些属于后一类。现在，命题的一般类型已在第四章的结论中阐明。它们所代表的命题划分如下：第一，主词为全称而谓词为特称的命题。

符号类型(IV.15)为

$$X = vY,$$

X 与 Y 满足二重性规律。消去 v，我们得到

$$X(1-Y) = 0, \tag{1}$$

并且这也将被发现满足同一规律。无需再通过平方过程进行进一步化简。第二，两项均为全称的命题，其符号类型为

$$X = Y,$$

X 与 Y 分别满足二重性规律。将方程写成形式 X – Y = 0，并平方，我们得到

$$\begin{gathered} X - 2XY + Y = 0, \\ \text{or, } X(1-Y) + Y(1-X) = 0. \end{gathered} \tag{2}$$

The first member of this equation satisfies the law of duality, as is evident from its very form.

We may arrive at the same equation in a different manner. The equation

$$X = Y$$

is equivalent to the two equations

$$X = vY,\ Y = vX,$$

(for to affirm that X's are identical with Y's is to affirm both that All X's are Y's, and that All Y's are X's). Now these equations give, on elimination of v,

$$X(1-Y) = 0,\ Y(1-X) = 0,$$

which added, produce (2).

3rd. Propositions of which both terms are particular. The form of such propositions is

$$vX = vY,$$

but v is not quite arbitrary, and therefore must not be eliminated. For v is the representative of *some*, which, though it may include in its meaning *all*, does not include *none*. We must therefore transpose the second member to the first side, and square the resulting equation according to the rule. The result will obviously be

$$vX(1-Y) + vY(l-X) = 0.$$

The above conclusions it may be convenient to embody in a Rule, which will serve for constant future direction.

8. RULE.— *The equations being so expressed as that the terms X and Y in the following typical forms obey the law of duality, change the equations*

$$X = vY \text{ into } X(1-Y) = 0,$$

$$X = Y \text{ into } X(1-Y) + Y(1-X) = 0.$$

$$vX = vY \text{ into } vX(1-Y) + vY(1-X) = 0.$$

Any equation which is given in the form $X = 0$ will not need transformation, and any equation which presents itself in the form $X = 1$ may be replaced by $1 - X = 0$, as appears from the second of the above transformations.

When the equations of the system have thus been reduced, any of them, as well as any equations derived from them by the process of elimination, may be combined by addition.

9. NOTE.–It has been seen in Chapter IV. that in literally translating the terms of a proposition, without attending to its real meaning, into the language of symbols, we may produce equations in which the terms X and Y do not obey the law of duality. The equation $w = st(p+r)$, given in (3) Prop. 3 of the chapter referred to, is of this kind. Such equations, however, as it has been seen,

此方程的第一个成员满足对偶律，这从其形式本身即可明显看出。

我们可以用另一种方式得到同一个方程。方程

$$X = Y$$

等价于以下两个方程

$$X = vY,\ Y = vX,$$

(for to affirm that X's are identical with Y's is to affirm both that All X's are Y's, and that All Y's are X's). Now these equations give, on elimination of v,

$$X(1-Y) = 0,\ Y(1-X) = 0,$$

which added, produce (2).

3rd. Propositions of which both terms are particular. The form of such propositions is

$$vX = vY,$$

但 v 并非完全任意，因此不应被消去。因为 v 是某个的代表，它虽然可以在其含义中包括全部，但不包括无。因此我们必须将第二个成员移到第一边，并按照规则对所得方程进行平方。结果显然将是

$$vX(1-Y) + vY(l-X) = 0.$$

The above conclusions it may be convenient to embody in a Rule, which will serve for constant future direction.

8. RULE.— *The equations being so expressed as that the terms X and Y in the following typical forms obey the law of duality, change the equations*

$$X = vY \text{ into } X(1-Y) = 0,$$

$$X = Y \text{ into } X(1-Y) + Y(1-X) = 0.$$

$$vX = vY \text{ into } vX(1-Y) + vY(1-X) = 0.$$

任何以形式 X = 0 给出的方程将不需要变换，而任何以形式 X = 1 呈现的方程都可以替换为 1 – X = 0 ，这从上述第二个变换中可以看出。当系统的方程被如此化简后，其中任何一个，以及通过消元过程从它们导出的任何方程，都可以通过相加来组合。

9.注意。--在第四章中已经看到，在将命题的项字面翻译为记号语言而不注意其真实含义时，我们可能产生其中项 X 和 Y 不遵守对偶律的方程。在所指章节的命题3的(3)中给出的方程 w = st(p + r) 就属于这种类型。然而，这样的方程，正如已经看到的，

have a meaning. Should it, for curiosity, or for any other motive, be determined to employ them, it will be best to reduce them by the Rule (VI. 5).

10. Ex. 2.–Let us take the following Propositions of Elementary Geometry:

1st. Similar figures consist of all whose corresponding angles are equal, and whose corresponding sides are proportional.

2nd. Triangles whose corresponding angles are equal have their corresponding sides proportional, and *vice versâ*.

To represent these premises, let us make

s = similar.
t = triangles.
q = having corresponding angles equal.
r = having corresponding sides proportional.

Then the premises are expressed by the following equations:

$$s = qr, \tag{1}$$
$$tq = tr. \tag{2}$$

Reducing by the Rule, or, which amounts to the same thing, bringing the terms of these equations to the first side, squaring each equation, and then adding, we have

$$s + qr - 2qrs + tq + tr - 2tqr = 0. \tag{3}$$

Let it be required to deduce a description of dissimilar figures formed out of the elements expressed by the terms, *triangles*, having corresponding angles equal, having corresponding sides proportional.

We have from (3),

$$s = \frac{tq + qr + rt - 2tqr}{2qr - 1},$$
$$\therefore 1 - s = \frac{qr - tq - rt + 2tqr - 1}{2qr - 1}. \tag{4}$$

And fully developing the second member, we find

$$\begin{aligned} 1 - s = {} & 0tqr + 2tq(1-r) + 2tr(1-q) + t(1-q)(1-r) \\ & + 0(1-t)qr + (1-t)q(1-r) + (1-t)r(1-q) \\ & + (1-t)(1-q)(1-r). \end{aligned} \tag{5}$$

In the above development two of the terms have the coefficient 2, these must be equated to 0 by the Rule, then those terms whose coefficients are 0 being rejected, we have

$$1 - s = t(1-q)(1-r) + (1-t)q(1-r) + (1-t)r(1-q)$$
$$+ (1-t)(1-q)(1-r); \tag{6}$$
$$tq(1-r) = 0; \tag{7}$$
$$tr(1-q) = 0; \tag{8}$$

是有含义的。如果出于好奇或任何其他动机决定使用它们，最好按照规则(VI.5)对它们进行化简。

10. 例2--让我们取以下初等几何的命题：

1. 相似图形由所有对应角相等且
其对应边成比例。

第二。对应角相等的三角形，其对应
边成比例，且反之亦然。

为表示这些前提，让我们令 s = 相似的。t = 三角形。q = 对应角相等。r = 对应边成比例。

于是，前提由以下方程表示：

$$s = qr, \tag{1}$$
$$tq = tr. \tag{2}$$

按照规则化简，或者，与此相同，将这些方程的项移到第一边，对每个方程平方，然后相加，我们得到

$$s + qr - 2qrs + tq + tr - 2tqr = 0. \tag{3}$$

设要求推导出由项三角形、对应角相等、对应边成比例所表达的元素构成的非相似图形的描述。

由(3)可得，

$$s = \frac{tq + qr + rt - 2tqr}{2qr - 1},$$
$$\therefore 1 - s = \frac{qr - tq - rt + 2tqr - 1}{2qr - 1}. \tag{4}$$

And fully developing the second member, we find

$$\begin{aligned} 1 - s = {} & 0tqr + 2tq(1-r) + 2tr(1-q) + t(1-q)(1-r) \\ & + 0(1-t)qr + (1-t)q(1-r) + (1-t)r(1-q) \\ & + (1-t)(1-q)(1-r). \end{aligned} \tag{5}$$

在上述展开式中，有两项的系数为2，这些必须按照规则等于0，然后那些系数为0的项被舍弃，我们得到

$$1 - s = t(1-q)(1-r) + (1-t)q(1-r) + (1-t)r(1-q)$$
$$+ (1-t)(1-q)(1-r); \tag{6}$$
$$tq(1-r) = 0; \tag{7}$$
$$tr(1-q) = 0; \tag{8}$$

the direct interpretation of which is

1st. *Dissimilar figures consist of all triangles which have not their corresponding angles equal and sides proportional, and of all figures not being triangles which have either their angles equal, and sides not proportional, or their corresponding sides proportional, and angles not equal, or neither their corresponding angles equal nor corresponding sides proportional.*

2nd. *There are no triangles whose corresponding angles are equal. and sides not proportional.*

3rd. *There are no triangles whose corresponding sides are proportional and angles not equal.*

11. Such are the immediate interpretations of the final equation. It is seen, in accordance with the general theory, that in deducing a description of a particular class of objects, viz., dissimilar figures, in terms of certain other elements of the original premises, we obtain also the independent relations which exist among those elements in virtue of the same premises. And that this is not superfluous information, even as respects the immediate object of inquiry, may easily be shown. For example, the independent relations may always be made use of to reduce, if it be thought desirable, to a briefer form, the expression of that relation which is directly sought. Thus if we write (7) in the form

$$0 = tq(l - r),$$

and add it to (6), we get, since

$$t(1-q)(1-r) + tq(1-r) = t(1-r),$$
$$1 - s = t(1-r) + (1-t)q(1-r) + (1-t)r(1-q)$$
$$+(1-t)(1-q)(1-r),$$

which, on interpretation, would give for the first term of the description of dissimilar figures, "Triangles whose corresponding sides are not proportional," instead of the fuller description originally obtained. A regard to convenience must always determine the propriety of such reduction.

12. A reduction which is always advantageous (VII. 15) consists in collecting the terms of the immediate description sought, as of the second member of (5) or (6), into as few groups as possible. Thus the third and fourth terms of the second member of (6) produce by addition the single term $(1-t)(1-q)$. If this reduction be combined with the last, we have

$$1 - s = t(1-r) + (1-t)q(1-r) + (1-t)(1-q),$$

the interpretation of which is

Dissimilar figures consist of all triangles whose corresponding sides are not proportional, and all figures not being triangles which have either their corresponding angles unequal, or their corresponding angles equal, but sides not proportional.

The fulness of the general solution is therefore not a superfluity. While it gives us all the information that we seek, it provides us also with the means of expressing that information in the mode that is most advantageous.

其直接解释是

1. 不相似的图形包括所有对应角不相等且边不成比例的三角形，以及所有不是三角形但其角相等而边不成比例，或对应边成比例而角不相等，或对应角既不相等且对应边也不成比例的图形。

2. 不存在对应角相等而边不成比例的三角形。

3. 不存在对应边成比例而角不相等的三角形。

11. 以上就是最终方程的直接解释。可以看出，按照一般理论，在根据原前提的某些其他元素推演出对某一特定类对象(即不相似图形)的描述时，我们同时也获得了那些元素之间凭借同一前提而存在的独立关系。而且，即使就眼前的研究对象而言，这也不是多余的信息，这一点很容易表明。例如，独立关系总是可以被用来--如果认为可取的话--将直接寻求的那种关系的表达式化为更简短的形式。因此，如果我们把(7)写成如下形式

$$0 = tq(l - r),$$

将其加到(6)上，我们得到，因为

$$t(1-q)(1-r) + tq(1-r) = t(1-r),$$
$$1 - s = t(1-r) + (1-t)q(1-r) + (1-t)r(1-q)$$
$$+(1-t)(1-q)(1-r),$$

这在解释时，将给出非相似图形描述的第一项为"对应边不成比例的三角形"，而不是最初获得的更完整的描述。对便利性的考虑必须始终决定这种化简是否恰当。

12.一种总是有利的化简(VII.15)在于将所寻求的直接描述的项，如(5)或(6)的第二个成员的项，收集成尽可能少的组。因此，(6)的第二个成员的第三项和第四项通过相加产生单项(1 – t)(1 – q)。如果将此化简与上一次化简结合，我们得到

$$1 - s = t(1-r) + (1-t)q(1-r) + (1-t)(1-q),$$

the interpretation of which is

非相似图形由所有对应边不成比例的三角形，以及所有不是三角形但对应角不相等，或对应角相等但边不成比例的图形组成。

因此，一般解的充分性并非多余。它在给我们提供我们所寻求的全部信息的同时，也为我们提供了以最有利的方式表达该信息的手段。

13. Another observation, illustrative of a principle which has already been stated, remains to be made. Two of the terms in the full development of $1-s$ in (5) have 2 for their coefficients, instead of $\frac{1}{0}$. It will hereafter be shown that this circumstance indicates that the two premises were not independent. To verify this, let us resume the equations of the premises in their reduced forms, viz.,

$$\begin{aligned} s(1-qr)+qr(1-s) &= 0, \\ tq(1-r)+tr(1-q) &= 0. \end{aligned}$$

Now if the first members of these equations have any common constituents, they will appear on multiplying the equations together. If we do this we obtain

$$stq(1-r)+str(1-q)=0.$$

Whence there will result

$$stq(1-r)=0,\ str(1-q)=0,$$

these being equations which are deducible from either of the primitive ones. Their interpretations are—

Similar triangles which have their corresponding angles equal have their corresponding sides proportional.

Similar triangles which have their corresponding sides proportional have their corresponding angles equal.

And these conclusions are equally deducible from either premiss *singly*. In this respect, according to the definitions laid down, the premises are not independent.

14. Let us, in conclusion, resume the problem discussed in illustration of the first method of this chapter, and endeavour to ascertain, by the present method, what may be concluded from the presence of the property C, with reference to the properties A and B.

We found on eliminating the symbols v the following equations, viz.:

$$xy(wz+\bar{w}\bar{z})=0, \tag{1}$$
$$yz(x\bar{w}+\bar{x}w)=0, \tag{2}$$
$$\bar{x}\bar{y}=\bar{w}\bar{z}. \tag{3}$$

From these we are to eliminate w and determine z. Now (1) and (2) already satisfy the condition $V(1-V)=0$. The third equation gives, on bringing the terms to the first side, and squaring

$$\bar{x}\bar{y}(1-\bar{w}\bar{z})+\bar{w}\bar{z}(1-\bar{x}\bar{y})=0. \tag{4}$$

Adding (1) (2) and (4) together, we have

$$xy(wz+\bar{w}\bar{z})+yz(x\bar{w}+\bar{x}w)+\bar{x}\bar{y}(1-\bar{w}\bar{z})+\bar{w}\bar{z}(1-\bar{x}\bar{y})=0.$$

13. 还有一点观察需要作出，它说明了一条已经陈述过的原则。在(5)中 1 – s 的完整展开式里，有两项以 2 为系数，而不是1⁰。这一点将在以后被表明，即这种情况表明两个前提并非独立。为验证这一点，让我们重新列出各前提的约化形式方程，即，

$$\begin{aligned} s(1-qr)+qr(1-s) &= 0, \\ tq(1-r)+tr(1-q) &= 0. \end{aligned}$$

现在，如果这些方程的第一个成员有任何共同的成分，它们将在方程相乘时出现。如果我们这样做，我们得到

$$stq(1-r)+str(1-q)=0.$$

由此将得出

$$stq(1-r)=0,\ str(1-q)=0,$$

这些是可以从任一原始方程导出的方程。它们的解释是--

对应角相等的相似三角形具有其
对应边成比例。

对应边成比例的相似三角形具有其
同位角相等。

而这些结论同样可以从任一前提单独导出。在这方面，根据所规定的定义，这些前提不是独立的。

14.最后，让我们重新讨论为说明本章第一种方法而讨论的问题，并努力通过当前方法确定，从性质 C 的存在可以就性质 A 和 B 得出什么结论。

我们消去符号v后得到以下方程，即：

$$xy(wz+\bar{w}\bar{z})=0, \tag{1}$$
$$yz(x\bar{w}+\bar{x}w)=0, \tag{2}$$
$$\bar{x}\bar{y}=\bar{w}\bar{z}. \tag{3}$$

从这些中我们要消去 w 并确定 z。现在(1)和(2)已经满足条件 V(1 – V) = 0。第三个方程在将
将各项移至第一边，然后平方

$$\bar{x}\bar{y}(1-\bar{w}\bar{z})+\bar{w}\bar{z}(1-\bar{x}\bar{y})=0. \tag{4}$$

将(1)(2)与(4)相加，我们得到

$$xy(wz+\bar{w}\bar{z})+yz(x\bar{w}+\bar{x}w)+\bar{x}\bar{y}(1-\bar{w}\bar{z})+\bar{w}\bar{z}(1-\bar{x}\bar{y})=0.$$

Eliminating w, we get

$$(xyz + yz\bar{x} + \bar{x}\bar{y})\{xy\bar{z} + yzx + \bar{x}\bar{y}z + \bar{z}(1 - \bar{x}\bar{y})\} = 0.$$

Now, on multiplying the terms in the second factor by those in the first successively, observing that

$$x\bar{x} = 0,\ y\bar{y} = 0,\ z\bar{z} = 0,$$

nearly all disappear, and we have only left

$$xyz + \bar{x}\bar{y}z = 0; \tag{5}$$

whence

$$\begin{aligned} z &= \frac{0}{xy + \bar{x}\bar{y}} \\ &= 0xy + \frac{0}{0}x\bar{y} + \frac{0}{0}\bar{x}y + 0\bar{x}\bar{y} \\ &= \frac{0}{0}x\bar{y} + \frac{0}{0}\bar{x}y, \end{aligned}$$

furnishing the interpretation. *Wherever the property C is found, either the property A or the property B will be found with it, but not both of them together.*

From the equation (5) we may readily deduce the result arrived at in the previous investigation by the method of arbitrary constant multipliers, as well as any other proposed forms of the relation between x, y, and z; e. g. *If the property B is absent, either A and C will be jointly present, or C will be absent.* And conversely, *If A and C are jointly present, B will be absent.* The converse part of this conclusion is founded on the presence of a term xz with unity for its coefficient in the developed value of $\bar{y}$.

消去w，我们得到

$$(xyz + yz\bar{x} + \bar{x}\bar{y})\{xy\bar{z} + yzx + \bar{x}\bar{y}z + \bar{z}(1 - \bar{x}\bar{y})\} = 0.$$

现在，将第二个因子中的项与第一个因子中的项依次相乘，注意到

$$x\bar{x} = 0,\ y\bar{y} = 0,\ z\bar{z} = 0,$$

几乎全部消失，我们只剩下

$$xyz + \bar{x}\bar{y}z = 0; \tag{5}$$

由此

$$\begin{aligned} z &= \frac{0}{xy + \bar{x}\bar{y}} \\ &= 0xy + \frac{0}{0}x\bar{y} + \frac{0}{0}\bar{x}y + 0\bar{x}\bar{y} \\ &= \frac{0}{0}x\bar{y} + \frac{0}{0}\bar{x}y, \end{aligned}$$

提供了解释。凡发现该性质之处，将同时发现该性质或该性质，但不会两者同时被发现。

由方程(5)，我们可以容易地推出前次研究中通过任意常数乘数方法所得的结果，以及所提出的x、y与z之间关系的任何其他形式；例如，如果该性质不存在，那么要么与将共同存在，要么将不存在。反之，如果与共同存在，将不存在。本结论的逆部分建立在 y的展开值中存在一个系数为一的项xz这一事实之上。

Chapter IX

ON CERTAIN METHODS OF ABBREVIATION.

1. Though the three fundamental methods of development, elimination, and reduction, established and illustrated in the previous chapters, are sufficient for all the practical ends of Logic, yet there are certain cases in which they admit, and especially the method of elimination, of being simplified in an important degree; and to these I wish to direct attention in the present chapter. I shall first demonstrate some propositions in which the principles of the above methods of abbreviation are contained, and I shall afterwards apply them to particular examples.

Let us designate as class terms any terms which satisfy the fundamental law $V(1-V)=0$. Such terms will individually be constituents; but, when occurring together, will not, as do the terms of a development, necessarily involve the same symbols in each. Thus $ax+bxy+cyz$ may be described as an expression consisting of three class terms, x, xy, and yz, multiplied by the coefficients a, b, c respectively. The principle applied in the two following Propositions, and which, in some instances, greatly abbreviates the process of elimination, is that of the *rejection of superfluous class terms*; those being regarded as superfluous which do not add to the constituents of the final result.

Proposition I.

2. *From any equation, $V=0$, in which V consists of a series of class terms having positive coefficients, we are permitted to reject any term which contains another term as a factor, and to change every positive coefficient to unity.*

For the significance of this series of positive terms depends only upon the number and nature of the constituents of its final expansion, i.e. of its expansion with reference to all the symbols which it involves, and not at all upon the actual values of the coefficients (VI. 5). Now let x be any term of the series, and xy any other term having x as a factor. The expansion of x with reference to the symbols x and y will be

$$xy+x\left(1-y\right),$$

and the expansion of the sum of the terms x and xy will be

$$2xy+x\left(1-y\right).$$

第九章

论某些简略方法。

1.尽管前几章所建立并阐明的展开、消元与归约这三种基本方法足以满足逻辑学的一切实际目的，然而在某些情况下，它们--尤其是消元方法--可以在重要程度上得到简化；本章我想提请注意这些情况。我将首先证明一些包含上述简化方法原理的命题，然后再将它们应用于具体例子。

让我们把满足基本法则V(1 – V) = 0的任何项称为类项。这样的项各自将是成分；但当它们一起出现时，并不像展开式的项那样必然在每一项中涉及相同的记号。因此，ax + bxy + cyz可以描述为一个由三个类项x、xy和yz分别乘以系数a、b、c所组成的表达式。以下两个命题中所应用的原理，在某些情况下大大缩短了消元过程，那就是舍弃多余类项的原理；那些不增加最终结果的成分的类项被视为多余。

命题一。

2. 从任何方程 V = 0 ，其中由一系列具有正系数的类项组成，我们被允许拒绝任何以另一项为因子的项，并将每个正系数变为一。

因为这系列正项的意义仅取决于其最终展开式，即其关于所涉及的所有记号的展开式的成分的数目和性质，而完全不取决于系数的实际值(VI.5)。现在令 x 为该系列中的任一项，xy 为任何其他以 x 为因子的项。x 关于记号 x 和 y 的展开式将是

$$xy+x\left(1-y\right),$$

and the expansion of the sum of the terms x and xy will be

$$2xy+x\left(1-y\right).$$

But by what has been said, these expressions occurring in the first member of an equation, of which the second member is 0, and of which all the coefficients of the first member are positive, are equivalent; since there must exist simply the two constituents xy and $x\,(1-y)$ in the final expansion, whence will simply arise the resulting equations

$$xy = 0,\ x\,(1-y) = 0.$$

And, therefore, the aggregate of terms $x + xy$ may be replaced by the single term x.

The same reasoning applies to all the cases contemplated in the Proposition. Thus, if the term x is repeated, the aggregate $2x$ may be replaced by x, because under the circumstances the equation $x = 0$ must appear in the final reduction.

PROPOSITION II.

3. *Whenever in the process of elimination we have to multiply together two factors, each consisting solely of positive terms, satisfying the fundamental law of logical symbols, it is permitted to reject from both factors any common term, or from either factor any term which is divisible by a term in the other factor; provided always, that the rejected term be added to the product of the resulting factors.*

In the enunciation of this Proposition, the word "divisible" is a term of convenience, used in the algebraic sense, in which xy and $x\,(1-y)$ are said to be divisible by x.

To render more clear the import of this Proposition, let it be supposed that the factors to be multiplied together are $x + y + z$ and $x + yw + t$. It is then asserted, that from these two factors We may reject the term x, and that from the second factor we may reject the term yw, provided that these terms be transferred to the final product. Thus, the resulting factors being $y + z$ and t, if to their product $yt + zt$ we add the terms x and yw, we have

$$x + yw + yt + zt,$$

as an expression equivalent to the product of the given factors $x + y + z$ and $x + yw + t$; equivalent namely in the process of elimination.

Let us consider, first, the case in which the two factors have a common term x, and let us represent the factors by the expressions $x + P$, $x + Q$, supposing P in the one case and Q in the other to be the sum of the positive terms additional to x.

Now,

$$(x + P)(x + Q) = x + xP + xQ + PQ. \tag{1}$$

But the process of elimination consists in multiplying certain factors together, and equating the result to 0. Either then the second member of the above equation is to be equated to 0, or it is a factor of some expression which is to be equated to 0.

但根据已所述，这些表达式出现在一个方程的第一个成员中，该方程的第二个成员为 0，且其第一个成员的所有系数为正，它们是等价的；因为在最终展开式中必定仅存在两个成分 xy 和 x(1 – y)，由此将仅产生结果方程

$$xy = 0,\ x\,(1-y) = 0.$$

因此，项的聚合 x + xy 可以用单项 x 替换。

同样的推理适用于该命题中所考虑的所有情况。因此，如果项 x 被重复，聚合 2 x 可以用 x 替换，因为在这种情况下，方程 x = 0 必定出现在最终化简中。

命题二。

3. 在消元过程中，每当我们必须将两个因子相乘，而每个因子都仅由正项组成并满足逻辑记号的基本定律时，允许从两个因子中舍弃任何共同的项，或从任一因子中舍弃任何可被另一因子中的项整除的项；但始终须将所舍弃的项加到所得因子的乘积上。

在本命题的表述中，“可整除”一词是一个方便用语，用于代数意义，在此意义下，xy 和 x(1 – y) 被称为可被 x 整除。

为使本命题的含义更加清楚，设相乘的因子为x + y + z和x + yw + t。于是可以断言，我们可以从这两个因子中舍弃项x，并且可以从第二个因子中舍弃项yw，只要把这些项转移到最终乘积中。因此，所得因子为y + z和t，若在其乘积yt + zt上加上项x和yw，我们便得到

$$x + yw + yt + zt,$$

作为一个表达式，它等价于给定因子 x + y + z 与 x + yw + t 的乘积；即在消元过程中等价。

让我们首先考虑两个因子有一个共同项 x 的情形，并用表达式 x + P、x + Q 表示这两个因子，假定在一种情形下 P、在另一种情形下 Q 是附加于 x 的正项之和。

现在，

$$(x + P)(x + Q) = x + xP + xQ + PQ. \tag{1}$$

但消元过程在于将某些因子相乘，并令结果等于 0。那么，或者上述方程的第二边须等于 0，或者它是某个须等于 0 的表达式的因子。

If the former alternative be taken, then, by the last Proposition, we are permitted to reject the terms xP and xQ, inasmuch as they are positive terms having another term x as a factor. The resulting expression is

$$x + PQ,$$

which is what we should obtain by rejecting x from both factors, and adding it to the product of the factors which remain.

Taking the second alternative, the only mode in which the second member of (1) can affect the final result of elimination must depend upon the number and nature of its constituents, both which elements are unaffected by the rejection of the terms xP and xQ. For that development of x includes all possible constituents of which x is a factor.

Consider finally the case in which one of the factors contains a term, as xy, divisible by a term, x, in the other factor.

Let $x + P$ and $xy + Q$ be the factors. Now

$$(x + P)(xy + Q) = xy + xQ + xyP + PQ.$$

But by the reasoning of the last Proposition, the term xyP may be rejected as containing another positive term xy as a factor, whence we have

$$\begin{aligned} &xy + xQ + PQ \\ &= xy + (x + P)Q. \end{aligned}$$

But this expresses the rejection of the term xy from the second factor, and its transference to the final product. Wherefore the Proposition is manifest.

PROPOSITION III.

4. *If t be any symbol which is retained in the final result of the elimination of any other symbols from any system of equations, the result of such elimination may be expressed in the form*

$$Et + E\,(1 - t) = 0,$$

in which E is formed by making in the proposed system $t = 1$, and eliminating the same other symbols; and E' by making in the proposed system $t = 0$, and eliminating the same other symbols.

For let $\phi\,(t) = 0$ represent the final result of elimination. Expanding this equation, we have

$$\phi\,(1)\,t + \phi\,(0)\,(1 - t) = 0.$$

Now by whatever process we deduce the function $\phi\,(t)$ from the proposed system of equations, by the same process should we deduce $\phi\,(1)$, if in those equations t were changed into 1; and by the same process should we deduce $\phi\,(0)$, if in the same equations t were changed into 0. Whence the truth of the proposition is manifest.

若采取前一种选择，那么根据上一命题，我们允许舍弃项 xP 和 xQ，因为它们是以另一项 x 为因子的正项。所得表达式为

$$x + PQ,$$

这正是我们通过从两个因子中舍弃 x 并将其加到剩余因子的乘积上所应得到的结果。

采取第二种选择时，(1) 的第二边能够影响消元的最终结果的唯一方式，必定取决于其构成项的数量和性质，而这两个要素都不受舍弃项 xP 和 xQ 的影响。因为 x 的那个展开式包含以 x 为因子的所有可能的构成项。

Consider finally the case in which one of the factors contains a term, as xy, divisible by a term, x, in the other factor.

Let $x + P$ and $xy + Q$ be the factors. Now

$$(x + P)(xy + Q) = xy + xQ + xyP + PQ.$$

但根据上一命题的推理，项 xyP 可以作为包含另一正项 xy 为因子而被舍弃，由此我们得到

$$\begin{aligned} &xy + xQ + PQ \\ &= xy + (x + P)Q. \end{aligned}$$

但这表达的是从第二个因子中舍弃项 xy，并将其转移到最终乘积中。因此本命题是显然的。

命题三。

4. 若是从任何方程组中消去任何其他记号后保留在最终结果中的任何记号，则这种消元的结果可以表示为如下形式

$$Et + E\,(1 - t) = 0,$$

其中是通过在所给系统中 t = 1 并消去同样的其他记号而形成的；而是通过在所给系统中 t = 0 并消去同样的其他记号而形成的。

For let $\phi\,(t) = 0$ represent the final result of elimination. Expanding this equation, we have

$$\phi\,(1)\,t + \phi\,(0)\,(1 - t) = 0.$$

现在，无论我们通过什么过程从所给方程组推出函数 φ(t)，我们都应通过同一过程推出 φ(1)，如果在那些方程中 t 被变为 1；并且我们应通过同一过程推出 φ(0)，如果在同样的方程中 t 被变为 0。由此本命题的真理性是显然的。

5. Of the three propositions last proved, it may be remarked, that though quite unessential to the strict development or application of the general theory, they yet accomplish important ends of a practical nature. By Prop. 1 we can simplify the results of addition; by Prop. 2 we can simplify those of multiplication; and by Prop. 3 we can break up any tedious process of elimination into two distinct processes, which will in general be of a much less complex character. This method will be very frequently adopted, when the final object of inquiry is the determination of the value of t, in terms of the other symbols which remain after the elimination is performed.

6. Ex. 1.—Aristotle, in the Nicomachean Ethics, Book II. Cap. 3, having determined that actions are virtuous, not as possessing in themselves a certain character, but as implying a certain condition of mind in him who performs them, viz., that he perform them knowingly, and with deliberate preference, and for their own sakes, and upon fixed principles of conduct, proceeds in the two following chapters to consider the question, whether virtue is to be referred to the genus of Passions, or Faculties, or Habits, together with some other connected points. He grounds his investigation upon the following premises, from which, also, he deduces the general doctrine and definition of moral virtue, of which the remainder of the treatise forms an exposition.

PREMISES.

1. Virtue is either a passion (πάθος), or a faculty (δύναμις), or a habit (ἕξις).

2. Passions are not things according to which we are praised or blamed, or in which we exercise deliberate preference.

3. Faculties are not things according to which we are praised or blamed, and which are accompanied by deliberate preference.

4. Virtue is something according to which we are praised or blamed, and which is accompanied by deliberate preference.

5. Whatever art or science makes its work to be in a good state avoids extremes, and keeps the mean in view relative to human nature (τὸ μέσον . . . πρὸς ἡμᾶς)

6. Virtue is more exact and excellent than any art or science. This is an argument *à fortiori*. If science and true art shun defect and extravagance alike, much more does virtue pursue the undeviating line of moderation. If *they* cause their work to be in a good state, much more reason have to we to say that Virtue causeth her peculiar work to be "in a good state." Let the final premiss be thus interpreted. Let us also pretermit all reference to praise or blame, since the mention of these in the premises accompanies only the mention of deliberate preference, and this is an element which we purpose to retain. We may then assume as our representative symbols–

5.关于最后所证明的三个命题，可以指出，尽管它们对于一般理论的严格展开或应用完全无关紧要，然而它们却实现了具有实际性质的重要目的。通过命题1，我们可以简化加法的结果；通过命题2，我们可以简化乘法的结果；通过命题3，我们可以把任何冗长的消元过程分解为两个不同的过程，而这两个过程一般具有小得多的复杂性。当研究的最终目的是在消元之后所留下的其他记号中确定t的值时，这种方法将被非常频繁地采用。

6. 例1。--亚里士多德在《尼各马可伦理学》第二卷第三章中，已经确定行为之所以为有德，并非因其自身具有某种性质，而是因为行为者具有某种心灵状态，即他明知而为、出于审慎的抉择、为其自身之故、并依据固定的行为原则而行；随后他在接下来的两章中着手考察这样一个问题：德性应归于激情、能力还是习惯这一属，以及若干其他相关之点。他以如下前提作为其考察的基础，并由此推出道德德性的一般学说与定义，该论著余下部分则构成对此的阐述。

前提。

1. 德性要么是情感(πὰθος)，要么是能力(δύναμις)，要么是习惯(ἕξις)。

2.情感不是我们据以受赞扬或责备、或我们据以作出 deliberate preference 的事物。

3.能力不是我们据以受赞扬或责备、并且伴有 deliberate preference 的事物。

4.德性是我们据以受赞扬或责备、并且伴有 deliberate preference 的某种事物。

5.任何使自己的作品处于良好状态的艺术或科学都避免极端，并着眼于相对于人性的中道(τὸμὲσον...πρὸς ἡμ῀ ας)

6. 德性比任何技艺或科学都更为精确、更为卓越。这是一个`*a fortiori*的论证。如果科学与真正的技艺同样避免缺陷与过度，那么德性就更要遵循不偏不倚的中道。如果它们使其作品处于良好状态，那么我们更有理由说，德性使其特有的作品"处于良好状态"。让最后的前提如此解释。我们也将略去一切关于赞扬或责备的提及，因为前提中对这些的提及只与审慎的偏好相伴随，而这是我们打算保留的一个元素。于是我们可以假定以下记号作为我们的代表--

v = virtue.
p = passions.
f = faculties.
h = habits.
d = things accompanied by deliberate preference.
g = things causing their work to be in a good state.
m = things keeping the mean in view relative to human nature.

Using, then, q as an indefinite class symbol, our premises will be expressed by the following equations:

$$\begin{aligned} v &= q\{p(1-f)(1-h)+f(1-p)(1-h)+h(1-p)(1-f)\}. \\ p &= q(1-d). \\ f &= q(1-d). \\ v &= qd. \\ g &= qm. \\ v &= qg. \end{aligned}$$

And separately eliminating from these the symbols q,

$$v\{1-p(1-f)(1-h)-f(1-p)(1-h)-h(1-p)(1-f)\}=0. \quad (1)$$
$$pd=0. \quad (2)$$
$$fd=0. \quad (3)$$
$$v(1-d)=0. \quad (4)$$
$$g(1-m)=0. \quad (5)$$
$$v(1-g)=0. \quad (6)$$

We shall first eliminate from (2), (3), and (4) the symbol d, and then determine v in relation to p, f, and h. Now the addition of (2), (3), and (4) gives

$$(p+f)d+v(1-d)=0.$$

From which, eliminating d in the ordinary way, we find

$$(p+f)v=0. \quad (7)$$

Adding this to (1), and determining v, we find

$$v=\frac{0}{p+f+1-p(1-f)(1-h)-f(1-p)(1-h)-h(1-f)(1-p)}.$$

Whence by development,

$$v=\frac{0}{0}h(1-f)(1-p).$$

The interpretation of this equation is: *Virtue is a habit, and not a faculty or a passion.*

v = virtue.
p = passions.
f = faculties.
h = habits.

d = 伴有 deliberate preference 的事物。g = 使其作品处于良好状态的事物。m = 着眼于相对于人性的中道的事物。

那么，使用 q 作为一个不定类记号，我们的前提将由以下方程表达：

$$\begin{aligned} v &= q\{p(1-f)(1-h)+f(1-p)(1-h)+h(1-p)(1-f)\}. \\ p &= q(1-d). \\ f &= q(1-d). \\ v &= qd. \\ g &= qm. \\ v &= qg. \end{aligned}$$

并从中分别消去符号q,

$$v\{1-p(1-f)(1-h)-f(1-p)(1-h)-h(1-p)(1-f)\}=0. \quad (1)$$
$$pd=0. \quad (2)$$
$$fd=0. \quad (3)$$
$$v(1-d)=0. \quad (4)$$
$$g(1-m)=0. \quad (5)$$
$$v(1-g)=0. \quad (6)$$

我们将首先从(2)、(3)和(4)中消去记号 d，然后确定 v 与 p、f 和 h 的关系。现在，(2)、(3)和(4)相加得到

$$(p+f)d+v(1-d)=0.$$

由此，按通常方式消去 d，我们得到

$$(p+f)v=0. \quad (7)$$

将此式加到(1)上，并确定v，我们得到

$$v=\frac{0}{p+f+1-p(1-f)(1-h)-f(1-p)(1-h)-h(1-f)(1-p)}.$$

由此通过发展，

$$v=\frac{0}{0}h(1-f)(1-p).$$

这个方程的解释是：德性是一种习惯，而不是一种能力或情感。

Next, we will eliminate f, p, and g from the original system of equations, and then determine v in relation to h, d, and m. We will in this case eliminate p and f together. On addition of (1), (2), and (3), we get

$$v\{1-p(1-f)(1-h)-f(1-p)(1-h)-h(1-p)(1-f)\} \\ +pd+fd=0.$$

Developing this with reference to p and f, we have

$$(v+2d)pf+(vh+d)p(1-f)+(vh+d)(1-p)f \\ +v(1-h)(1-p)(1-f)=0.$$

Whence the result of elimination will be

$$(v+2d)(vh+d)(vh+d)v(1-h)=0.$$

Now $v+2d=v+d+d$, which by Prop. I. is reducible to $v+d$. The product of this and the second factor is

$$(v+d)(vh+d),$$

which by Prop. II. reduces to $d+v(vh)$ or $vh+d$.

In like manner, this result, multiplied by the third factor, gives simply $vh+d$. Lastly, this multiplied by the fourth factor, $v(1-h)$, gives, as the final equation,

$$vd(l-h)=0 \tag{8}$$

It remains to eliminate g from (5) and (6). The result is

$$v(1-m)=0 \tag{9}$$

Finally, the equations (4), (8), and (9) give on addition

$$v(1-d)+vd(1-h)+v(1-m)=0$$

from which we have

$$v=\frac{0}{1-d+d(1-h)+1-m}.$$

And the development of this result gives

$$v=\frac{0}{0}hdm,$$

f which the interpretation is,–*Virtue is a habit accompanied by deliberate preference, and keeping in view the mean relative to human nature.*

Properly speaking, this is not a definition, but a description of virtue. It is *all*, however, that can be correctly inferred from the premises. Aristotle specially connects with it the necessity of prudence, to determine the safe and middle line

接下来，我们将从原方程组中消去 f、p 和 g，然后确定 v 与 h、d 和 m 的关系。在这种情况下，我们将一起消去 p 和 f。将(1)、(2)和(3)相加，我们得到

$$v\{1-p(1-f)(1-h)-f(1-p)(1-h)-h(1-p)(1-f)\} \\ +pd+fd=0.$$

参照 p 和 f 来展开这一内容，我们便得到

$$(v+2d)pf+(vh+d)p(1-f)+(vh+d)(1-p)f \\ +v(1-h)(1-p)(1-f)=0.$$

于是消元的结果将是

$$(v+2d)(vh+d)(vh+d)v(1-h)=0.$$

现在 v + 2 d = v + d + d，根据命题 I.，这可归约为 v + d。它与第二个因子的乘积是

$$(v+d)(vh+d),$$

which by Prop. II. reduces to $d+v(vh)$ or $vh+d$.

In like manner, this result, multiplied by the third factor, gives simply $vh+d$. Lastly, this multiplied by the fourth factor, $v(1-h)$, gives, as the final equation,

$$vd(l-h)=0 \tag{8}$$

剩下的就是从(5)和(6)中消去g。结果是

$$v(1-m)=0 \tag{9}$$

最后，将方程(4)、(8)和(9)相加可得

$$v(1-d)+vd(1-h)+v(1-m)=0$$

由此我们得到

$$v=\frac{0}{1-d+d(1-h)+1-m}.$$

而这一结果的发展给出

$$v=\frac{0}{0}hdm,$$

f 其解释是，--德性是一种伴有 *deliberate preference*、并着眼于相对于人性的中道的习惯。

严格说来，这不是一个定义，而是对德性的一种描述。然而，它却是全部能够从前提出正确推断出来的东西。亚里士多德特别将审慎的必然性与它联系起来，以确定安全和中间路线

of action; and there is no doubt that the ancient theories of virtue generally partook more of an intellectual character than those (the theory of utility excepted) which have most prevailed in modern days. Virtue was regarded as consisting in the right state and habit of the whole mind, rather than in the single supremacy of conscience or the moral facility. And to some extent those theories were undoubtedly right. For though unqualified obedience to the dictates of conscience is an essential element of virtuous conduct, yet the conformity of those dictates with those unchanging principles of rectitude (αἰώνια δίκαια)which are founded in, or which rather are themselves the foundation of the constitution of things, is another element. And generally this conformity, in any high degree at least, is inconsistent with a state of ignorance and mental hebetude. Reverting to the particular theory of Aristotle, it will probably appear to most that it is of too negative a character, and that the shunning of extremes does not afford a sufficient scope for the expenditure of the nobler energies of our being. Aristotle seems to have been imperfectly conscious of this defect of his system, when in the opening of his seventh book he spoke of an "heroic virtue"[1] rising above the measure of human nature.

7. I have already remarked (VIII. 1) that the theory of single equations or propositions comprehends questions which cannot be fully answered, except in connexion with the theory of systems of equations. This remark is exemplified when it is proposed to determine from a given single equation the relation, not of some single elementary class, but of some compound class, involving in its expression more than one element, in terms of the remaining elements. The following particular example, and the succeeding general problem, are of this nature.

Ex. 2.—Let us resume the symbolical expression of the definition of wealth employed in Chap, VII., viz.,

$$w = st\left\{p + r\left(1 - p\right)\right\},$$

wherein, as before,
w = wealth,
s = things limited in supply,
t = things transferable,
p = things productive of pleasure,
r = things preventive of pain;
and suppose it required to determine hence the relation of things transferable and productive of pleasure, to the other elements of the definition, viz., wealth, things limited in supply, and things preventive of pain.

The expression for things transferable and productive of pleasure is tp. Let us represent this by a new symbol y. We have then the equations

$$\begin{aligned} w &= st\left\{p + r\left(1 - p\right)\right\}, \\ y &= tp, \end{aligned}$$

[1] τὴν ὑπὲρ ἡμᾶς ἀρετὴν ἠρωϊκήν τινα και θειαν–Nic. Eth. Book vii.

行动；毫无疑问，古代关于德性的理论总体上比那些(效用理论除外)在近代最为流行的理论更具理智的特征。德性被视为在于整个心灵的正当状态和习惯，而非在于良心的单一至上或道德才能。并且在某种程度上，那些理论无疑是正确的。因为尽管对良心命令的无条件服从是德性行为的一个本质要素，但那些命令与那些不变的正直原则(αἰώνια δίκαια)--它们奠基于事物的构造之中，或者更确切地说，它们本身就是事物构造的基础--的一致性，则是另一个要素。而且一般来说，这种一致性，至少在任何高度上，都与无知和心智迟钝的状态不相容。回到亚里士多德的具体理论，在大多数人看来，它可能具有过于否定的特征，而且回避极端并不能为施展我们存在中更高贵的能量提供充分的空间。亚里士多德似乎对他的体系的这一缺陷认识不足，当他在其第七卷的开篇谈到一种超越人性尺度的"英雄德性"[1]时。

7. 我已经指出(VIII.1)，单一方程或命题的理论包含了一些问题，这些问题除非与方程组理论相关联，否则无法得到完全解答。当提出从一给定的单一方程确定不是某个单一基本类、而是某个复合类--其表达式中涉及多于一个元素--与其余元素的关系时，这一评述便得到了例证。以下的特殊例子以及随后的普遍问题，即属此种性质。

例2--让我们重新采用第七章中所使用的财富定义的符号表达式，即，

$$w = st\left\{p + r\left(1 - p\right)\right\},$$

wherein, as before,
w = wealth,

s = 供给有限的事物，t = 可转让的事物，p = 产生快乐的事物，r = 防止痛苦的事物；并假定要求由此确定可转让且产生快乐的事物与定义中其他元素，即财富、供给有限的事物和防止痛苦的事物之间的关系。

可转让且产生快乐的事物的表达式为tp。让我们用一个新的记号y来表示它。于是我们得到方程

$$\begin{aligned} w &= st\left\{p + r\left(1 - p\right)\right\}, \\ y &= tp, \end{aligned}$$

1 τὴν ὑπὲρ ἡμᾶς ἀρετὴν ἠρωϊκήν τινα και θειαν - 《尼各马可伦理学》第七卷

from which, if we eliminate t and p, we may determine y as a function of w, s, and r. The result interpreted will give the relation sought.

Bringing the terms of these equations to the first side, we have

$$\begin{aligned} w - stp - str\,(1-p) &= 0. \\ y - tp &= 0. \end{aligned} \tag{3}$$

And adding the squares of these equations together,

$$w + stp + str\,(1-p) - 2wstp - 2wstr\,(1-p) + y + tp - 2ytp = 0. \tag{4}$$

Developing the first member with respect to t and p, in order to eliminate those symbols, we have

$$\begin{aligned} &(w + s - 2ws + 1 - y)\,tp + (w + sr - 2wsr + y)\,t\,(1-p) \\ &\quad + (w+y)\,(1-t)\,p + (w+y)\,(1-t)\,(1-p)\,; \end{aligned} \tag{5}$$

and the result of the elimination of t and p will be obtained by equating to 0 the product of the four coefficients of

$$tp, t\,(1-p)\,, (1-t)\,p, \text{ and } (1-t)\,(1-p)\,.$$

Or, by Prop. 3, the result of the elimination of t and p from the above equation will be of the form

$$Ey + E'\,(1-y)\,,$$

wherein E is the result obtained by changing in the given equation y into 1, and then eliminating t and p; and E' the result obtained by changing in the same equation y into 0, and then eliminating t and p. And the mode in each case of eliminating t and p is to multiply together the coefficients of the four constituents tp, $t\,(1-p)$, &c.

If we make $y = 1$, the coefficients become–

1st. $w\,(1-s) + s\,(1-w)$

2nd. $1 + w\,(1-sr) + s\,(1-w)\,r$, equivalent to 1 by Prop. I.

3rd and 4th. $1 + w$, equivalent to 1 by Prop. I.

Hence the value of E will be

$$w\,(1-s) + s\,(1-w)\,.$$

Again, in (5) making $y = 0$, we have for the coefficients–

1st. $1 + w\,(1-s) + s\,(1-w)$, equivalent to 1.

2nd. $w\,(1-sr) + sr\,(1-w)$.

3rd and 4th. w.

The product of these coefficients gives

$$E' = w\,(1-sr)\,.$$

由此，如果我们消去t和p，我们就可以把y确定为w、s和r的函数。对结果加以解释，就会给出所求的关系。

将上述方程的各项移至第一边，我们得到

$$\begin{aligned} w - stp - str\,(1-p) &= 0. \\ y - tp &= 0. \end{aligned} \tag{3}$$

并将这些方程的平方相加，

$$w + stp + str\,(1-p) - 2wstp - 2wstr\,(1-p) + y + tp - 2ytp = 0. \tag{4}$$

为了消去那些记号，将第一个成员关于t和p展开，我们得到

$$\begin{aligned} &(w + s - 2ws + 1 - y)\,tp + (w + sr - 2wsr + y)\,t\,(1-p) \\ &\quad + (w+y)\,(1-t)\,p + (w+y)\,(1-t)\,(1-p)\,; \end{aligned} \tag{5}$$

而消去t和p的结果，将通过令以下四个系数的乘积等于0而得到

$$tp, t\,(1-p)\,, (1-t)\,p, \text{ and } (1-t)\,(1-p)\,.$$

或者，根据命题3，从上述方程中消去t和p的结果将具有如下形式

$$Ey + E'\,(1-y)\,,$$

其中E是通过在给定方程中把y变为1，然后消去t和p而得到的结果；而E ′是通过在同一方程中把y变为0，然后消去t和p而得到的结果。而在每种情况下消去t和p的方式，都是把四个成分tp、t(1 – p)等的系数相乘。

If we make $y = 1$, the coefficients become–

1st. $w\,(1-s) + s\,(1-w)$

2nd. $1 + w\,(1-sr) + s\,(1-w)\,r$, equivalent to 1 by Prop. I.

3rd and 4th. $1 + w$, equivalent to 1 by Prop. I.

Hence the value of E will be

$$w\,(1-s) + s\,(1-w)\,.$$

Again, in (5) making $y = 0$, we have for the coefficients–

1st. $1 + w\,(1-s) + s\,(1-w)$, equivalent to 1.

2nd. $w\,(1-sr) + sr\,(1-w)$.

3rd and 4th. w.

The product of these coefficients gives

$$E' = w\,(1-sr)\,.$$

The equation from which y is to be determined, therefore, is

$$\{w(1-s)+s(1-w)\}\,y+w(1-sr)(1-y)=0,$$
$$\therefore y=\frac{w(1-sr)}{w(1-sr)-w(1-s)-s(1-w)};$$

and expanding the second member,

$$\begin{aligned} y \quad &= \tfrac{0}{0}wsr+ws(1-r)+\tfrac{1}{0}w(1-s)r+\tfrac{1}{0}w(1-s)(1-r)\\ &\quad +0(1-w)sr+0(1-w)s(1-r)+\tfrac{0}{0}(1-w)(1-s)r\\ &\quad +\tfrac{0}{0}(1-w)(1-s)(1-r); \end{aligned}$$

whence reducing.

$$y=ws(1-r)+\frac{0}{0}wsr+\frac{0}{0}(1-w)(1-s), \tag{6}$$
$$\text{with } w(1-s)=0. \tag{7}$$

The interpretation of which is–

1st. *Things transferable and productive of pleasure consist of all wealth (limited in supply and) not preventive of pain, an indefinite amount of wealth (limited in supply and) preventive of pain, and an indefinite amount of what is not wealth and not limited in supply.*

2nd. *All wealth is limited in supply.*

I have in the above solution written in parentheses that part of the full description which is implied by the accompanying independent relation (7).

8. The following problem is of a more general nature, and will furnish an easy practical rule for problems such as the last.

General Problem.

Given any equation connecting the symbols x, y..w, z..

Required to determine the logical expression of any class expressed in any way by the symbols x, y.. in terms of the remaining symbols, w, z, &c.

Let us confine ourselves to the case in which there are but two symbols, x, y, and two symbols, w, z, a case sufficient to determine the general Rule.

Let $V=0$ be the given equation, and let $\phi(x,y)$ represent the class whose expression is to be determined.

Assume $t=\phi(x,y)$, then, from the above two equations, x and y are to be eliminated.

Now the equation $V=0$ may be expanded in the form

$$Axy+Bx(1-y)+C(1-x)y+D(1-x)(1-y)=0, \tag{1}$$

A, B, C, and D being functions of the symbols w and z.

Again, as $\phi(x,y)$ represents a class or collection of things, it must consist of a constituent, or series of constituents, whose coefficients are 1.

因此，用以确定y的方程是

$$\{w(1-s)+s(1-w)\}\,y+w(1-sr)(1-y)=0,$$
$$\therefore y=\frac{w(1-sr)}{w(1-sr)-w(1-s)-s(1-w)};$$

并展开第二项，

$$\begin{aligned} y \quad &= \tfrac{0}{0}wsr+ws(1-r)+\tfrac{1}{0}w(1-s)r+\tfrac{1}{0}w(1-s)(1-r)\\ &\quad +0(1-w)sr+0(1-w)s(1-r)+\tfrac{0}{0}(1-w)(1-s)r\\ &\quad +\tfrac{0}{0}(1-w)(1-s)(1-r); \end{aligned}$$

由此化简。

$$y=ws(1-r)+\frac{0}{0}wsr+\frac{0}{0}(1-w)(1-s), \tag{6}$$
$$\text{with } w(1-s)=0. \tag{7}$$

其解释为--第一。可转移且能产生快乐的东西，由全部(供给有限且)不防止痛苦的财富、(供给有限且)防止痛苦的无限量财富，以及无限量的既非财富又非供给有限的东西组成。

第二。一切财富的供给都是有限的。

在上述解答中，我把完整描述中由伴随的独立关系(7)所隐含的那一部分写在括号里。

8.下述问题具有更为一般的性质，并将为诸如上一个问题那样的问题提供一个简便的实用规则。

一般问题。

Given any equation connecting the symbols x, y..w, z..

要求用剩余的记号 w、z 等来确定由记号 x、y.. 以任何方式表达的任何类的逻辑表达式。让我们将自己限制在只有两个记号 x、y 和两个记号 w、z 的情形，这一情形足以确定一般规则。设 V = 0 为给定的方程，并设 φ(x, y) 表示其表达式有待确定的类。假设 t = φ(x, y)，那么，从上述两个方程中，x 和 y 应被消去。

现在方程 V = 0 可以展开为如下形式

$$Axy+Bx(1-y)+C(1-x)y+D(1-x)(1-y)=0, \tag{1}$$

A、B、C和D是记号w和z的函数。

再者，由于φ(x, y)表示一个类或事物的集合，它必定由一个成分或一系列系数为1的成分组成。

Wherefore if the *full* development of $\phi(x, y)$ be represented in the form

$$axy + bx(1-y) + c(1-x)y + d(1-x)(1-y),$$

the coefficients a, b, c, d must each be 1 or 0.

Now reducing the equation $t = \phi(x, y)$ by transposition and squaring, to the form

$$t1 - \phi(x, y) + \phi(x, y)(1-t) = 0;$$

and expanding with reference to x and y, we get

$$\begin{aligned} t(1-a) + a(1-t)xy + t(1-b) + b(1-t)x(1-y) \\ +t(1-c) + c(1-t)(1-x)y \\ +t(1-d) + d(1-t)(1-x)(1-y) = 0; \end{aligned}$$

whence, adding this to (1), we have

$$\begin{aligned} A + t(1-a) + a(1-t)xy \\ +B + t(l-b) + b(l-t)x(1-y) + \&c. = 0. \end{aligned}$$

Let the result of the elimination of x and y be of the form

$$Et + E'(1-t) = 0,$$

then E will, by what has been said, be the reduced product of what the coefficients of the above expansion become when $t = 1$, and E' the product of the same factors similarly reduced by the condition $t = 0$.

Hence E will be the reduced product

$$(A + 1 - a)(B + 1 - b)(C + 1 - c)(D + 1 - d).$$

Considering any factor of this expression, as $A + 1 - a$, we see that when $a = 1$ it becomes A, and when $a = 0$ it becomes $1 + A$, which reduces by Prop. I. to 1. Hence we may infer that E will be the product of the coefficients of those constituents in the development of V whose coefficients in the development of $\phi(x, y)$ are 1.

Moreover E' will be the reduced product

$$(A + a)(B + b)(C + c)(D + d).$$

Considering any one of these factors, as $A + a$, we see that this becomes A when $a = 0$, and reduces to 1 when $a = 1$; and so on for the others. Hence E will be the product of the coefficients of those constituents in the development of y, whose coefficients in the development $\phi(x, y)$ are 0. Viewing these cases together, we may establish the following Rule:

因此，如果φ(x, y)的完整展开式被表示为如下形式

$$axy + bx(1-y) + c(1-x)y + d(1-x)(1-y),$$

the coefficients a, b, c, d must each be 1 or 0.

Now reducing the equation $t = \phi(x, y)$ by transposition and squaring, to the form

$$t1 - \phi(x, y) + \phi(x, y)(1-t) = 0;$$

and expanding with reference to x and y, we get

$$\begin{aligned} t(1-a) + a(1-t)xy + t(1-b) + b(1-t)x(1-y) \\ +t(1-c) + c(1-t)(1-x)y \\ +t(1-d) + d(1-t)(1-x)(1-y) = 0; \end{aligned}$$

由此，将此加到(1)中，我们得到

$$\begin{aligned} A + t(1-a) + a(1-t)xy \\ +B + t(l-b) + b(l-t)x(1-y) + \&c. = 0. \end{aligned}$$

设消去 x 与 y 所得的结果具有如下形式

$$Et + E'(1-t) = 0,$$

那么，根据上述所说，E将是当t = 1时上述展开式的系数所变成的东西的约简乘积，而E ′则是同样受条件t = 0约简的相同因子的乘积。

因此，E 将是简化后的乘积

$$(A + 1 - a)(B + 1 - b)(C + 1 - c)(D + 1 - d).$$

考虑这个表达式的任一因子，如A + 1 – a，我们看到当a = 1时它变为A，而当a = 0时它变为1 + A，根据命题I这约简为1。因此我们可以推断，E将是V的展开式中那些在φ(x, y)的展开式中系数为1的成分的系数的乘积。

而且 E ′将是约化乘积

$$(A + a)(B + b)(C + c)(D + d).$$

考虑这些因子中的任何一个，如A + a，我们看到当a = 0时它变为A，而当a = 1时它约简为1；其余因子依此类推。因此E将是y的展开式中那些在φ(x, y)的展开式中系数为0的成分的系数的乘积。把这些情况合在一起看，我们可以建立以下规则：

9. *To deduce from a logical equation the relation of any class expressed by a given combination of the symbols x, y, &c, to the classes represented by any other symbols involved in the given equation.*

RULE.–*Expand the given equation with reference to the symbols x, y. Then form the equation*

$$Et + E'(1-t) = 0,$$

in which E is the product of the coefficients of all those constituents in the above development, whose coefficients in the expression of the given class are 1, and E' the product of the coefficients of those constituents of the development whose coefficients in the expression of the given class are 0. The value of t deduced from the above equation by solution and interpretation will be the expression required.

NOTE.–*Although in the demonstration of this Rule V is supposed to consist solely of positive terms, it may easily be shown that this condition is unnecessary, and the Rule general, and that no preparation of the given equation is really required.*

10. Ex. 3.–The same definition of wealth being given as in Example 2, required an expression for *things transferable, but not productive of pleasure*, $t(1-p)$, in terms of the other elements represented by w, s, and r.

The equation

$$w - stp - str(1-p) = 0,$$

gives, when squared,

$$w + stp + str(1-p) - 2wstp - 2wstr(1-p) = 0;$$

and developing the first member with respect to t and p,

$$(w + s - 2ws)tp + (w + sr - 2wsr)t(1-p) + w(1-t)p \\ + w(1-t)(1-p) = 0.$$

The coefficients of which it is best to exhibit as in the following equation;

$$w(1-s) + s(1-w)tp + w(1-sr) + sr(1-w)t(1-p) + w(1-t)p \\ + w(1-t)(1-p) = 0$$

Let the function $t(1-p)$ to be determined, be represented by z; then the full development of z in respect of t and p is

$$z = 0tp + t(1-p) + 0(1-t)p + 0(1-t)(1-p).$$

Hence, by the last problem, we have

$$Ez + E'(1-z) = 0;$$
$$\text{where } E = w(1-sr) + sr(1-w);$$
$$E' = w(1-s) + s(1-w) \times w \times w = w(1-s);$$
$$\therefore w(1-sr) + sr(1-w)z + w(1-s)(1-z) = 0.$$

9. 从逻辑方程推导出由记号、、等的给定组合所表达的任何类与给定方程中所涉及的任何其他记号所表示的类之间的关系。

RULE.–*Expand the given equation with reference to the symbols x, y. Then form the equation*

$$Et + E'(1-t) = 0,$$

其中是上述展开式中那些在给定类的表达式中系数为*1*的全部成分的系数的乘积，而是展开式中那些在给定类的表达式中系数为*0*的成分的系数的乘积。通过求解和解释从上述方程推出的值，将是所求的表达式。

注--尽管在本规则的证明中，被假定为仅由正项组成，但很容易表明这个条件是不必要的，而且该规则是普遍的，并且实际上不需要对给定方程作任何准备。

10.例3--给出与例2中相同的财富定义，要求用由w、s和r所表示的其他元素来表达可转让但不产生快乐的事物,t(1 – p)。方程

$$w - stp - str(1-p) = 0,$$

平方后给出，

$$w + stp + str(1-p) - 2wstp - 2wstr(1-p) = 0;$$

并对t和p展开第一个成员，

$$(w + s - 2ws)tp + (w + sr - 2wsr)t(1-p) + w(1-t)p \\ + w(1-t)(1-p) = 0.$$

The coefficients of which it is best to exhibit as in the following equation;

$$w(1-s) + s(1-w)tp + w(1-sr) + sr(1-w)t(1-p) + w(1-t)p \\ + w(1-t)(1-p) = 0$$

令待确定的函数t(1 – p)由z表示；那么
关于t和p的z的充分发展是

$$z = 0tp + t(1-p) + 0(1-t)p + 0(1-t)(1-p).$$

因此，根据上一个问题，我们有

$$Ez + E'(1-z) = 0;$$
$$\text{where } E = w(1-sr) + sr(1-w);$$
$$E' = w(1-s) + s(1-w) \times w \times w = w(1-s);$$
$$\therefore w(1-sr) + sr(1-w)z + w(1-s)(1-z) = 0.$$

Hence,

$$z = \frac{w(1-s)}{2wsr - ws - sr}$$
$$= \frac{0}{0}wsr + 0ws(1-r) + \frac{1}{0}w(1-s)r + \frac{1}{0}w(1-s)(1-r),$$
$$+0(1-w)sr + \frac{0}{0}(1-w)s(1-r) + \frac{0}{0}(1-w)(1-s)r$$
$$+\frac{0}{0}(1-w)(1-s)(1-r).$$
$$\text{Or, } z = \frac{0}{0}wsr + \frac{0}{0}(1-w)s(1-r) + \frac{0}{0}(1-w)(1-s),$$
$$\text{with } w(1-s) = 0.$$

Hence, *Things transferable and not productive of pleasure are either wealth (limited in supply and preventive of pain); or things which are not wealth, but limited in supply and not preventive of pain; or things which are not wealth, and are unlimited in supply.*

The following results, deduced in a similar manner, will be easily verified:

Things limited in supply and productive of pleasure which are not wealth,–are intransferable.

Wealth that is not productive of pleasure is transferable, limited in supply, and preventive of pain.

Things limited in supply which are either wealth, or are productive of pleasure, but not both,–are either transferable and preventive of pain, or intransferable.

11. From the domain of natural history a large number of curious examples might be selected. I do not, however, conceive that such applications would possess any independent value. They would, for instance, throw no light upon the true principles of classification in the science of zoology. For the discovery of these, some basis of positive knowledge is requisite,–some acquaintance with organic structure, with teleological adaptation; and this is a species of knowledge which can only be derived from the use of external means of observation and analysis. Taking, however, any collection of propositions in natural history, a great number of logical problems present themselves, without regard to the system of classification adopted. Perhaps in forming such examples, it is better to avoid, as superfluous, the mention of that property of a class or species which is immediately suggested by its name, e.g. the ring-structure in the annelida, a class of animals including the earth-worm and the leech.

Ex. 4.–1. The annelida are soft-bodied, and either naked or enclosed in a tube.

2. The annelida consist of all invertebrate animals having red blood in a double system of circulating vessels.

因此，

$$z = \frac{w(1-s)}{2wsr - ws - sr}$$
$$= \frac{0}{0}wsr + 0ws(1-r) + \frac{1}{0}w(1-s)r + \frac{1}{0}w(1-s)(1-r),$$
$$+0(1-w)sr + \frac{0}{0}(1-w)s(1-r) + \frac{0}{0}(1-w)(1-s)r$$
$$+\frac{0}{0}(1-w)(1-s)(1-r).$$
$$\text{Or, } z = \frac{0}{0}wsr + \frac{0}{0}(1-w)s(1-r) + \frac{0}{0}(1-w)(1-s),$$
$$\text{with } w(1-s) = 0.$$

因此，可转让且不产生快乐的事物，要么是财富(供给有限且防止痛苦)；要么是并非财富、但供给有限且不防止痛苦的事物；要么是并非财富、且供给无限的事物。

以下以类似方式推出的结果将很容易验证：供给有限且产生快乐但并非财富的事物--是不可转让的。

不产生快乐的财富是可转让的、供给有限的，并且防止痛苦。

供给有限且要么是财富、要么产生快乐但并非两者兼有的事物--要么是可转让且防止痛苦的，要么是不可转让的。

11. 从自然史的领域中可以选择大量奇特的例子。然而，我并不认为这类应用会具有任何独立的价值。例如，它们不会为动物学科学中分类的真正原则提供任何启示。因为要发现这些原则，需要某种实证知识的基础--对有机结构、对目的论适应的一些了解；而这是一种只能通过运用外部观察和分析手段才能获得的知识。然而，取自然史中任何一组命题，大量逻辑问题便会呈现出来，而不论采用何种分类系统。也许在构造这类例子时，最好避免提及一个类或物种的名称所直接暗示的那种属性，因为那是多余的，例如环节动物(一类包括蚯蚓和水蛭的动物)的环状结构。

例4.-1. 环节动物身体柔软，或裸露，或被包裹在
管。
2.环节动物由所有在双重循环血管系统中具有红色血液的无脊椎动物组成。

Assume a = annelida;
s = soft-bodied animals;
n = naked;
t = enclosed in a tube;
i = invertebrate;
r = having red blood, &c.

Then the propositions given will be expressed by the equations

$$a = vsn(1-t) + t(1-n); \tag{1}$$
$$a = ir; \tag{2}$$

to which we may add the implied condition,

$$nt = 0. \tag{3}$$

On eliminating v, and reducing the system to a single equation, we have

$$a[1 - sn(1-t) - st(1-n)] + a(1-ir) + ir(1-a) + nt = 0. \tag{4}$$

Suppose that we wish to obtain the relation in which soft-bodied animals enclosed in tubes arc placed (by virtue of the premises) with respect to the following elements, viz., the possession of red blood, of an external covering, and of a vertebral column.

We must first eliminate a. The result is

$$ir1 - sn(1-t) - st(1-n) + nt = 0.$$

Then (IX. 9) developing with respect to s and t, and reducing the first coefficient by Prop. 1, we have

$$nst + ir(1-n)s(1-t) + (ir+n)(1-s)t + ir(1-s)(1-t) = 0. \tag{5}$$

Hence, if $st = w$, we find

$$nw + ir(1-n) \times (ir+n) \times ir(1-w) = 0;$$

or,

$$nw + ir(1-n)(1-w) = 0;$$
$$\therefore w = \frac{ir(1-n)}{ir(1-n) - n}$$
$$= 0irn + ir(1-n) + 0i(1-r)n + \frac{0}{0}i(1-r)(1-n)$$
$$+0(1-i)rn + \frac{0}{0}(1-i)r(1-n) + 0(1-i)(1-r)n$$
$$+\frac{0}{0}(1-i)(1-r)(1-n);$$
$$\text{or, } w = ir(1-n) + \frac{0}{0}i(1-r)(1-n) + \frac{0}{0}(1-i)(1-n).$$

假设 a = 环节动物门；
s = 软体动物；n = 裸露的；t = 包被于管中的；i = 无脊椎的；r = 具有红色血液的，等等。

那么给定的命题将由方程来表达

$$a = vsn(1-t) + t(1-n); \tag{1}$$
$$a = ir; \tag{2}$$

我们可以补充隐含条件，

$$nt = 0. \tag{3}$$

消去v，并将方程组化为单一方程，我们得到

$$a[1 - sn(1-t) - st(1-n)] + a(1-ir) + ir(1-a) + nt = 0. \tag{4}$$

假定我们希望获得包被于管中的软体动物(凭借前提)相对于以下元素所处的关系，即，具有红色血液、具有外部覆盖物以及具有脊柱。

我们必须先消去 a。结果为

$$ir1 - sn(1-t) - st(1-n) + nt = 0.$$

Then (IX. 9) developing with respect to s and t, and reducing the first coefficient by Prop. 1, we have

$$nst + ir(1-n)s(1-t) + (ir+n)(1-s)t + ir(1-s)(1-t) = 0. \tag{5}$$

Hence, if $st = w$, we find

$$nw + ir(1-n) \times (ir+n) \times ir(1-w) = 0;$$

or,

$$nw + ir(1-n)(1-w) = 0;$$
$$\therefore w = \frac{ir(1-n)}{ir(1-n) - n}$$
$$= 0irn + ir(1-n) + 0i(1-r)n + \frac{0}{0}i(1-r)(1-n)$$
$$+0(1-i)rn + \frac{0}{0}(1-i)r(1-n) + 0(1-i)(1-r)n$$
$$+\frac{0}{0}(1-i)(1-r)(1-n);$$
$$\text{or, } w = ir(1-n) + \frac{0}{0}i(1-r)(1-n) + \frac{0}{0}(1-i)(1-n).$$

Hence, *soft-bodied animals enclosed in tubes consist of all invertebrate animals having red blood and not naked, and an indefinite remainder of invertebrate animals not having red blood and not naked, and of vertebrate animals which are not naked.*

And in an exactly similar manner, the following reduced equations, the interpretation of which is left to the reader, have been deduced from the development (5).

$$s(1-t) = irn + \frac{0}{0}i(1-n) + \frac{0}{0}(1-i)$$
$$(1-s)t = \frac{0}{0}(1-i)r(1-n) + \frac{0}{0}(1-r)(1-n)$$
$$(1-s)(1-t) = \frac{0}{0}i(1-r) + \frac{0}{0}(1-i).$$

In none of the above examples has it been my object to exhibit in any special manner the power of the method. That, I conceive, can only be fully displayed in connexion with the mathematical theory of probabilities. I would, however, suggest to any who may be desirous of forming a correct opinion upon this point, that they examine by the rules of ordinary logic the following problem, *before* inspecting its solution; remembering at the same time, that whatever complexity it possesses might be multiplied indefinitely, with no other effect than to render its solution by the method of this work more operose, but not less certainly attainable.

Ex. *5.* Let the observation of a class of natural productions be supposed to have led to the following general results.

1st, That in whichsoever of these productions the properties A and C are missing, the property E is found, together with one of the properties B and D, but not with both.

2nd, That wherever the properties A and D are found while E is missing, the properties B and C will either both be found, or both be missing.

3rd, That wherever the property A is found in conjunction with either B or E, or both of them, there either the property C or the property D will be found, but not both of them. And conversely, wherever the property C or D is found singly, there the property A will be found in conjunction with either B or E, or both of them.

Let it then be required to ascertain, first, what in any particular instance may be concluded from the ascertained presence of the property A, with reference to the properties B, C, and D; also whether any relations exist independently among the properties B, C, and D. Secondly, what may be concluded in like manner respecting the property B, and the properties A, C, and D.

It will be observed, that in each of the three data, the information conveyed respecting the properties A, B, C, and D, is complicated with another element, E, about which we desire to say nothing in our conclusion. It will hence be requisite to eliminate the symbol representing the property E from the system of equations, by which the given propositions will be expressed.

因此，包裹在管中的软体动物由所有具有红血且非裸露的无脊椎动物，以及不具有红血且非裸露的无脊椎动物的不确定剩余部分，以及非裸露的脊椎动物组成。

并且以完全类似的方式，以下约化方程已从展开式(5)中推导出来，其解释留给读者。

$$s(1-t) = irn + \frac{0}{0}i(1-n) + \frac{0}{0}(1-i)$$
$$(1-s)t = \frac{0}{0}(1-i)r(1-n) + \frac{0}{0}(1-r)(1-n)$$
$$(1-s)(1-t) = \frac{0}{0}i(1-r) + \frac{0}{0}(1-i).$$

在上述任何一个例子中，我的目的都不是以任何特殊方式展示该方法的力量。我认为，那种力量只有与概率的数学理论相关联才能充分展示。然而，我要向任何可能希望就此形成正确看法的人建议，让他们按照普通逻辑的规则来考察以下问题，然后再查看其解答；同时记住，无论它具有怎样的复杂性，这种复杂性都可以无限倍增，其唯一效果不过是使按照本著作的方法求解更为费力，但并不会减少其确实可达到的程度。

例*5*。假设对一类自然产物的观察导致了以下一般结果。

第一，在这些产物中的任何一个中，当性质A和C缺失时，性质E被发现，连同性质B和D之一，但不是两者同时。

第二，凡是在性质A和D被发现而E缺失的地方，性质B和C将要么同时被发现，要么同时缺失。

第三，凡是在性质A与B或E或两者同时被发现的地方，那里要么性质C要么性质D将被发现，但不是两者同时。反之，凡是在性质C或D单独被发现的地方，那里性质A将与B或E或两者同时被发现。

于是要求确定：第一，在任何特定情况下，从性质A的已确定存在可以得出关于性质B、C和D的什么结论；以及性质B、C和D之间是否存在任何独立的关系。第二，以类似方式关于性质B以及性质A、C和D可以得出什么结论。

将会注意到，在三个数据中的每一个中，所传达的关于性质A、B、C和D的信息都与另一个元素E交织在一起，关于这个元素我们在结论中不想说什么。因此，将有必要从表达给定命题的方程系统中消去代表性质E的记号。

Let us represent the property A by x, B by y, C by z, D by w, E by v. The data are

$$\bar{x}\bar{z} = qv(y\bar{w} + w\bar{y}); \quad (1)$$
$$\bar{v}xw = q(yz + \bar{y}\bar{z}); \quad (2)$$
$$xy + xv\bar{y} = w\bar{z} + z\bar{w}; \quad (3)$$

$\bar{x}$ standing for $1-x$, &c., and q being an indefinite class symbol. Eliminating q separately from the first and second equations, and adding the results to the third equation reduced by (5), Chap.VIII., we get

$$\bar{x}\bar{z}(1 - vy\bar{w} - vw\bar{y}) + \bar{v}xw(y\bar{z} + z\bar{y}) + (xy + xv\bar{y})(wz + \bar{w}\bar{z}) + (w\bar{z} + z\bar{w})(1 - xy - xv\bar{y}) = 0. \quad (4)$$

From this equation v must be eliminated, and the value of x determined from the result. For effecting this object, it will be convenient to employ the method of Prop. 3 of the present chapter.

Let then the result of elimination be represented by the equation

$$Ex + E'(l - x) = 0.$$

To find E make $x = 1$ in the first member of (4), we find

$$\bar{v}w(y\bar{z} + z\bar{y}) + (y + v\bar{y})(wz + \bar{w}\bar{z}) + (w\bar{z} + z\bar{w})\bar{v}\bar{y}.$$

Eliminating v, we have

$$(wz + \bar{w}\bar{z})\{w(y\bar{z} + z\bar{y}) + y(wz + \bar{w}\bar{z}) + \bar{y}(w\bar{z} + z\bar{w})\};$$

which, on actual multiplication, in accordance with the conditions $w\bar{w} = 0$, $z\bar{z} = 0$, &c., gives

$$E = wz + y\bar{w}\bar{z}$$

Next, to find E' make $x = 0$ in (4), we have

$$z(1 - vy\bar{w} - v\bar{y}w) + w\bar{z} + z\bar{w}.$$

whence, eliminating v, and reducing the result by Propositions 1 and 2, we find

$$E' = w\bar{z} + z\bar{w} + \bar{y}\bar{w}\bar{z};$$

and, therefore, finally we have

$$(wz + y\bar{w}\bar{z})x + (w\bar{z} + z\bar{w} + \bar{y}\bar{w}\bar{z})\bar{x} = 0; \quad (5)$$

from which

Let us represent the property A by x, B by y, C by z, D by w, E by v. The data are

$$\bar{x}\bar{z} = qv(y\bar{w} + w\bar{y}); \quad (1)$$
$$\bar{v}xw = q(yz + \bar{y}\bar{z}); \quad (2)$$
$$xy + xv\bar{y} = w\bar{z} + z\bar{w}; \quad (3)$$

¯x代表1 – x，等等，而q是一个不确定的类记号。消去
将q分别从第一和第二方程中消去，并将结果加到按(5)约化的第三方程，第八章，我们得到

$$\bar{x}\bar{z}(1 - vy\bar{w} - vw\bar{y}) + \bar{v}xw(y\bar{z} + z\bar{y}) + (xy + xv\bar{y})(wz + \bar{w}\bar{z}) + (w\bar{z} + z\bar{w})(1 - xy - xv\bar{y}) = 0. \quad (4)$$

从这个方程中必须消去v，并从结果中确定x的值。为实现这一目标，采用本章命题3的方法将是方便的。

那么，消元的结果可由方程表示

$$Ex + E'(l - x) = 0.$$

To find E make $x = 1$ in the first member of (4), we find

$$\bar{v}w(y\bar{z} + z\bar{y}) + (y + v\bar{y})(wz + \bar{w}\bar{z}) + (w\bar{z} + z\bar{w})\bar{v}\bar{y}.$$

消去 v，我们有

$$(wz + \bar{w}\bar{z})\{w(y\bar{z} + z\bar{y}) + y(wz + \bar{w}\bar{z}) + \bar{y}(w\bar{z} + z\bar{w})\};$$

which, on actual multiplication, in accordance with the conditions $w\bar{w} = 0$, $z\bar{z} = 0$, &c., gives

$$E = wz + y\bar{w}\bar{z}$$

接下来，求E′，在(4)中令x = 0，我们有

$$z(1 - vy\bar{w} - v\bar{y}w) + w\bar{z} + z\bar{w}.$$

whence, eliminating v, and reducing the result by Propositions 1 and 2, we find

$$E' = w\bar{z} + z\bar{w} + \bar{y}\bar{w}\bar{z};$$

因此，最终我们得到

$$(wz + y\bar{w}\bar{z})x + (w\bar{z} + z\bar{w} + \bar{y}\bar{w}\bar{z})\bar{x} = 0; \quad (5)$$

从哪一个

$$x = \frac{w\bar{z} + z\bar{w} + \bar{y}\bar{w}\bar{z}}{w\bar{z} + z\bar{w} + \bar{y}\bar{w}\bar{z} - wz - y\bar{w}\bar{z}}$$

wherefore, by development,

$$\begin{aligned} x = {} & 0yzw + yz\bar{w} + y\bar{z}w + 0y\bar{z}\bar{w} \\ & + 0\bar{y}zw + \bar{y}z\bar{w} + \bar{y}\bar{z}w + \bar{y}\bar{z}\bar{x}; \end{aligned}$$

or, collecting the terms in vertical columns,

$$x = z\bar{w} + \bar{z}w + \bar{y}\bar{z}\bar{w}; \tag{6}$$

the interpretation of which is–

In whatever substances the property A is found, there will also be found either the property C or the property D, but not both, or else the properties B, C, and D, will all be wanting. And conversely, where either the property C or the property D is found singly, or the properties B, C, and D, are together missing, there the property A will be found.

It also appears that there is no independent relation among the properties B, C, and D.

Secondly, we are to find y. Now developing (5) with respect to that symbol,

$$(xwz + x\bar{w}\bar{z} + \bar{x}w\bar{z} + \bar{x}z\bar{w})y + (xwz + \bar{x}w\bar{z} + \bar{x}z\bar{w} + \bar{x}\bar{z}\bar{w})\bar{y} = 0;$$

whence, proceeding as before,

$$y = \bar{x}\bar{w}\bar{z} + \frac{0}{0}(\bar{x}wz + xw\bar{z} + xz\bar{w}), \tag{7}$$

$$xzw = 0; \tag{8}$$

$$\bar{x}z\bar{z}w = 0; \tag{9}$$

$$\bar{x}z\bar{w} = 0; \tag{10}$$

From (10) reduced by solution to the form

$$\bar{x}z = \frac{0}{0}w;$$

we have the independent relation,–*If the property A is absent and C present, D is present.* Again, by addition and solution (8) and (9) give

$$xz + \bar{x}\bar{z} = \frac{0}{0}\bar{w}.$$

Whence we have for the general solution and the remaining independent relation:

$$x = \frac{w\bar{z} + z\bar{w} + \bar{y}\bar{w}\bar{z}}{w\bar{z} + z\bar{w} + \bar{y}\bar{w}\bar{z} - wz - y\bar{w}\bar{z}}$$

wherefore, by development,

$$\begin{aligned} x = {} & 0yzw + yz\bar{w} + y\bar{z}w + 0y\bar{z}\bar{w} \\ & + 0\bar{y}zw + \bar{y}z\bar{w} + \bar{y}\bar{z}w + \bar{y}\bar{z}\bar{x}; \end{aligned}$$

或者，将各项按竖列合并，

$$x = z\bar{w} + \bar{z}w + \bar{y}\bar{z}\bar{w}; \tag{6}$$

其解释为--

无论性质在什么物质中被发现，那里也将被发现要么性质要么性质，但不是两者同时，否则性质、、和将全部缺失。反之，凡是在性质或性质单独被发现的地方，或者性质、、和一起缺失的地方，那里性质将被发现。

It also appears that there is no independent relation among the properties B, C, and D.

Secondly, we are to find y. Now developing (5) with respect to that symbol,

$$(xwz + x\bar{w}\bar{z} + \bar{x}w\bar{z} + \bar{x}z\bar{w})y + (xwz + \bar{x}w\bar{z} + \bar{x}z\bar{w} + \bar{x}\bar{z}\bar{w})\bar{y} = 0;$$

由此，如前所述，

$$y = \bar{x}\bar{w}\bar{z} + \frac{0}{0}(\bar{x}wz + xw\bar{z} + xz\bar{w}), \tag{7}$$

$$xzw = 0; \tag{8}$$

$$\bar{x}z\bar{z}w = 0; \tag{9}$$

$$\bar{x}z\bar{w} = 0; \tag{10}$$

由(10)经求解化简为如下形式

$$\bar{x}z = \breve{\frac{0}{0}}w;$$

我们有独立关系，--如果性质缺失而存在，
D 存在。再次，通过相加和求解，(8) 和 (9) 给出

$$xz + \bar{x}\bar{z} = \breve{\frac{0}{0}}\bar{w}.$$

由此我们得到通解以及其余独立的
关系：

1st. *If the property B be present in one of the productions, either the properties A, C, and D, are all absent, or some one alone of them is absent.* And conversely, *if they are all absent it may be concluded that the property A is present* (7).

2nd. *If A and C are both present or both absent, D will be absent, quite independently of the presence or absence of B* (8) and (9).

I have not attempted to verify these conclusions.

1. 如果该属性出现在其中一个生成物中，那么属性、、和要么全部缺失，要么其中仅有一个缺失。反之，如果它们全部缺失，则可以断定属性*A*存在(7)。

2. 如果和同时存在或同时缺失，则将缺失，这完全与 B(8)和(9)的存在或缺失无关。

我未曾尝试去验证这些结论。

Chapter X

OF THE CONDITIONS OF A PERFECT METHOD.

1. The subject of Primary Propositions has been discussed at length, and we are about to enter upon the consideration of Secondary Propositions. The interval of transition between these two great divisions of the science of Logic may afford a fit occasion for us to pause, and while reviewing some of the past steps of our progress, to inquire what it is that in a subject like that with which we have been occupied constitutes perfection of method. I do not here speak of that perfection only which consists in power, but of that also which is founded in the conception of what is fit and beautiful. It is probable that a careful analysis of this question would conduct us to some such conclusion as the following, viz., that a perfect method should not only be an efficient one, as respects the accomplishment of the objects for which it is designed, but should in all its parts and processes manifest a certain unity and harmony. This conception would be most fully realized if even the very forms of the method were suggestive of the fundamental principles, and if possible of the one fundamental principle, upon which they are founded. In applying these considerations to the science of Reasoning, it may be well to extend our view beyond the mere analytical processes, and to inquire what is best as respects not only the mode or form of deduction, but also the system of data or premises from which the deduction is to be made.

2. As respects mere power, there is no doubt that the first of the methods developed in Chapter VIII. is, within its proper sphere, a perfect one. The introduction of arbitrary constants makes us independent of the forms of the premises, as well as of any conditions among the equations by which they are represented. But it seems to introduce a foreign element, and while it is a more laborious, it is also a less elegant form of solution than the second method of reduction demonstrated in the same chapter. There are, however, conditions under which the latter method assumes a more perfect form than it otherwise bears. To make the one fundamental condition expressed by the equation

$$x(1-x) = 0,$$

the universal type of form, would give a unity of character to both processes and results, which would not else be attainable. Were brevity or convenience the

第十章

论完善方法的条件。

1. 关于初级命题的主题已经详细讨论过了，我们即将进入次级命题的考察。逻辑科学这两大部分之间的过渡间隙，可以为我们提供一个合适的时机来暂停一下，在回顾我们已走过的某些步骤的同时，探究在像我们所从事的这样一个主题中，究竟是什么构成了方法的完善。我在这里所说的完善，不仅仅是那种在于力量的完善，而且也是那种奠基于对适宜与美好之概念的理解的完善。对这一问题的仔细分析，很可能将我们引向如下这样的结论，即：一种完善的方法不仅在其所要达成的目标方面应当是有效的，而且在其所有的部分和过程中都应当体现出某种统一与和谐。如果甚至连方法的形式本身都能暗示那些基本原则，并且如果可能的话暗示它们所依据的那一个基本原则，那么这一概念就会得到最充分的实现。在将这些考虑应用于推理科学时，我们不妨将视野扩展到单纯的分析过程之外，不仅就演绎的模式或形式而言，而且就据以进行演绎的数据或前提的体系而言，探究什么是最好的。

2. 就单纯的力量而言，毫无疑问，第八章中展开的第一种方法在其适当的范围内是一种完美的方法。任意常数的引入使我们不依赖于前提的形式，也不依赖于用以表示前提的方程之间的任何条件。但它似乎引入了一个异质的元素，而且虽然它是一种更为费力的解法形式，它也是一种不如同一章中所论证的第二种归约方法优雅的解法形式。然而，在某些条件下，后一种方法会呈现出比它本来所具有的更为完美的形式。为了使由方程所表达的那个基本条件

$$x(1-x) = 0,$$

the universal type of form, would give a unity of character to both processes and results, which would not else be attainable. Were brevity or convenience the

only valuable quality of a method, no advantage would flow from the adoption of such a principle. For to impose upon every step of a solution the character above described, would involve in some instances no slight labour of preliminary reduction. But it is still interesting to know that this can be done, and it is even of some importance to be acquainted with the conditions under which such a form of solution would spontaneously present itself. Some of these points will be considered in the present chapter.

PROPOSITION I.

3. *To reduce any equation among logical symbols to the form* $V = 0$, *in which* V *satisfies the law of duality,*

$$V(1 - V) = 0.$$

It is shown in Chap. V. Prop. 4, that the above condition is satisfied whenever V is the sum of a series of constituents. And it is evident from Prop. 2, Chap. VI. that all equations are equivalent which, when reduced by transposition to the form $V = 0$, produce, by development of the first member, the same series of constituents with coefficients which do not vanish; the particular numerical values of those coefficients being immaterial.

Hence the object of this Proposition may always be accomplished by bringing all the terms of an equation to the first side, fully expanding that member, and changing in the result all the coefficients which do not vanish into unity, except such as have already that value.

But as the development of functions containing many symbols conducts us to expressions inconvenient from their great length, it is desirable to show how, in the only cases which do practically offer themselves to our notice, this source of complexity may be avoided.

The great primary forms of equations have already been discussed in Chapter VIII. They are–

$$\begin{aligned} X &= vY, \\ X &= Y, \\ vX &= vY. \end{aligned}$$

Whenever the conditions $X(1-X) = 0, Y(1-Y) = 0$, are satisfied, we have seen that the two first of the above equations conduct us to the forms

$$\begin{aligned} X(1-Y) &= 0, && (1) \\ X(1-Y) &+ Y(1-X) = 0; && (2) \end{aligned}$$

and under the same circumstances it may be shown that the last of them gives

一种方法唯一有价值的性质，采纳这样一种原则并不会带来任何好处。因为要在求解的每一步上都赋予上述特征，在某些情况下会涉及不小的初步归约工作。但知道这是可以做到的仍然是有趣的，而且了解这种解法形式会自发呈现的条件甚至具有一定的重要性。其中一些要点将在本章中加以考虑。

命题一。

3. 将逻辑记号间的任何方程化为形式 V = 0 ，其中满足对偶律，

$$V(1 - V) = 0.$$

第五章命题4已表明，只要V是一系列成分之和，上述条件便得到满足。又由第六章命题2显然可知，所有方程，当经移项化为V = 0的形式后，通过对第一项作展开而产生相同的成分系列、且其系数不为零者，都是等价的；那些系数的具体数值无关紧要。因此，本命题的目的总可以通过以下方式达成：将方程的所有项移到第一边，充分展开该边，并在结果中把所有不为零的系数改为1，但已经具有该值者除外。然而，由于含有许多记号的函数的展开式会使我们得到因过长而不便的表达式，因此有必要说明，在实际呈现于我们注意力的唯一一些情形中，如何避免这一复杂性的来源。

方程的那些重要的基本形式已经在第 VIII. 它们是--

$$\begin{aligned} X &= vY, \\ X &= Y, \\ vX &= vY. \end{aligned}$$

只要条件X(1 – X) = 0 , Y(1 – Y) = 0得到满足，我们就有既然上述方程中的前两个把我们引向这些形式

$$\begin{aligned} X(1-Y) &= 0, && (1) \\ X(1-Y) &+ Y(1-X) = 0; && (2) \end{aligned}$$

在相同情况下，可以表明它们中的最后一个给出

$$v(X(1-Y)+Y(1-X))=0; \tag{3}$$

all which results obviously satisfy, in their first members, the condition

$$V(1-V)=0.$$

Now as the above are the forms and conditions under which the equations of a logical system properly expressed do actually present themselves, it is always possible to reduce them by the above method into subjection to the law required. Though, however, the separate equations may thus satisfy the law, their equivalent sum (VIII. 4) may not do so, and it remains to show how upon it also the requisite condition may be imposed.

Let us then represent the equation formed by adding the several reduced equations of the system together, in the form

$$v+v'+v'',\&c.=0, \tag{4}$$

this equation being singly equivalent to the system from which it was obtained. We suppose v, v', v'', &c. to be class terms (IX. 1) satisfying the conditions

$$v(1-v)=0, v'(1-v')=0, \&c.$$

Now the full interpretation of (4) would be found by developing the first member with respect to all the elementary symbols x, y, &c. which it contains, and equating to 0 all the constituents whose coefficients do not vanish; in other words, all the constituents which are found in either v, v', v'', &c. But those constituents consist of–1st, such as are found in v; 2nd, such as are not found in v, but are found in v'; 3rd, such as are neither found in v nor v', but are found in v'', and so on. Hence they will be such as are found in the expression

$$v+(1-v)v'+(1-v)(1-v)v''+\&c., \tag{5}$$

an expression in which no constituents are repeated, and which obviously satisfies the law $V(1-V)=0$.

Thus if we had the expression

$$(1-t)+v+(1-z)+tzw,$$

in which the terms $1-t$, $1-z$ are bracketed to indicate that they are to be taken as single class terms, we should, in accordance with (5), reduce it to an expression satisfying the condition $V(1-V)=0$, by multiplying all the terms after the first by t, then all after the second by $1-v$; lastly, the term which remains after the third by z; the result being

$$1-t+tv+t(1-v)(1-z)+t(1-v)zw. \tag{6}$$

4. All logical equations then are reducible to the form $V=0$, V satisfying the law of duality. But it would obviously be a higher degree of perfection

$$v(X(1-Y)+Y(1-X))=0; \tag{3}$$

all which results obviously satisfy, in their first members, the condition

$$V(1-V)=0.$$

既然上述就是逻辑系统的方程在得到恰当表达时实际呈现的形式和条件，那么总是可以通过上述方法将它们归约为服从所需定律的形式。然而，尽管各个方程可以由此满足该定律，它们的等价和(VIII.4)却可能不满足，因此还有待说明如何也对它施加所需的条件。那么，让我们把将系统中各个已归约的方程相加而成的方程表示为如下形式

$$v+v'+v'',\&c.=0, \tag{4}$$

这个方程与从中获得它的系统单独等价。我们假设v, v′, v″等是满足条件的类项(IX.1)

$$v(1-v)=0, v'(1-v')=0, \&c.$$

现在，(4)的完整解释将通过就其所包含的所有基本记号x、y等展开第一项，并将所有系数不为零的成分等于0来求得；换言之，即在v、v′、v″等中所找到的所有成分。但这些成分包括--第一，在v中所找到的那些；第二，不在v中所找到、但在v′中所找到的那些；第三，既不在v中也不在v′中所找到、但在v″中所找到的那些，依此类推。因此它们将是在表达式中找到的那些

$$v+(1-v)v'+(1-v)(1-v)v''+\&c., \tag{5}$$

一个其中没有成分被重复的表达式，它显然满足定律V(1 – V) = 0。

因此，如果我们有表达式

$$(1-t)+v+(1-z)+tzw,$$

其中项1 – t、1 – z被括起来以表示它们应被视为单个类项，我们应按照(5)将其约化为满足条件V(1 – V) = 0的表达式，方法是将第一项之后的所有项乘以t，然后将第二项之后的所有项乘以1 – v；最后，将第三项之后剩余的项乘以z；结果为

$$1-t+tv+t(1-v)(1-z)+t(1-v)zw. \tag{6}$$

4.那么所有逻辑方程都可约化为形式V = 0,V满足对偶律。但显然更高程度的完善将是

if equations always presented themselves in such a form, without preparation of any kind, and not only exhibited this form in their original statement, but retained it unimpaired after those additions which are necessary in order to reduce systems of equations to single equivalent forms. That they do not spontaneously present this feature is not properly attributable to defect of method, but is a consequence of the fact that our premises are not always complete, and accurate, and independent. They are not complete when they involve material (as distinguished from formal) relations, which are not expressed. They are not accurate when they imply relations which are not intended. But setting aside these points, with which, in the present instance, we are less concerned, let it be considered in what sense they may fail of being independent.

5. A system of propositions may be termed independent, when it is not possible to deduce from any portion of the system a conclusion deducible from any other portion of it. Supposing the equations representing those propositions all reduced to the form

$$V = 0,$$

then the above condition implies that no constituent which can be made to appear in the development of a particular function V of the system, can be made to appear in the development of any other function V' of the same system. When this condition is not satisfied, the equations of the system are not independent. This may happen in various cases. Let all the equations satisfy in their first members the law of duality, then if there appears a positive term x in the expansion of one equation, and a term xy in that of another, the equations are not independent, for the term x is further developable into $xy + x(1 - y)$, and the equation

$$xy = 0$$

is thus involved in both the equations of the system. Again, let a term xy appear in one equation, and a term xz in another. Both these may be developed so as to give the common constituent xyz. And other cases may easily be imagined in which premises which appear at first sight to be quite independent are not really so. Whenever equations of the form $V = 0$ are thus not truly independent, though individually they may satisfy the law of duality,

$$V(1 - V) = 0,$$

the equivalent equation obtained by adding them together will not satisfy that condition, unless sufficient reductions by the method of the present chapter have been performed. When, on the other hand, the equations of a system both satisfy the above law, and are independent of each other, their sum will also satisfy the same law. I have dwelt upon these points at greater length than would otherwise have been necessary, because it appears to me to be important to endeavour to form to ourselves, and to keep before us in all our investigations, the pattern of an ideal perfection,—the object and the guide of future efforts. In the present class of inquiries the chief aim of improvement of method should be

如果方程总是以这样的形式呈现自身，无需任何准备，并且不仅在它们的原始陈述中表现出这种形式，而且在那些为了使方程组化为单一等价形式所必需的添加之后仍然保持不变。它们并非自发地呈现这一特征，这不能恰当地归因于方法的缺陷，而是我们的前提并不总是完备、准确和独立这一事实的结果。当前提涉及未表达出来的(与形式的相区别的)实质关系时，它们是不完备的。当前提蕴含并非意图中的关系时，它们是不准确的。但撇开这些在当前情况下我们较少关切的要点，让我们考虑它们在何种意义上可能不是独立的。

5.一个命题系统可以被称为独立的，当不可能从系统的任何部分推出可从其任何其他部分推出的结论时。假设代表那些命题的方程都约化为形式

$$V = 0,$$

那么上述条件意味着，能够出现在系统某一特称函数V的展开式中的任何成分，都不能出现在同一系统的任何其他函数V ′的展开式中。当这一条件不被满足时，该系统的方程不是独立的。这可能在各种情况下发生。设所有方程在其第一项中都满足对偶律，那么如果一个方程的展开式中出现一个正项x，而另一个方程的展开式中出现一个项xy，则这些方程不是独立的，因为项x可进一步展开为xy + x(1 – y)，而方程

$$xy = 0$$

因此涉及系统中的两个方程。再者，设一项xy出现在一个方程中，而一项xz出现在另一个方程中。两者都可以展开以给出共同成分xyz。还可以容易地想象其他情况，其中初看似乎完全独立的前提实际上并非如此。每当形式为V = 0的方程如此并非真正独立时，尽管它们个别可能满足对偶律，

$$V(1 - V) = 0,$$

将它们相加所得的等价方程将不满足该条件，除非已通过本章的方法进行了充分的化简。另一方面，当一个系统的方程既满足上述定律又彼此独立时，它们的和也将满足同一定律。我之所以在这些要点上论述得比原本必要的更为详尽，是因为在我看来，努力为我们自己形成并在一切研究中始终置于眼前的，是一种理想完善性的典范--这是未来努力的目标和指南。在当前这一类探究中，方法改进的主要目标应当是

to facilitate, as far as is consistent with brevity, the transformation of equations, so as to make the fundamental condition above adverted to universal.

In connexion with this subject the following Propositions are deserving of attention.

Proposition II.

If the first member of any equation $V = 0$ satisfy the condition $V(1 - V) = 0$, and if the expression of any symbol t of that equation be determined as a developed function of the other symbols, the coefficients of the expansion can only assume the forms 1, 0, $\frac{0}{0}$, $\frac{1}{0}$.

For if the equation be expanded with reference to t, we obtain as the result,

$$Et + E'(1 - t), \tag{1}$$

E and E' being what V becomes when t is successively changed therein into 1 and 0. Hence E and E' will themselves satisfy the conditions

$$E(1 - E) = 0, \quad E'(1 - E') = 0. \tag{2}$$

Now (1) gives

$$t = \frac{E'}{E' - E},$$

the second member of which is to be expanded as a function of the remaining symbols. It is evident that the only numerical values which E and E' can receive in the calculation of the coefficients will be 1 and 0. The following cases alone can therefore arise:

1st. $E' = 1$, $E = 1$, then $\frac{E'}{E' - E} = \frac{1}{0}$.

2nd. $E' = 1$, $E = 0$, then $\frac{E'}{E' - E} = 1$.

3rd. $E' = 0$, $E = 1$, then $\frac{E'}{E' - E} = 0$.

4th. $E' = 0$, $E = 0$, then $\frac{E'}{E' - E} = \frac{0}{0}$.

Whence the truth of the Proposition is manifest.

6. It may be remarked that the forms 1, 0, and $\frac{0}{0}$ appear in the solution of equations independently of any reference to the condition $V(1 - V) = 0$. But it is not so with the coefficient $\frac{1}{0}$. The terms to which this coefficient is attached when the above condition is satisfied may receive any other value except the three values 1, 0, and $\frac{0}{0}$, when that condition is not satisfied. It is permitted, and it would conduce to uniformity, to change any coefficient of a development not presenting itself in any of the four forms referred to in this Proposition into $\frac{1}{0}$, regarding this as the symbol proper to indicate that the coefficient to which it is attached should be equated to 0. This course I shall frequently adopt.

Proposition III.

在尽可能与简洁一致的范围内，促进方程的变换，以使上述基本条件普遍化。

与此主题相关，以下命题值得注意。

命题二。

如果任何方程的第一个成员 V = 0 满足条件 V(1 – V) = 0 ，并且如果该方程中任何记号的表达式被确定为其他记号的展开函数，则展开式的系数可以

仅取形式 1 , 0 ,0 0 ,1 – 0.

因为如果方程关于t展开，我们得到结果，

$$Et + E'(1 - t), \tag{1}$$

E 和 E′是当 t 在其中相继变为 1 和 0 时 V 所变成的东西。因此 E 和 E′本身将满足这些条件

$$E(1 - E) = 0, \quad E'(1 - E') = 0. \tag{2}$$

现在(1)给出

$$t = \frac{E'}{E' - E},$$

其第二项须作为其余记号的函数展开。显然，在计算系数时 E′和 E′所能取的唯一数值将是 1 和 0。因此只可能出现以下情形：

1st. $E' = 1$, $E = 1$, then $\frac{E'}{E' - E} = \frac{1}{0}$.

2nd. $E' = 1$, $E = 0$, then $\frac{E'}{E' - E} = 1$.

3rd. $E' = 0$, $E = 1$, then $\frac{E'}{E' - E} = 0$.

4th. $E' = 0$, $E = 0$, then $\frac{E'}{E' - E} = \frac{0}{0}$.

Whence the truth of the Proposition is manifest.

6. 可以指出，形式 1、0 和00出现于方程的解中，与条件 V(1 –V) = 0 的任何指涉无关。但系数10.则并非如此。当上述条件得到满足时，该系数所附的项可以取除 1、0 和00,当该条件不被满足的以外的任何其他值。允许并且将有助于一致性的是，把不呈现本命题所提及的四种形式中任何一种的展开式的任何系数改为10,将此视为表示记号所适于表明其所附的系数应被等于 0。我将经常采用这一做法。

命题三。

7. *The result of the elimination of any symbols x, y, &c. from an equation $V = 0$, of which the first member identically satisfies the law of duality,*

$$V(1 - V) = 0,$$

may be obtained by developing the given equation with reference to the other symbols, and equating to 0 *the sum of those constituents whose coefficients in the expansion are equal to unity.*

Suppose that the given equation $V = 0$ involves but three symbols, x, y, and t, of which x and y are to be eliminated. Let the development of the equation, with respect to t, be

$$At + B(1 - t) = 0, \tag{1}$$

A and B being free from the symbol t.

By Chap. IX. Prop. 3, the result of the elimination of x and y from the given equation will be of the form

$$Et + E'(1 - t) = 0, \tag{2}$$

in which E is the result obtained by eliminating the symbols x and y from the equation $A = 0$, E' the result obtained by eliminating from the equation $B = 0$.

Now A and B must satisfy the condition

$$A(1 - A) = 0, B(1 - B) = 0$$

Hence A (confining ourselves for the present to this coefficient) will either be 0 or 1, or a constituent, or the sum of a part of the constituents which involve the symbols x and y. If $A = 0$ it is evident that $E = 0$; if A is a single constituent, or the sum of a part of the constituents involving x and y, E will be 0. For the *full* development of A, with respect to x and y, will contain terms with vanishing coefficients, and E is the product of all the coefficients. Hence when $A = 1$, E is equal to A, but in other cases E is equal to 0. Similarly, when $B = 1$, E is equal to B, but in other cases E vanishes. Hence the expression (2) will consist of that part, if any there be, of (1) in which the coefficients A, B are unity. And this reasoning is general. Suppose, for instance, that V involved the symbols x, y, z, t, and that it were required to eliminate x and y. Then if the development of V, with reference to z and t, were

$$zt + xz(1 - t) + y(1 - z)t + (1 - z)(1 - t),$$

the result sought would be

$$zt + (1 - z)(1 - t) = 0,$$

this being that portion of the development of which the coefficients are unity.

Hence, if from any system of equations we deduce a single equivalent equation $V = 0$, V satisfying the condition

$$V(1 - V) = 0,$$

7. 从任一方程 V = 0 中消去任何记号、、&c.的结果，该方程的第一项恒满足对偶律，

$$V(1 - V) = 0,$$

可以通过参照其他记号展开给定方程，并令展开式中系数等于1的那些成分之和等于0来得到。假设给定方程V = 0只涉及三个记号x、y和t，其中x和y需要被消去。设该方程关于t的展开式为

$$At + B(1 - t) = 0, \tag{1}$$

A and B being free from the symbol t.

By Chap. IX. Prop. 3, the result of the elimination of x and y from the given equation will be of the form

$$Et + E'(1 - t) = 0, \tag{2}$$

其中E是从方程A = 0中消去记号x和y所得的结果，E′是从方程B = 0中消去所得的结果。

现在 A 和 B 必须满足该条件

$$A(1 - A) = 0, B(1 - B) = 0$$

因此，A(目前我们只限于这个系数)将是0或1，或一个构成项，或包含记号x和y的构成项的一部分之和。如果A=0，显然E=0；如果A是单个构成项，或包含x和y的构成项的一部分之和，E将为0。因为A关于x和y的完整展开式将包含系数为零的项，而E是所有系数的乘积。因此当A=1时，E等于A，但在其他情况下E等于0。同样，当B=1时，E等于B，但在其他情况下E消失。因此表达式(2)将包含(1)中系数A、B为1的那一部分(如果有的话)。这一推理是普遍的。例如，假设V涉及记号x、y、z、t，并且要求消去x和y。那么如果V关于z和t的展开式为

$$zt + xz(1 - t) + y(1 - z)t + (1 - z)(1 - t),$$

the result sought would be

$$zt + (1 - z)(1 - t) = 0,$$

这就是展开式中系数为1的那一部分。因此，如果从任何方程组我们推出一个单一的等价方程V = 0,V满足条件

$$V(1 - V) = 0,$$

the ordinary processes of elimination may be entirely dispensed with, and the single process of development made to supply their place.

8. It may be that there is no practical advantage in the method thus pointed out, but it possesses a theoretical unity and completeness which render it deserving of regard, and I shall accordingly devote a future chapter (XIV.) to its illustration. The progress of applied mathematics has presented other and signal examples of the reduction of systems of problems or equations to the dominion of some central but pervading law.

9. It is seen from what precedes that there is one class of propositions to which all the special appliances of the above methods of preparation are unnecessary. It is that which is characterized by the following conditions:

First, That the propositions are of the ordinary kind, implied by the use of the copula *is* or *are*, the predicates being particular.

Secondly, That the terms of the proposition are intelligible without the supposition of any understood relation among the elements which enter into the expression of those terms.

Thirdly, That the propositions are independent.

We may, if such speculation is not altogether vain, permit ourselves to conjecture that these are the conditions which would be obeyed in the employment of language as an instrument of expression and of thought, by unerring beings, declaring simply what they mean, without suppression on the one hand, and without repetition on the other. Considered both in their relation to the idea of a perfect language, and in their relation to the processes of an exact method, these conditions are equally worthy of the attention of the student.

通常的消元过程可以完全省去，而由单一的展开过程来取代它们的位置。

8.如此指出的方法或许没有实际的好处，但它具有理论上的统一性和完备性，这使它值得重视，因此我将用未来的一章(第十四章)来阐明它。应用数学的进展已经提供了其他显著的例子，说明问题或方程的系统可以归入某个中心但普遍渗透的定律的支配之下。

9.从上述可见，有一类命题不需要上述准备方法的全部特殊手段。这就是以下列条件为特征的那一类：

第一，这些命题是普通类型的，由系词是或是的使用所隐含，谓词是特称的。

第二，命题的项是可理解的，无需假设进入这些项的表达式的元素之间存在任何被理解的关系。

第三，这些命题是独立的。

如果这种思辨并非全然徒劳，我们可以允许自己猜想：这些就是那些无误的存在者在将语言用作表达和思维的工具时所遵循的条件--他们只陈述自己的意思，一方面不压抑，另一方面不重复。无论从它们与完美语言观念的关系来看，还是从它们与精确方法过程的关系来看，这些条件都同样值得研究者注意。

Chapter XI

OF SECONDARY PROPOSITIONS, AND OF THE PRINCIPLES OF THEIR SYMBOLICAL EXPRESSION.

1. The doctrine has already been established in Chap. IV., that every logical proposition may be referred to one or the other of two great classes, viz., Primary Propositions and Secondary Propositions. The former of these classes has been discussed in the preceding chapters of this work, and we are now led to the consideration of Secondary Propositions, i.e. of Propositions concerning, or relating to, other propositions regarded as true or false. The investigation upon which we are entering will, in its general order and progress, resemble that which we have already conducted. The two inquiries differ as to the subjects of thought which they recognise, not as to the formal and scientific laws which they reveal, or the methods or processes which are founded upon those laws. Probability would in some measure favour the expectation of such a result. It consists with all that we know of the uniformity of Nature, and all that we believe of the immutable constancy of the Author of Nature, to suppose, that in the mind, which has been endowed with such high capabilities, not only for converse with surrounding scenes, but for the knowledge of itself, and for reflection upon the laws of its own constitution, there should exist a harmony and uniformity not less real than that which the study of the physical sciences makes known to us. Anticipations such as this are never to be made the primary rule of our inquiries, nor are they in any degree to divert us from those labours of patient research by which we ascertain what is the actual constitution of things within the particular province submitted to investigation. But when the grounds of resemblance have been properly and independently determined, it is not inconsistent, even with purely scientific ends, to make that resemblance a subject of meditation, to trace its extent, and to receive the intimations of truth, yet undiscovered, which it may seem to us to convey. The necessity of a final appeal to fact is not thus set aside, nor is the use of analogy extended beyond its proper sphere,–the suggestion of relations which independent inquiry must either verify or cause to be rejected.

2. *Secondary Propositions are those which concern or relate to Propositions considered as true or false.* The relations of *things* we express by primary propositions. But we are able to make Propositions themselves also the subject

第十一章

论次级命题，以及论 它们符号表达的原则。

1. 第四章已经确立了这一学说：每一个逻辑命题都可以归入两大类之一，即主命题和次命题。前一类已在本著前几章中讨论过，我们现在被引向对次命题的考察，即关于、或涉及被视为真或假的其他命题的命题。我们即将进行的探究，在其一般顺序和进展上，将类似于我们已经进行过的探究。这两项研究的不同在于它们所承认的思维对象，而不在于它们所揭示的形式的和科学的规律，也不在于建立在这些规律之上的方法或过程。概率会在某种程度上支持对这种结果的预期。它与我们所知的一切关于自然界的齐一性、以及我们所相信的一切关于自然界作者的不可改变的恒常性相一致，可以设想：在这样一个被赋予了如此之高能力的心灵中--不仅用于与周围景象交谈，而且用于认识自身，用于反思自身构造的规律--应当存在着一种和谐与齐一，其真实性不亚于物理科学的研究向我们揭示的那种和谐与齐一。诸如此类的预期绝不能成为我们探究的首要规则，也绝不能在任何程度上使我们偏离那些耐心研究的劳作，正是通过这些劳作我们才能确定提交给探究的特定领域内事物的实际构造。但是，当相似性的依据已被恰当而独立地确定之后，即使对于纯粹科学的目的而言，将这种相似性作为沉思的主题、追溯其范围、并接受它似乎向我们传达的尚未发现的真理的暗示，也并非不相宜。对事实作最终诉诸的必要性并不因此被搁置，类比的使用也不超出其应有的范围--即提示那些独立探究必须加以验证或予以拒绝的关系。

2. 次级命题是那些涉及或关涉被视为真或假的命题的命题。我们通过初级命题来表达事物的关系。但我们也能够使命题本身成为主词

of thought, and to express our judgments concerning them. The expression of any such judgment constitutes a secondary proposition. There exists no proposition whatever of which a competent degree of knowledge would not enable us to make one or the other of these two assertions, viz., either that the proposition is true, or that it is false; and each of these assertions is a secondary proposition. "It is true that the sun shines;" "It is not true that the planets shine by their own light;" are examples of this kind. In the former example the Proposition "The sun shines," is asserted to be true. In the latter, the Proposition, "The planets shine by their own light," is asserted to be false. Secondary propositions also include all judgments by which we express a relation or dependence among propositions. To this class or division we may refer conditional propositions, as, "If the sun shine the day will be fair." Also most disjunctive propositions, as, "Either the sun will shine, or the enterprise will be postponed." In the former example we express the dependence of the truth of the Proposition, "The day will be fair," upon the truth of the Proposition, "The sun will shine." In the latter we express a relation between the two Propositions, "The sun will shine," "The enterprise will be postponed," implying that the truth of the one excludes the truth of the other. To the same class of secondary propositions we must also refer all those propositions which assert the simultaneous truth or falsehood of propositions, as, "It is not true both that 'the sun will shine' and that 'the journey will be postponed.' " The elements of distinction which we have noticed may even be blended together in the same secondary proposition. It may involve both the disjunctive element expressed by *either*, *or*, and the conditional element expressed by *if*; in addition to which, the connected propositions may themselves be of a compound character. *If* "the sun shine," *and* "leisure permit," then *either* "the enterprise shall be commenced," *or* "some preliminary step shall be taken." In this example a number of propositions are connected together, not arbitrarily and unmeaningly, but in such a manner as to express a *definite* connexion between them,–a connexion having reference to their respective truth or falsehood. This combination, therefore, according to our definition, forms a Secondary Proposition.

The theory of Secondary Propositions is deserving of attentive study, as well on account of its varied applications, as for that close and harmonious analogy, already referred to, which it sustains with the theory of Primary Propositions. Upon each of these points I desire to offer a few further observations.

3. I would in the first place remark, that it is in the form of secondary propositions, at least as often as in that of primary propositions, that the reasonings of ordinary life are exhibited. The discourses, too, of the moralist and the metaphysician are perhaps less often concerning things and their qualities, than concerning principles and hypotheses, concerning truths and the mutual connexion and relation of truths. The conclusions which our narrow experience suggests in relation to the great questions of morals and society yet unsolved, manifest, in more ways than one, the limitations of their human origin; and though the existence of universal principles is not to be questioned, the partial formulae which comprise our knowledge of their application are subject to conditions, and exceptions, and failure. Thus, in those departments of inquiry

思维，并表达我们关于它们的判断。任何此类判断的表达都构成一个次命题。不存在任何命题，以至于适当程度的知识不能使我们作出以下两种断言之一，即要么该命题是真的，要么该命题是假的；而这两种断言中的每一个都是次命题。"太阳照耀是真的；""行星靠自身的光照耀不是真的；"就是这类例子。在前一例中，命题"太阳照耀"被断言为真。在后一例中，命题"行星靠自身的光照耀"被断言为假。次命题还包括我们借以表达命题之间的关系或依赖性的所有判断。我们可以将条件命题归入这一类或 division，如"如果太阳照耀，天气将是晴朗的。"还有大多数析取命题，如"要么太阳将照耀，要么计划将被推迟。"在前一例中，我们表达命题"天气将是晴朗的"的真对命题"太阳将照耀"的真的依赖。在后一例中，我们表达两个命题"太阳将照耀"、"计划将被推迟"之间的关系，意味着一个的真排斥另一个的真。我们同样必须将那些断言命题同时为真或同时为假的命题归入这同一类次命题，如"并非'太阳将照耀'和'旅程将被推迟'两者同时为真。"我们所注意到的区分要素甚至可以在同一个次命题中混合在一起。它可能既包含由要么、要么表达的析取要素，又包含由如果表达的条件要素；除此之外，相联系的命题本身还可能是复合性质的。如果"太阳照耀",并且"闲暇允许"，那么要么"计划应当开始",要么"应当采取某种预备步骤"。在这个例子中，若干命题被联结在一起，不是任意地和无意义地，而是以这样的方式表达它们之间的确定的联系--一种涉及它们各自真假性的联系。因此，按照我们的定义，这种组合构成一个次命题。

次级命题的理论值得认真研究，既因为它的多种应用，也因为前面已经提到的它与初级命题理论之间那种紧密而和谐的类比。对于这些要点中的每一个，我都希望进一步提出几点观察。

3. 我首先要指出，日常生活中的推理至少与以主命题形式出现一样频繁地以次命题形式出现。道德家和形而上学家的论述，也许较少关于事物及其性质，而较多关于原则和假说，关于真理以及真理之间的相互联系和关系。我们狭隘的经验就道德和社会那些尚未解决的重大问题所提示的结论，以不止一种方式表明了它们的人类起源的局限性；尽管普遍原则的存在不容置疑，但包含我们对它们应用之知识的那些局部公式却受制于条件、例外和失败。因此，在那些探究部门中

which, from the nature of their subject-matter, should be the most interesting of all, much of our actual knowledge is hypothetical. That there has been a strong tendency to the adoption of the same forms of thought in writers on speculative philosophy, will hereafter appear. Hence the introduction of a general method for the discussion of hypothetical and the other varieties of secondary propositions, will open to us a more interesting field of applications than we have before met with.

4. The discussion of the theory of Secondary Propositions is in the next place interesting, from the close and remarkable analogy which it bears with the theory of Primary Propositions. It will appear, that the formal laws to which the operations of the mind are subject, are identical in expression in both cases. The mathematical processes which are founded on those laws are, therefore, identical also. Thus the methods which have been investigated in the former portion of this work will continue to be available in the new applications to which we are about to proceed. But while the laws and processes of the method remain unchanged, the rule of interpretation must be adapted to new conditions. Instead of classes of things, we shall have to substitute propositions, and for the relations of classes and individuals, we shall have to consider the connexions of propositions or of events. Still, between the two systems, however differing in purport and interpretation, there will be seen to exist a pervading harmonious relation, an analogy which, while it serves to facilitate the conquest of every yet remaining difficulty, is of itself an interesting subject of study, and a conclusive proof of that unity of character which marks the constitution of the human faculties.

Proposition I.

5. *To investigate the nature of the connexion of Secondary Propositions with the idea of Time.*

It is necessary, in entering upon this inquiry, to state clearly the nature of the analogy which connects Secondary with Primary Propositions.

Primary Propositions express relations among things, viewed as component parts of a universe within the limits of which, whether coextensive with the limits of the actual universe or not, the matter of our discourse is confined. The relations expressed are essentially *substantive*. Some, or all, or none, of the members of a given class, are also members of another class. The subjects to which primary propositions refer–the relations among those subjects which they express–are all of the above character.

But in treating of secondary propositions, we find ourselves concerned with another class both of subjects and relations. For the subjects with which we have to do are themselves propositions, so that the question may be asked,–Can we regard these subjects also as *things*, and refer them, by analogy with the previous case, to a universe of their own? Again, the relations among these subject propositions are relations of coexistent truth or falsehood, not of substantive equivalence. We do not say, when expressing the connexion of two distinct propositions, that the one *is* the other, but use some such forms of speech as

从其主题的性质来看，这本应是最有趣的，但我们实际知识中有许多是假设性的。以后将会看到，思辨哲学的作者们有一种强烈的倾向，采用相同的思维形式。因此，引入一种一般方法来讨论假设性命题和其他种类的次级命题，将为我们开辟一个比我们以前遇到的更有趣的应用领域。

4. 其次，关于次命题理论的讨论之所以有趣，是因为它与原始命题理论有着密切而显著的类比关系。将会看到，心灵活动所服从的形式规律在两种情形下具有相同的表达。因此，建立在这些规律之上的数学过程也是相同的。这样，在本著作前一部分所研究的方法，在我们即将着手的新应用中将继续可用。然而，尽管方法的规律和过程保持不变，解释的规则必须适应新的条件。我们将不得不以命题代替事物的类，并且对于类和个体的关系，我们将不得不考虑命题或事件的联系。尽管如此，在两个系统之间，无论在意旨和解释上如何不同，将会看到存在一种普遍贯通的和谐关系，一种类比，它既有助于克服每一个尚存的困难，本身又是一个有趣的研究主题，并且是标志着人类官能构造的那种性格统一的conclusive proof。

命题一。

5. 研究次级命题与时间观念之间联系的性质。

在进入这一研究时，有必要清楚地说明连接次级命题与初级命题的类比的性质。

原始命题表达事物之间的关系，这些事物被视为一个宇宙的组成部分，而我们的论域材料就局限在这个宇宙的界限之内，无论这个宇宙是否与实际宇宙的界限同延。所表达的关系本质上是实体性的。一个给定类的成员中的一些、全部或没有一个，也是另一个类的成员。原始命题所涉及的主词--它们所表达的那些主词之间的关系--全都具有上述性质。

但在处理次命题时，我们发现自己涉及的是另一类主词和关系。因为我们所要处理的主词本身就是命题，所以可以问--我们是否也能把这些主词视为事物，并按照前一种情形的类比，将它们归入一个它们自己的宇宙?再者，这些主词命题之间的关系是共存真理或虚假的关系，而不是实体等价的关系。当表达两个不同命题的联系时，我们不说一个是另一个，而是使用诸如此类的说法：

the following, according to the meaning which we desire to convey: "*Either* the proposition X is true, *or* the proposition Y is true;" "If the proposition X is true, the proposition Y is true;" "The propositions X and Y are jointly true;" and so on.

Now, in considering any such relations as the above, we are not called upon to inquire into the whole extent of their possible meaning (for this might involve us in metaphysical questions of causation, which are beyond the proper limits of science); but it suffices to ascertain some meaning which they undoubtedly possess, and which is adequate for the purposes of logical deduction. Let us take, as an instance for examination, the conditional proposition, "If the proposition X is true, the proposition Y is true." An undoubted meaning of this proposition is, that the *time* in which the proposition X is true, is *time* in which the proposition Y is true. This indeed is only a relation of coexistence, and may or may not exhaust the meaning of the proposition, but it is a relation really involved in the statement of the proposition, and further, it suffices for all the purposes of logical inference.

The language of common life sanctions this view of the essential connexion of secondary propositions with the notion of time. Thus we limit the application of a primary proposition by the word "some," but that of a secondary proposition by the word "sometimes." To say, "Sometimes injustice triumphs," is equivalent to asserting that there are times in which the proposition "Injustice now triumphs," is a true proposition. There are indeed propositions, the truth of which is not thus limited to particular periods or conjunctures; propositions which are true throughout all time, and have received the appellation of "eternal truths." The distinction must be familiar to every reader of Plato and Aristotle, by the latter of whom, especially, it is employed to denote the contrast between the abstract verities of science, such as the propositions of geometry which are always true, and those contingent or phænomenal relations of things which are sometimes true and sometimes false. But the forms of language in which both kinds of propositions are expressed manifest a common dependence upon the idea of time; in the one case as limited to some finite duration, in the other as stretched out to eternity.

6. It may indeed be said, that in ordinary reasoning we are often quite unconscious of this notion of time involved in the very language we are using. But the remark, however just, only serves to show that we commonly reason by the aid of words and the forms of a well-constructed language, without attending to the ulterior grounds upon which those very forms have been established. The course of the present investigation will afford an illustration of the very same principle. I shall avail myself of the notion of time in order to determine the laws of the expression of secondary propositions, as well as the laws of combination of the symbols by which they are expressed. But when those laws and those forms are once determined, this notion of time (essential, as I believe it to be, to the above end) may practically be dispensed with. We may then pass from the forms of common language to the closely analogous forms of the symbolical instrument of thought here developed, and use its processes, and interpret its results, without any conscious recognition of the idea of time whatever.

根据我们希望传达的意义，如下："或者命题X是真的，或者命题Y是真的；""如果命题X是真的，那么命题Y是真的；""命题X和Y共同是真的；"等等。

现在，在考虑诸如此类的关系时，我们并不被要求探究它们可能意义的全部范围(因为这可能使我们陷入因果性的形而上学问题，而这些问题超出了科学的正当界限)；但只需确定它们无疑具有的某种意义，而这种意义足以满足逻辑演绎的目的。让我们以一个条件命题作为考察的实例："如果命题 X 是真的，那么命题 Y 是真的。"这个命题的一个无疑意义是：命题 X 为真的时间，就是命题 Y 为真的时间。这确实只是一种共存关系，可能并未穷尽该命题的意义，但它是该命题陈述中确实涉及的一种关系，而且进一步说，它足以满足逻辑推论的一切目的。

日常生活的语言认可这种关于次命题与时间观念本质联系的观点。因此，我们用"有些"一词来限制原始命题的应用，但用"有时"一词来限制次命题的应用。说"有时不义得胜"，等同于断言存在一些时间，在这些时间中命题"不义现在得胜"是一个真命题。确实有一些命题，其真理并非如此局限于特定时期或时机；这些命题在所有时间中都是真的，并获得了"永恒真理"的称号。这一区分对每个读过柏拉图(Plato)和亚里士多德(Aristotle)的人来说必定是熟悉的，尤其是后者，用它来表示科学的抽象真理--例如几何学命题，它们总是真的--与那些有时真有时假的事物之偶然的或现象的关系之间的对比。但是，表达这两种命题的语言形式都表现出对时间观念的共同依赖；在一种情形中局限于某个有限延续，在另一种情形中则延伸至永恒。

6. 确实可以说，在通常的推理中，我们往往完全没有意识到我们所使用的语言本身所包含的这种时间观念。但这一评论无论多么公正，也只是表明我们通常借助语词和结构良好的语言形式进行推理，而不去关注那些形式赖以确立的更深层依据。当前的研究进程将说明这同一原则。我将借助时间观念来确定次级命题表达式的规律，以及表达它们的记号组合的规律。但一旦那些规律和那些形式被确定下来，这种时间观念(我认为它对上述目的是本质性的)在实践中就可以被弃置不用。于是我们就可以从日常语言的形式过渡到这里所发展的思维符号工具的密切类似的形式，并使用其过程、解释其结果，而完全不必有意识地承认时间观念。

Proposition II.

7. *To establish a system of notation for the expression of Secondary Propositions, and to show that the symbols which it involves are subject to the same laws of combination as the corresponding symbols employed in the expression of Primary Propositions.*

Let us employ the capital letters X, Y, Z, to denote the elementary propositions concerning which we desire to make some assertion touching their truth or falsehood, or among which we seek to express some relation in the form of a secondary proposition. And let us employ the corresponding small letters x, y, z, considered as expressive of mental operations, in the following sense, viz.: Let x represent an act of the mind by which we fix our regard upon that portion of time for which the proposition X is true; and let this meaning be understood when it is asserted that x *denotes* the time for which the proposition X is true. Let us further employ the connecting signs $+$, $-$, $=$, &c., in the following sense, viz.: Let $x + y$ denote the aggregate of those portions of time for which the propositions X and Y are respectively true, those times being entirely separated from each other. Similarly let $x - y$ denote that remainder of time which is left when we take away from the portion of time for which X is true, that (by supposition) included portion for which Y is true. Also, let $x = y$ denote that the time for which the proposition X is true, is identical with the time for which the proposition Y is true. We shall term x; the *representative symbol* of the proposition X, &c.

From the above definitions it will follow, that we shall always have

$$x + y = y + x,$$

for either member will denote the same aggregate of time.

Let us further represent by xy the performance in succession of the two operations represented by y and x, i.e. the whole mental operation which consists of the following elements, viz., 1st, The mental selection of that portion of time for which the proposition Y is true. 2ndly, The mental selection, out of that portion of time, of such portion as it contains of the time in which the proposition X is true,–the result of these successive processes being the fixing of the mental regard upon the whole of that portion of time for which the propositions X and Y are both true.

From this definition it will follow, that we shall always have

$$xy = yx. \tag{1}$$

For whether we select mentally, first that portion of time for which the proposition Y is true, then out of the result that contained portion for which X is true; or first, that portion of time for which the proposition X is true, then out of the result that contained portion of it for which the proposition Y is true; we shall arrive at the same final result, viz., that portion of time for which the propositions X and Y are both true.

命题二。

7. 建立一个记号系统来表达次级命题，并表明它所涉及的记号服从与表达初级命题所用的相应记号相同的组合律。

让我们用大写字母 X、Y、Z 来表示那些我们想要就其真或假作出某种断言的、或试图在它们之间以次级命题的形式表达某种关系的初始命题。让我们用相应的小写字母 x、y、z 来表示心理操作，其意义如下，即：让 x 表示心灵的一种活动，通过它我们把注意力固定于命题 X 为真的那部分时间；当断言 x 表示命题 X 为真的时间时，就应作此理解。让我们进一步在如下意义下使用连接符号 +、-、= 等，即：让 x + y 表示命题 X 和 Y 分别为真的那些时间部分的总和，这些时间彼此完全分离。类似地，让 x – y 表示当我们从命题 X 为真的那部分时间中减去(据假设)命题 Y 为真的那包含部分后所剩下的那部分时间。又，让 x = y 表示命题 X 为真的时间与命题 Y 为真的时间相同。我们将称 x 为命题 X 的代表符号，等等。

由上述定义可以推出，我们将始终有

$$x + y = y + x,$$

对任一成员而言，都将表示同一时间集合。

让我们进一步用 xy 表示相继执行由 y 和 x 所表示的两个操作，即由以下元素构成的整个心理操作，即：第一，对命题 Y 为真的那部分时间的心理选择。第二，从该部分时间中，对其中所包含的命题 X 为真的那部分时间的心理选择--这些相继过程的结果是把注意力固定于命题 X 和 Y 均为真的那部分时间的整体。

由这一定义可以推出，我们将始终有

$$xy = yx. \tag{1}$$

因为无论我们首先在思想上选出命题Y为真的那部分时间，然后从结果中选出X为真的那部分时间；还是首先选出命题X为真的那部分时间，然后从结果中选出命题Y为真的那部分时间；我们都将得到相同的最终结果，即命题X和Y都为真的那部分时间。

By continuing this method of reasoning it may be established, that the laws of combination of the symbols x, y, z, &c., in the species of interpretation here assigned to them, are identical in expression with the laws of combination of the same symbols, in the interpretation assigned to them in the first part of this treatise. The reason of this final identity is apparent. For in both cases it is the same faculty, or the same combination of faculties, of which we study the operations; operations, the essential character of which is unaffected, whether we suppose them to be engaged upon that universe of things in which all existence is contained, or upon that whole of time in which all events are realized, and to some part, at least, of which all assertions, truths, and propositions, refer.

Thus, in addition to the laws above stated, we shall have by (4), Chap, II., the law whose expression is

$$x(y+z) = xy + xz; \tag{2}$$

and more particularly the fundamental law of duality (2) Chap, II., whose expression is

$$x^2 = x, or, x(1-x) = 0; \tag{3}$$

a law, which while it serves to distinguish the system of thought in Logic from the system of thought in the science of quantity, gives to the processes of the former a completeness and a generality which they could not otherwise possess.

8. Again, as this law (3) (as well as the other laws) is satisfied by the symbols 0 and 1, we are led, as before, to inquire whether those symbols do not admit of interpretation in the present system of thought. The same course of reasoning which we before pursued shows that they do, and warrants us in the two following positions, viz.:

1st, That in the expression of secondary propositions, 0 represents *nothing* in reference to the element of time.

2nd, That in the same system 1 represents the universe, or whole of time, to which the discourse is supposed in any manner to relate.

As in primary propositions the universe of discourse is sometimes limited to a small portion of the actual universe of things, and is sometimes co-extensive with that universe; so in secondary propositions, the universe of discourse may be limited to a single day or to the passing moment, or it may comprise the whole duration of time. It may, in the most literal sense, be "eternal." Indeed, unless there is some limitation expressed or implied in the nature of the discourse, the proper interpretation of the symbol 1 in secondary propositions is "eternity;" even as its proper interpretation in the primary system is the actually existent universe.

9. Instead of appropriating the symbols x, y, z, to the representation of the truths of propositions, we might with equal propriety apply them to represent the occurrence of events. In fact, the occurrence of an event both implies, and is implied by, the truth of a proposition, viz., of the proposition which asserts the occurrence of the event. The one signification of the symbol x

通过继续这种推理方法可以确立：记号 x、y、z 等在此处赋予它们的解释种类下的组合规律，与同一记号在本论著第一部分赋予它们的解释下的组合规律，在表达式上是相同的。这种最终同一性的理由是明显的。因为在两种情况下，我们所研究的都是同一种能力，或同一种能力的组合；这些操作的本质特征不受影响，无论我们假设它们作用于包含一切存在的那个事物论域，还是作用于一切事件得以实现的整个时间，而一切断言、真理和命题至少都涉及这整个时间的某一部分。

因此，除了上述定律之外，我们还将根据第二章第(4)节得到其表达式为的定律

$$x(y+z) = xy + xz; \tag{2}$$

尤其是第二章第(2)节的基本对偶律，其表达式为

$$x^2 = x, or, x(1-x) = 0; \tag{3}$$

这一条定律，虽然它有助于区分逻辑中的思想系统与量科学中的思想系统，却赋予前者的过程一种它们否则不可能具有的完备性和一般性。

8.再者，由于这条定律(3)(以及其他定律)被记号0和1所满足，我们如前一样被引导去追问这些记号在当前思想系统中是否容许解释。我们先前所进行的同一推理过程表明它们确实容许，并使我们有理由采取以下两个立场，即：

第一，在次级命题的表达式中，0 相对于时间元素表示无。

第二，在同一系统中，1 表示论域，即该论述以任何方式被认为所涉及的全部时间。

正如在初级命题中论域有时被限制于实际事物论域的一小部分，有时又与那个论域同广延；同样在次级命题中，论域可以被限制于单独一天或流逝的瞬间，也可以包含时间的全部持续。它可以在最字面的意义上成为"永恒的"。事实上，除非话语的性质中表达或暗示了某种限制，次级命题中记号 1 的恰当解释就是"永恒"；正如它在初级系统中的恰当解释是实际存在的论域。

9.我们不必将记号x、y、z专门用于表示命题的真，而可以同样恰当地用它们来表示事件的发生。事实上，事件的发生既蕴含又被告命题的真所蕴含，即断言该事件发生的命题的真。记号x的一个意义

necessarily involves the other. It will greatly conduce to convenience to be able to employ our symbols in either of these really equivalent interpretations which the circumstances of a problem may suggest to us as most desirable; and of this liberty I shall avail myself whenever occasion requires. In problems of pure Logic I shall consider the symbols x, y, &c. as representing elementary propositions, among which relation is expressed in the premises. In the mathematical theory of probabilities, which, as before intimated (I. 12), rests upon a basis of Logic, and which it is designed to treat in a subsequent portion of this work, I shall employ the same symbols to denote the simple events, whose implied or required frequency of occurrence it counts among its elements.

Proposition III.

10. *To deduce general Rules for the expression of Secondary Propositions.*

In the various inquiries arising out of this Proposition, fulness of demonstration will be the less necessary, because of the exact analogy which they bear with similar inquiries already completed with reference to primary propositions. We shall first consider the expression of terms; secondly, that of the propositions by which they are connected.

As 1 denotes the whole duration of time, and x that portion of it for which the proposition X is true, $1 - x$ will denote that portion of time for which the proposition X is false.

Again, as xy denotes that portion of time for which the propositions X and Y are both true, we shall, by combining this and the previous observation, be led to the following interpretations, viz.:

The expression $x(1-y)$ will represent the time during which the proposition X is true, and the proposition Y false. The expression $(1-x)(1-y)$ will represent the time during which the propositions X and Y are simultaneously false.

The expression $x(1-y)+y(1-x)$ will express the time during which either X is true or Y true, but not both; for that time is the sum of the times in which they are singly and exclusively true. The expression $xy+(1-x)(1-y)$ will express the time during which X and Y are either both true or both false.

If another symbol z presents itself, the same principles remain applicable. Thus xyz denotes the time in which the propositions X, Y, and Z are simultaneously true; $(1-x)(1-y)(1-z)$ the time in which they are simultaneously false; and the sum of these expressions would denote the time in which they are either true or false together.

The general principles of interpretation involved in the above examples do not need any further illustrations or more explicit statement.

11. The laws of the expression of propositions may now be exhibited and studied in the distinct cases in which they present themselves. There is, however, one principle of fundamental importance to which I wish in the first place to direct attention. Although the principles of expression which have been laid down are perfectly general, and enable us to limit our assertions of the truth or falsehood of propositions to any particular portions of that whole of time

必然包含另一者。能够以这两种真正等价的解释中的任何一种来使用我们的记号，而问题的情境可能向我们表明哪一种最为可取，这将大大有助于便利；凡遇场合需要，我都要利用这一自由。在纯逻辑的问题中，我将把记号 x、y 等视为代表基本命题，前提所表达的关系即在这些命题之间。在概率的数学理论中--如前所述(I.12)，它建立在逻辑的基础之上，并且本书打算在后续部分加以处理--我将使用同样的记号来表示简单事件，这些事件所隐含的或所要求的出现频率被它算作其元素之一。

命题三。

10. 推导表达次级命题的一般规则。在由这一命题产生的各种研究中，充分论证将不那么必要，因为它们与已经完成的关于初级命题的类似研究具有精确的类比。我们将首先考虑项的表达；其次考虑连接它们的命题的表达。

由于1表示时间的整个持续，而x表示其中命题X为真的那部分，1 – x将表示命题X为假的那部分时间。

再者，由于xy表示命题X和Y都为真的那部分时间，我们将通过结合这一点和先前的观察，得出以下解释，即：

表达式x(1 – y)将表示命题X为真而命题Y为假的那段时间。表达式(1 – x)(1 – y)将表示命题X和Y同时为假的那段时间。

表达式x(1 – y) + y(1 – x)将表示X为真或Y为真但不同时为真的那段时间；因为那段时间是它们各自单独为真的时间之和。表达式xy +(1 – x)(1 – y)将表示X和Y或者都为真或者都为假的那段时间。

如果出现另一个记号z，同样的原则仍然适用。因此xyz表示命题X、Y和Z同时为真的时间；(1 – x)(1 – y)(1 – z)表示它们同时为假的时间；这些表达式之和将表示它们或者一起为真或者一起为假的时间。

上述例子中所涉及的解释的一般原则不需要进一步的说明或更明确的陈述。

11. 命题表达式的规律现在可以在它们所呈现的不同情形中展示和研究。然而，有一个具有根本重要性的原则，我希望首先提请注意。尽管已经奠定的表达原则是完全一般的，并且使我们能够将关于命题真假的断言限制在整个时间中的任何特定部分

(whether it be an unlimited eternity, or a period whose beginning and whose end are definitely fixed, or the passing moment) which constitutes the universe of our discourse, yet, in the actual procedure of human reasoning, such limitation is not commonly employed. When we assert that a proposition is true, we generally mean that it is true throughout the whole duration of the time to which our discourse refers; and when different assertions of the unconditional truth or falsehood of propositions are jointly made as the premises of a logical demonstration, it is to the same universe of time that those assertions are referred, and not to particular and limited parts of it. In that necessary matter which is the object or field of the exact sciences every assertion of a truth may be the assertion of an "eternal truth." In reasoning upon transient phænomena (as of some social conjuncture) each assertion may be qualified by an immediate reference to the present time, "Now." But in both cases, unless there is a distinct expression to the contrary, it is to the same period of duration that each separate proposition relates. The cases which then arise for our consideration are the following:

1st. *To express the Proposition, "The proposition X is true."*

We are here required to express that within those limits of time to which the matter of our discourse is confined the proposition X is true. Now the time for which the proposition X is true is denoted by x, and the extent of time to which our discourse refers is represented by 1. Hence we have

$$x = 1 \tag{4}$$

as the expression required.

2nd. *To express the Proposition, "The proposition X is false."*

We are here to express that within the limits of time to which our discourse relates, the proposition X is false; or that within those limits there is no portion of time for which it is true. Now the portion of time for which it is true is x. Hence the required equation will be

$$x = 0. \tag{5}$$

This result might also be obtained by equating to the whole duration of time 1, the expression for the time during which the proposition X is false, viz., $1-x$. This gives

$$\begin{aligned} 1 - x &= 1, \\ \text{whence } x &= 0. \end{aligned}$$

3rd. *To express the disjunctive Proposition, "Either the proposition X is true or the proposition Y is true;" it being thereby implied that the said propositions are mutually exclusive, that is to say, that one only of them is true.*

The time for which either the proposition X is true or the proposition Y is true, but not both, is represented by the expression $x(1-y)+y(1-x)$. Hence we have

(无论它是一个无限的永恒，还是一个起点和终点都已明确固定的时期，还是流逝的瞬间)构成我们论域的全域，然而，在人类推理的实际进程中，这种限制通常并不被采用。当我们断言一个命题为真时，我们一般是指它在我们论域所指的整个时间持续期内为真；而当关于命题的无条件真或假的种种不同断言被共同作出，作为逻辑论证的前提时，这些断言所指向的是同一个时间全域，而不是它的特殊而有限的部分。在那作为精确科学的对象或领域的必然质料中，对真理的每一断言都可能是对"永恒真理"的断言。在关于短暂现象的推理中(例如关于某种社会局势)，每一断言都可以通过直接参照当前时间"现在"而得到限定。但在两种情形下，除非有明确的相反表述，每个单独的命题都与同一持续时期相关。于是，进入我们考察的情形如下：

第一，表达命题"该命题为真"。

我们在这里需要表达的是，在我们话语主题所限定的那些时间界限之内，命题 X 是真的。现在，命题 X 为真的时间由 x 表示，而我们话语所指的时间范围由 1 表示。因此我们有

$$x = 1 \tag{4}$$

作为所需的表达式。2nd.要表达命题，"该命题是假的。"

我们在这里要表达的是，在我们话语所涉及的时间界限之内，命题 X 是假的；或者在这些界限之内，没有任何一段时间使它为真。现在，它为真的那部分时间是 x。因此所需的方程将是

$$x = 0. \tag{5}$$

这一结果也可以通过将命题 X 为假的时间表达式，即 1 – x，等于整个时间长度 1 来获得。这给出

$$\begin{aligned} 1 - x &= 1, \\ \text{whence } x &= 0. \end{aligned}$$

3rd.要表达析取命题，"要么该命题是真的，要么该命题是真的；"由此暗示所述命题相互排斥，也就是说，其中只有一个是真的。

命题 X 为真或命题 Y 为真但不同时为真的时间，由表达式 x(1 – y) + y(1 – x) 表示。因此我们有

$$x(1-y)+y(1-x)=1, \tag{6}$$

for the equation required.

If in the above Proposition the particles *either*, *or*, are supposed not to possess an absolutely disjunctive power, so that the possibility of the simultaneous truth of the propositions X and Y is not excluded, we must add to the first member of the above equations the term xy. We shall thus have

$$\begin{gathered} xy+x(1-y)+(1-x)y=1, \\ \text{or } x+(1-x)y=1. \end{gathered} \tag{7}$$

4th. *To express the conditional Proposition, "If the proposition Y is true, the proposition X is true."*

Since whenever the proposition Y is true, the proposition X is true, it is necessary and sufficient here to express, that the time in which the proposition Y is true is time in which the proposition X is true; that is to say, that it is some indefinite portion of the whole time in which the proposition X is true. Now the time in which the proposition Y is true is y, and the whole time in which the proposition X is true is x. Let v be a symbol of time indefinite, then will vx represent an indefinite portion of the whole time x. Accordingly, we shall have

$$y=vx$$

as the expression of the proposition given.

12. When v is thus regarded as a symbol of time indefinite, vx may be understood to represent the whole, or an indefinite part, or no part, of the whole time x; for any one of these meanings may be realized by a particular determination of the arbitrary symbol v. Thus, if v be determined to represent a time in which the whole time x is included, vx will represent the whole time x. If v be determined to represent a time, some part of which is included in the time x, but which does not fill up the measure of that time, vx will represent a part of the time x. If, lastly, v is determined to represent a time, of which no part is common with any part of the time x, vx will assume the value 0, and will be equivalent to "no time," or "never."

Now it is to be observed that the proposition, "If Y is true, X is true," contains no assertion of the truth of either of the propositions X and Y. It may equally consist with the supposition that the truth of the proposition Y is a condition indispensable to the truth of the proposition X, in which case we shall have $v=1$; or with the supposition that although Y expresses a condition which, when realized, assures us of the truth of X, yet X may be true without implying the fulfilment of that condition, in which case v denotes a time, some part of which is contained in the whole time x; or, lastly, with the supposition that the proposition Y is not true at all, in which case v represents some time,

$$x(1-y)+y(1-x)=1, \tag{6}$$

for the equation required.

如果在上面的命题中，小品词*either*、*or*被假定不具有绝对的析取力量，从而不排除命题 X 和 Y 同时为真的可能性，我们就必须在上面的方程的第一个成员中加上项 xy。这样我们就有

$$\begin{gathered} xy+x(1-y)+(1-x)y=1, \\ \text{or } x+(1-x)y=1. \end{gathered} \tag{7}$$

4th.要表达条件命题，*"*如果该命题是真的，那么该命题是真的。*"*

既然每当命题 Y 为真时，命题 X 就为真，那么在这里就必然且充分地要表达：命题 Y 为真的时间，是命题 X 为真的时间；也就是说，它是命题 X 为真的全部时间中的某个不确定的部分。现在，命题 Y 为真的时间是 y，而命题 X 为真的全部时间是 x。设 v 为不确定时间的记号，那么 vx 将表示全部时间 x 的一个不确定的部分。据此，我们将有

$$y=vx$$

正如所给命题的表达式。

12. 当 v 被如此视为不确定时间的记号时，vx 可以被理解为表示全部时间 x 的整体、或不确定的部分、或无部分；因为通过对任意记号 v 的特定确定，这些意义中的任何一种都可以实现。于是，如果确定 v 表示一个包含全部时间 x 的时间，那么 vx 将表示全部时间 x。如果确定 v 表示这样一个时间，它的一部分包含在时间 x 之中，但并不充满该时间的度量，那么 vx 将表示时间 x 的一部分。最后，如果确定 v 表示这样一个时间，它没有任何部分与时间 x 的任何部分共同，那么 vx 将取值 0，并将等价于"无时间"，或"永不"。

现在应当注意，命题"如果 Y 为真，则 X 为真"并不包含对命题 X 和 Y 中任何一个为真的断言。它同样可以与如下假定相一致：命题 Y 的真对于命题 X 的真是一个不可或缺的条件，在这种情况下我们将有 v = 1；或者与如下假定相一致：尽管 Y 表达了一个条件，当它实现时便使我们确信 X 为真，然而 X 仍可能为真而不蕴含该条件的实现，在这种情况下 v 表示一个时间，它的一部分包含在全部时间 x 之中；或者最后，与如下假定相一致：命题 Y 根本不为真，在这种情况下 v 表示某个时间，

no part of which is common with any part of the time x. All these cases are involved in the general supposition that v is a symbol of time indefinite.

5th. *To express a proposition in which the conditional and the disjunctive characters both exist.*

The general form of a conditional proposition is, "If Y is true, X is true," and its expression is, by the last section, $y = vx$. We may properly, in analogy with the usage which has been established in primary propositions, designate Y and X as the terms of the conditional proposition into which they enter; and we may further adopt the language of the ordinary Logic, which designates the term Y, to which the particle *if* is attached, the "antecedent" of the proposition, and the term X the "consequent."

Now instead of the terms, as in the above case, being simple propositions, let each or either of them be a disjunctive proposition involving different terms connected by the particles *either*, *or*, as in the following illustrative examples, in which X, Y, Z, &c. denote simple propositions.

1st. If either X is true or Y is true, then Z is true.

2nd. If X is true, then either Y is true or Z true.

3rd. If either X is true or Y is true, then either Z and W are both true, or they are both false.

It is evident that in the above cases the relation of the antecedent to the consequent is not affected by the circumstance that one of those terms or both are of a disjunctive character. Accordingly it is only necessary to obtain, in conformity with the principles already established, the proper expressions for the antecedent and the consequent, to affect the latter with the indefinite symbol v, and to equate the results. Thus for the propositions above stated we shall have the respective equations,

$$\text{1st } x(1-y)+(1-x)y = vz.$$
$$\text{2nd.} x = v\{y(1-z)+z(1-y)\}.$$
$$\text{3rd.} x(1-y)+y(1-x) = v\{zw+(1-z)(1-w)\}$$

The rule here exemplified is of general application.

Cases in which the disjunctive and the conditional elements enter in a manner different from the above into the expression of a compound proposition, are conceivable, but I am not aware that they are ever presented to us by the natural exigencies of human reason, and I shall therefore refrain from any discussion of them. No serious difficulty will arise from this omission, as the general principles which have formed the basis of the above applications are perfectly general, and a slight effort of thought will adapt them to any imaginable case.

13. In the laws of expression above stated those of interpretation are implicitly involved. The equation

$$x = 1$$

must be understood to express that the proposition X is true; the equation

$$x = 0,$$

其中没有任何部分与时间 x 的任何部分共同。所有这些情况都包含在 v 是不定时间符号这一一般假定之中。

5th.要表达一个同时具有条件和析取性质的命题。

条件命题的一般形式是："如果Y为真，则X为真"，其表达式按上一节为y = vx。我们完全可以类比于主命题中已确立的用法，将Y和X称为它们所进入的条件命题的项；我们还可以进一步采用普通逻辑的语言，将附有虚词如果的项Y称为该命题的"前件"，将项X称为"后件"。

现在，不要像上述情况那样让各项都是简单命题，而让每一项或其中任一项是一个析取命题，包含由小品词*either*、*or*连接的不同项，如下面的说明性例子所示，其中 X、Y、Z 等表示简单命题。

第一。如果 X 为真或 Y 为真，那么 Z 为真。
第二，若X为真，则或者Y为真，或者Z为真。
第三。如果 X 为真或 Y 为真，那么 Z 和 W 或者都为真，或者都为假。

显然，在上述情形中，前件与后件的关系并不因这些项之一或两者具有析取性质而受到影响。因此，只需按照已确立的原则，获得前件和后件的恰当表达式，以后件乘以不定符号v，并令结果相等即可。于是，对于上述命题，我们将分别得到方程，

$$\text{1st } x(1-y)+(1-x)y = vz.$$
$$\text{2nd.} x = v\{y(1-z)+z(1-y)\}.$$
$$\text{3rd.} x(1-y)+y(1-x) = v\{zw+(1-z)(1-w)\}$$

The rule here exemplified is of general application.

可以设想，析取元素和条件元素以上述不同的方式进入复合命题的表达式中，但我不知道人类理性的自然需求是否曾向我们呈现过这类情形，因此我将不对它们进行任何讨论。这一省略不会引起严重的困难，因为构成上述应用基础的一般原则是完全普遍的，稍加思考即可将它们适用于任何可以想象的情形。

13. 在上述表达规律中，解释的规律隐含地包含在内。方程

$$x = 1$$

必须被理解为表达命题 X 是真的；方程

$$x = 0,$$

that the proposition X is false. The equation

$$xy = 1$$

will express that the propositions X and Y are both true together; and the equation

$$xy = 0$$

that they are not both together true.

In like manner the equations

$$\begin{aligned} x(1-y)+y(1-x) &= 1, \\ x(1-y)+y(1-x) &= 0, \end{aligned}$$

will respectively assert the truth and the falsehood of the disjunctive Proposition, "Either X is true or Y is true." The equations

$$\begin{aligned} y &= vx \\ y &= v(1-x) \end{aligned}$$

will respectively express the Propositions, "If the proposition Y is true, the proposition X is true." "If the proposition Y is true, the proposition X is false."

Examples will frequently present themselves, in the succeeding chapters of this work, of a case in which some terms of a particular member of an equation are affected by the indefinite symbol v, and others not so affected. The following instance will serve for illustration. Suppose that we have

$$y = xz + vx(1-z).$$

Here it is implied that the time for which the proposition Y is true consists of all the time for which X and Z are together true, together with an indefinite portion of the time for which X is true and Z false. From this it may be seen, 1st, That if Y is true, either X and Z are together true, or X is true and Z false; 2ndly, If X and Z are together true, Y is true. The latter of these may be called the reverse interpretation, and it consists in taking the antecedent out of the second member, and the consequent from the first member of the equation. The existence of a term in the second member, whose coefficient is unity, renders this latter mode of interpretation possible. The general principle which it involves may be thus stated:

14. PRINCIPLE.–*Any constituent term or terms in a particular member of an equation which have for their coefficient unity, may be taken as the antecedent of a proposition, of which all the terms in the other member form the consequent.*

Thus the equation

$$y = xz + vx(1-z) + (1-x)(1-z)$$

该命题X为假。方程

$$xy = 1$$

will express that the propositions X and Y are both true together; and the equation

$$xy = 0$$

它们并非同时为真。

以同样的方式，方程

$$\begin{aligned} x(1-y)+y(1-x) &= 1, \\ x(1-y)+y(1-x) &= 0, \end{aligned}$$

将分别断言析取命题"或者 X 是真的，或者 Y 是真的"的真和假。方程

$$\begin{aligned} y &= vx \\ y &= v(1-x) \end{aligned}$$

将分别表达命题："如果命题 Y 是真的，则命题 X 是真的。""如果命题 Y 是真的，则命题 X 是假的。"

在本著作接下来的各章中，将经常出现这样的例子：方程某一特定成员的某些项受到不定符号 v 的影响，而另一些项则不受其影响。下面的实例将用以说明。假设我们有

$$y = xz + vx(1-z).$$

这里隐含的是，命题Y为真的时间，由X和Z同时为真的全部时间，加上X为真而Z为假的时间中的不定部分所组成。由此可以看出：第一，如果Y为真，则或者X和Z同时为真，或者X为真而Z为假；第二，如果X和Z同时为真，则Y为真。后者可称为反向解释，它在于从方程的第二项中取出前件，从第一项中取出后件。第二项中存在系数为一的项，使得这后一种解释方式成为可能。它所涉及的一般原则可表述如下：

14. 原则。--方程某一特定成员中任何一个或一些以其系数为单位的构成项，可以被当作一个命题的前件，而另一成员中的所有项构成后件。

因此方程

$$y = xz + vx(1-z) + (1-x)(1-z)$$

would have the following interpretations:

DIRECT INTERPRETATION.–*If the proposition Y is true, then either X and Z are true, or X is true and Z false, or X and Z are both false.*

REVERSE INTERPRETATION.–*If either X and Z are true, or X and Z are false, Y is true.*

The aggregate of these partial interpretations will express the whole significance of the equation given.

15. We may here call attention again to the remark, that although the idea of time appears to be an essential element in the theory of the interpretation of secondary propositions, it may practically be neglected as soon as the laws of expression and of interpretation are definitely established. The forms to which those laws give rise seem, indeed, to correspond with the forms of a perfect language. Let us imagine any known or existing language freed from idioms and divested of superfluity, and let us express in that language any given proposition in a manner the most simple and literal,–the most in accordance with those principles of pure and universal thought upon which all languages are founded, of which all bear the manifestation, but from which all have more or less departed. The transition from such a language to the notation of analysis would consist of no more than the substitution of one set of signs for another, without essential change either of form or character. For the elements, whether things or propositions, among which relation is expressed, we should substitute letters; for the disjunctive conjunction we should write +; for the connecting copula or sign of relatioin, we should write =. This analogy I need not pursue. Its reality and completeness will be made more apparent from the study of those forms of expression which will present themselves in subsequent applications of the present theory, viewed in more immediate comparison with that imperfect yet noble instrument of thought–the English language.

16. Upon the general analogy between the theory of Primary and that of Secondary Propositions, I am desirous of adding a few remarks before dismissing the subject of the present chapter.

We might undoubtedly, have established the theory of Primary Propositions upon the simple notion of space, in the same way as that of secondary propositions has been established upon the notion of time. Perhaps, had this been done, the analogy which we are contemplating would have been in somewhat closer accordance with the view of those who regard space and time as merely "forms of the human understanding," conditions of knowledge imposed by the very constitution of the mind upon all that is submitted to its apprehension. But this view, while on the one hand it is incapable of demonstration, on the other hand ties us down to the recognition of "place," τὸ ποῦ ν, as an essential category of existence. The question, indeed, whether it is so or not, lies, I apprehend, beyond the reach of our faculties; but it may be, and I conceive has been, established, that the formal processes of reasoning in primary propositions do not require, as an essential condition, the manifestation in space of the things about which we reason; that they would remain applicable, with equal strictness of demonstration, to forms of existence, if such there be, which lie beyond the realm of sensible extension. It is a fact, perhaps, in some

会有以下解释：

直接解释。--如果该命题是真的，那么或者和是真的，或者是真的而是假的，或者和都是假的。

反向解释。--如果或者和是真的，或者和是假的，则是真的。

这些部分解释的总和将表达所给方程的整个意义。

15. 我们可以在此再次提请注意这一评述：尽管时间的观念似乎是次级命题解释理论中的一个本质要素，但一旦表达和解释的规律被明确确立，它在实践上就可以被忽略。这些规律所产生的形式，似乎确实与一种完美语言的形式相对应。让我们设想任何一种已知的或现存的语言，摆脱了习语，去除了冗余，让我们用那种语言以最简明、最字面的方式表达任何给定的命题--最符合那些纯粹而普遍的思想原则的方式，所有语言都建立在那些原则之上，所有语言都体现着它们，但所有语言都或多或少偏离了它们。从这样一种语言过渡到分析的记号，只不过是用一组符号替换另一组符号，在形式或性质上都没有本质的变化。对于其间表达着关系的元素，无论是事物还是命题，我们应当用字母来替换；对于析取连词，我们应当写+；对于连接系词或关系符号，我们应当写=。这一类比我无需继续展开。它的真实性和完整性，将从研究那些在本书理论的后续应用中出现、并与那种不完美却高贵的思想工具--英语--进行更直接比较的表达形式中，变得更加明显。

16. 关于初级命题理论与次级命题理论之间的一般类比，我想在结束本章主题之前补充几点意见。

我们无疑可以像建立关于时间的次级命题理论那样，仅仅依据空间这一简单观念来建立关于初级命题的理论。也许，若果真如此做了，我们所考察的这种类比就会在某种程度上更贴近那些把空间和时间仅仅视为"人类知性的形式"、视为心灵的构造本身加于一切被提交给其领会之物上的知识条件的人的观点。但这种观点一方面无法得到证明，另一方面又把我们束缚于承认"处所",τὸποΎν，为存在的一个本质范畴。这个问题究竟是否如此，我理解，超出了我们官能所及的范围；但可以--而且我认为已经--确立的是：初级命题中的形式推理过程并不需要以我们所推理之事物在空间中的显现作为本质条件；这些过程将同样严格地适用于证明那些或许存在于可感广延领域之外的存在形式。这或许在某种

degree analogous to this, that we are able in many known examples in geometry and dynamics, to exhibit the formal analysis of problems founded upon some intellectual conception of space different from that which is presented to us by the senses, or which can be realized by the imagination. [1] I conceive, therefore, that the idea of space is not essential to the development of a theory of primary propositions, but am disposed, though desiring to speak with diffidence upon a question of such extreme difficulty, to think that the idea of time is essential to the establishment of a theory of secondary propositions. There seem to be grounds for thinking, that without any change in those faculties which are concerned in *reasoning*, the manifestation of space to the human mind might have been different from what it is, but not (at least the same) grounds for supposing that the manifestation of time could have been otherwise than we perceive it to be. Dismissing, however, these speculations as possibly not altogether free from presumption, let it be affirmed that the real ground upon which the symbol 1 represents in primary propositions the universe of things, and not the space they occupy, is, that the sign of identity = connecting the members of the corresponding equations, implies that the things which they represent are identical, not simply that they are found in the same portion of space. Let it in like manner be affirmed, that the reason why the symbol 1 in secondary propositions represents, not the universe of events, but the eternity in whose successive moments and periods they are evolved, is, that the same sign of identity connecting the logical members of the corresponding equations implies, not that the events which those members represent are identical, but that the times of their occurrence are the same. These reasons appear to me to be decisive of the immediate question of interpretation. In a former treatise on this subject (Mathematical Analysis of Logic, p. 49), following the theory of Wallis respecting the Reduction of Hypothetical Propositions, I was led to interpret the symbol 1 in secondary propositions as the universe of "cases" or "conjunctures of circumstances;" but this view involves the necessity of a definition of what is meant by a "case," or "conjuncture of circumstances;" and it is certain, that whatever is involved in the term beyond the notion of time is alien to the objects, and restrictive of the processes, of formal Logic.

[1]Space is presented to us in perception, as possessing the three dimensions of length, breadth, and depth. But in a large class of problems relating to the properties of curved surfaces, the rotations of solid bodies around axes, the vibrations of elastic media, &c., this limitation appears in the analytical investigation to be of an arbitrary character, and if attention were paid to the processes of solution alone, no reason could be discovered why space should not exist in four or in any greater number of dimensions. The intellectual procedure in the imaginary world thus suggested can be apprehended by the clearest light of analogy.

The existence of space in three dimensions, and the views thereupon of the religious and philosophical mind of antiquity, are thus set forth by Aristotle:– Μεγέθος δὲ τὸ μὲν ἐφ ̈ἑν, γραμμή τὸ δ΄ ἐπὶ δ΄νο ἐπίπεδον, τὸ δ΄ ἐπὶ τρία σῶμα΄ Καί παρὰ τα΄ντα ο΄νκ ἔστιν ἄλλο μέγεθος, διὰ τὸ τριά πάντα εἶναι καὶ τὸ τρὶς πάντη. Κάθαπερ γάρ φασι καὶ οἱ Πνθαγόρειοι, τὸ πᾶν καὶ τὰ πάντα τοῖς τρισὶν ̈ωρισται. Τελεντὴ γὰρ καὶ μέσον καὶ ἀρκὴ τὸν ἀριθμὸν ̈ἐκει τὸν το΄ν παντός΄ τα΄ντα δὲ τὸν τῆς τριάδος. Διὸ παρὰ τῆς φ΄νσεως εἰληφότες ̈ωσπερ νόμονς ἐκείνης, καὶ πρὸς τὰς ἀγιστείας κρώμεθα τῶν θεῶν τ΄ψ ἀριθμ΄ψ το΄ντ΄ψ.–*De Caelo*, 1.

程度上与此类似：在许多已知的几何学与动力学例子中，我们能够对基于某种不同于感官所呈现、或想象力所能实现的理智空间概念的问题作出形式分析。[1]因此我认为，空间观念对于初级命题理论的发展并非本质性的，但我倾向于--尽管在如此极端困难的问题上希望谨慎发言--认为时间观念对于次级命题理论的建立是本质性的。似乎有理由认为，在不改变那些与推理相关的官能的情况下，空间向人类心灵的显现本可能不同于其实际所是，但没有(至少同样的)理由假定时间的显现本可能不同于我们所感知的那样。然而，暂且把这些或许并非完全不带僭妄的思辨搁置一旁，让我们确认：在初级命题中，记号1代表事物的全域而非它们所占据的空间，其真正根据在于，连接相应方程各成员的同一性符号=意味着它们所代表的事物是同一的，而不仅仅是它们被发现于空间的同一部分。让我们以同样方式确认：在次级命题中，记号1所代表的不是事件的全域，而是那些事件在其相继时刻与时期中展开于其中的永恒，其理由在于，连接相应方程逻辑成员的同一性符号所意味的不是那些成员所代表的事件是同一的，而是它们发生的时间是相同的。这些理由在我看来对当前的解释问题具有决定性。在先前一部论此主题的著作(《逻辑的数学分析》，第49页)中，我遵循沃利斯关于假言命题还原的理论，曾把次级命题中的记号1解释为"情况"或"境况组合"的全域；但这种观点必然要求对"情况"或"境况组合"之所指作出定义；而且可以肯定，该词中超出时间观念之外所包含的一切，都与形式逻辑的对象无关，并限制着形式逻辑的过程。

1 空间在知觉中呈现给我们时具有长、宽、深三个维度。但在涉及曲面性质、固体绕轴旋转、弹性介质振动等一大类问题中，这一限制在分析研究中显得具有任意性；而且若只关注求解过程本身，便无法发现任何理由说明空间为何不能存在于四个或任何更多维度之中。如此提示的想象世界中的理智程序，可以借类比的最清晰之光加以领会。

空间在三个维度中的存在，以及古代宗教与哲学心灵对此的看法，由亚里士多德如此阐述：- Μεγέθος δὲτὸμὲν ἐφ ̈ἑν, γραμμή τὸδ΄ ἐπὶδ΄νο ἐπίπεδον, τὸδ΄ ἐπὶτρία σνῶμα΄ Καί παρὰτα΄ντα ο΄νκ ἔσντιν ̈ἄλλο μέγεθος, διὰτὸτριά πάντα εἶναι καὶτὸτρὶς πάντη.Κάθαπερ γάρ φασνι καὶοἱΠνθαγόρειοι, τὸπᾶν καὶτὰ πάντα τοῖς τρισνὶν ̈ωρισνται.Τελεντὴγὰρ καὶμέσνον καὶἀρκὴτὸν ἀριθμὸν ̈ἐκει τὸν το΄ν παντός΄ τα΄ντα δὲτὸν τῆς τριάδος.Διὸπαρὰτῆς φ΄νσεως εἰληφότες ̈ωσνπερ νόμονς ἐκείνης, καὶπρὸς τὰς ἀγισντείας κρώμεθα τῶν θεῶν τ΄ψ ἀριθμ΄ψ το΄ντ΄ψ.- 《论天》,1。

Chapter XII

OF THE METHODS AND PROCESSES TO BE ADOPTED IN THE TREATMENT OF SECONDARY PROPOSITIONS.

1. It has appeared from previous researches (XI. 7) that the laws of combination of the literal symbols of Logic are the same, whether those symbols are employed in the expression of primary or in that of secondary propositions, the sole existing difference between the two cases being a difference of interpretation. It has also been established (V. 6), that whenever distinct systems of thought and interpretation are connected with the same system of formal laws, i.e., of laws relating to the combination and use of symbols, the attendant processes, intermediate between the expression of the primary conditions of a problem and the interpretation of its symbolical solution, are the same in both. Hence, as between the systems of thought manifested in the two forms of primary and of secondary propositions, this community of formal law exists, the processes which have been established and illustrated in our discussion of the former class of propositions will, without any modification, be applicable to the latter.

2. Thus the laws of the two fundamental processes of elimination and development are the same in the system of secondary as in the system of primary propositions. Again, it has been seen (Chap. VI. Prop. 2) how, in primary propositions, the interpretation of any proposed equation devoid of fractional forms may be effected by developing it into a series of constituents, and equating to 0 every constituent whose coefficient does not vanish. To the equations of secondary propositions the same method is applicable, and the interpreted result to which it finally conducts us is, as in the former case (VI. 6), a system of co-existent denials. But while in the former case the force of those denials is expended upon the existence of certain classes of things, in the latter it relates to the truth of certain combinations of the elementary propositions involved in the *terms* of the given premises. And as in primary propositions it was seen that the system of denials admitted of conversion into various other forms of propositions (VI. 7), &c., such conversion will be found to be possible here also, the sole difference consisting not in the forms of the equations, but in the nature of their interpretation.

3. Moreover, as in primary propositions, we can find the expression of any

第十二章

论所应采用的方法与过程 在次级命题的处理中被采用。

1. 由先前的研究(XI.7)已经表明，逻辑的字面记号的组合规律是相同的，无论这些记号是用于表达初级命题还是用于表达次级命题，两种情形之间唯一存在的差别是解释上的差别。同样已经确立(V.6)，每当不同的思想体系与解释体系同同一套形式规律--即关于记号之组合与使用的规律--相联系时，伴随的过程，即介于问题之初级条件的表达与其符号解的解释之间的过程，在两者中是相同的。因此，既然在初级命题与次级命题这两种形式中所表现的思想体系之间存在这种形式规律的共同性，那么我们在讨论前一类命题时所已确立并阐明过的过程，将无需任何修改而适用于后一类命题。

2. 因此，消元与展开这两个基本过程的规律，在次命题系统中与在初命题系统中是相同的。再者，已经看到(第六章命题2)，在初命题中，任何不含分数形式的给定方程的解释，都可以通过将它展开为一系列成分，并令每个系数不为零的成分等于0来实现。同样的方法也适用于次命题的方程，而它最终引导我们得到的解释结果，正如前一种情形(VI.6)一样，是一个共存否定的系统。但是，在前一种情形中，那些否定的力量消耗在某些事物类的存在上，而在后一种情形中，它则涉及包含在给定前提的项中的基本命题的某些组合的真。并且，正如在初命题中已经看到，否定系统可以转换为各种其他形式的命题(VI.7)等等，这里也会发现这种转换是可能的，唯一的区别不在于方程的形式，而在于其解释的性质。

3. 此外，如同在初始命题中一样，我们可以求得任一

element entering into a system of equations, in terms of the remaining elements (VI. 10), or of any selected number of the remaining elements, and interpret that expression into a logical inference, the same object can be accomplished by the same means, difference of interpretation alone excepted, in the system of secondary propositions. The elimination of those elements which we desire to banish from the final solution, the reduction of the system to a single equation, the algebraic solution and the mode of its development into an interpretable form, differ in no respect from the corresponding steps in the discussion of primary propositions.

To remove, however, any possible difficulty, it may be desirable to collect under a general Rule the different cases which present themselves in the treatment of secondary propositions.

RULE.–*Express symbolically the given propositions (XI. 11).*

Eliminate separately from each equation in which it is found the indefinite symbol v (VII. 5).

Eliminate the remaining symbols which it is desired to banish from the final solution: always before elimination 'reducing to a single equation those equations in which the symbol or symbols to be eliminated are found (VIII. 7). Collect the resulting equations into a single equation $V = 0$.

Then proceed according to the particular form in which it is desired to express the final relation, as–

1st. *If in the form of a denial, or system of denials, develop the function* V*, and equate to 0 all those constituents whose coefficients do not vanish.*

2ndly. *If in the form of a disjunctive proposition, equate to 1 the sum of those constituents whose coefficients vanish.*

3rdly. *If in the form of a conditional proposition having a simple element, as* x *or* $1-x$*, for its antecedent, determine the algebraic expression of that element, and develop that expression.*

4thly. *If in the form of a conditional proposition having a compound expression, as* xy*,* $xy + (1 - x)(1 - y)$*, &c., for its antecedent, equate that expression to a new symbol* t*, and determine* t *as a developed function of the symbols which are to appear in the consequent, either by ordinary methods or by the special method* (IX. 9).

5thly. *Interpret the results by* (XI. 13, 14).

If it only be desired to ascertain whether a particular elementary proposition x *is true or false, we must eliminate all the symbols but* x*; then the equation* $x = 1$ *will indicate that the proposition is true,* $x = 0$ *that it is false,* $0 = 0$ *that the premises are insufficient to determine whether it is true or false.*

4. Ex. 1.–The following prediction is made the subject of a curious discussion in Cicero's fragmentary treatise, De Fato:–"Si quis (Fabius) natus est oriente Canicula, is in mari non morietur." I shall apply to it the method of this chapter. Let y represent the proposition, "Fabius was born at the rising of the dogstar;" x the proposition, "Fabius will die in the sea." In saying that x *represents* the proposition, "Fabius, &c.," it is only meant that x is a symbol so appropriated (XI. 7) to the above proposition, that the equation $x = 1$ declares, and the equation $x = 0$ denies, the truth of that proposition. The equation we have to

元素进入一个方程组，用其余元素(VI.10)或任意选定数目的其余元素来表示，并将该表达式解释为一个逻辑推论，同样的目标可以通过同样的手段达成，只是解释不同而已，在次命题的系统中也是如此。将我们想要从最终解答中排除的那些元素消去，将系统归约为单个方程，代数求解及其展开为可解释形式的方式，与在讨论原命题时相应步骤没有任何不同。

然而，为了消除任何可能的困难，或许宜将处理次命题时所呈现的各种不同情形归入一条一般规则之下。

规则。--用符号表示给定的命题(XI.11)。

从含有不定记号 v的每一个方程中分别消去该记号(VII.5)。

消去那些希望从最终解答中排除的其余记号：在消元之前，始终先将那些含有待消去记号或诸记号的方程化为单一方程*(VIII.7)*。将所得各方程合并为单一方程 V = 0。

然后按照希望表达最终关系所采取的特定形式进行，如下--

1. 若以否定或否定系统的形式，展开函数，并将所有系数不为零的构成项等于*0*。

2. 若以析取命题的形式，将系数为零的那些构成项之和等于*1*。

第三，若采取具有简单元素的条件命题形式，以其为前件，如或 1 – x ，则确定该元素的代数表达式，并展开该表达式。

第四，若采取具有复合表达式的条件命题形式，以其为前件，如， xy +(1 – x)(1 – y) ，等等，则将该表达式等于一个新记号，并确定其作为将在后件中出现的诸记号的展开函数，或通过普通方法，或通过特殊方法(IX.9)。

第五，按(XI.13, 14)解释结果。

若仅欲确定某一特定基本命题为真或为假，则必须消去除以外的所有记号；于是方程 x = 1 将表明该命题为真， x = 0 表明其为假， 0 = 0 表明前提不足以确定其为真或为假。

4. 例1。--以下预言在西塞罗(Cicero)残篇《论命运》(De Fato)中成为一场奇特讨论的主题：--"Si quis(Fabius) natus est oriente Canicula, is in mari non morietur."我将把本章的方法应用于它。令 y 表示命题"法比乌斯(Fabius)生于天狼星升起之时";x 表示命题"法比乌斯将死于海中"。在说 x 表示命题"法比乌斯，等等"时，意思只是说 x 是一个如此被指定(XI.7)给上述命题的记号，以至于方程 x = 1 肯定、方程 x = 0 否定该命题的真理性。我们所得到的方程

discuss will be

$$y = v(1 - x). \tag{1}$$

And, first, let it be required to reduce the given proposition to a negation or system of negations (XII. 3). We have, on transposition,

$$y - v(1 - x) = 0.$$

Eliminating v,

$$\begin{aligned} y\{y - (1 - x)\} &= 0, \\ \text{or, } y - y(1 - x) &= 0, \\ \text{or, } yx &= 0. \end{aligned} \tag{2}$$

The interpretation of this result is:–"It is not true that Fabius was born at the rising of the dogstar, and will die in the sea." Cicero terms this form of proposition, "Conjunctio ex repugnantibus;" and he remarks that Chrysippus thought in this way to evade the difficulty which he imagined to exist in contingent assertions respecting the future: "Hoc loco Chrysippus aestuans falli sperat Chaldaeos casterosque divinos, neque eos usuros esse conjunctionibus ut ita sua percepta pronuntient: Si quis natus est oriente Canicula is in mari non morietur; sed potius ita dicant: Non et natus est quis oriente Caniculâ, et in mari morietur. O licentiam jocularem! ... Multa genera sunt enuntiandi, nec ullum distortius quam hoc quo Chrysippus sperat Chaldaeos contentos Stoicorum causa fore."–*Cic. De Fato*, 7, 8.

5. To reduce the given proposition to a disjunctive form. The constituents not entering into the first member of (2) are

$$x(1 - y), (1 - x)y, (1 - x)(1 - y).$$

Whence we have

$$y(1 - x) + x(1 - y) + (1 - x)(1 - y) = 1. \tag{3}$$

The interpretation of which is:–*Either Fabius was born at the rising of the dogstar, and will not perish in the sea; or he was not born at the rising of the dogstar, and will perish in the sea; or he was not born at the rising of the dogstar, and will not perish in the sea.*

In cases like the above, however, in which there exist constituents differing from each other only by a single factor, it is, as we have seen (VII. 15), most convenient to collect such constituents into a single term. If we thus connect the first and third terms of (3), we have

$$(1 - y)x + 1 - x = 1;$$

and if we similarly connect the second and third, we have

$$y(1 - x) + 1 - y = 1.$$

discuss will be

$$y = v(1 - x). \tag{1}$$

And, first, let it be required to reduce the given proposition to a negation or system of negations (XII. 3). We have, on transposition,

$$y - v(1 - x) = 0.$$

Eliminating v,

$$\begin{aligned} y\{y - (1 - x)\} &= 0, \\ \text{or, } y - y(1 - x) &= 0, \\ \text{or, } yx &= 0. \end{aligned} \tag{2}$$

这个结果的解释是：--"法比乌斯生于天狼星升起之时并将死于海中，这不是真的。"西塞罗把这种形式的命题称为"Conjunctio ex repugnantibus"；并且他评论说，克律西波(Chrysippus)以为用这种方式就能回避他想象中存在于关于未来的偶然断言中的困难："Hoc loco Chrysippus aestuans falli sperat Chaldaeos casterosque divinos, neque eos usuros esse conjunctionibus ut ita sua percepta pronuntient: Si quis natus est oriente Canicula is in mari non morietur; sed potius ita dicant: Non et natus est quis oriente Caniculˆ a, et in mari morietur.O licentiam jocularem!...Multa genera sunt enuntiandi, nec ullum distortius quam hoc quo Chrysippus sperat Chaldaeos contentos Stoicorum causa fore."--西塞罗《论命运》,7,8。

5.将给定命题化为析取形式。不进入(2)的第一项的那些成分为

$$x(1 - y), (1 - x)y, (1 - x)(1 - y).$$

由此我们得到

$$y(1 - x) + x(1 - y) + (1 - x)(1 - y) = 1. \tag{3}$$

其解释为：——要么法比乌斯是在天狼星升起时出生的，并且不会在海中丧生；要么他并非在天狼星升起时出生，并且会在海中丧生；要么他并非在天狼星升起时出生，并且不会在海中丧生。然而，在像上述这样的情况中，即存在仅因单一因素而彼此不同的成分时，正如我们所见（VII. 15），最方便的做法是将这些成分归入一个项。如果我们这样连接（3）的第一项和第三项，我们便得到

$$(1 - y)x + 1 - x = 1;$$

而如果我们同样连接第二和第三，我们就有

$$y(1 - x) + 1 - y = 1.$$

These forms of the equation severally give the interpretations–

Either Fabius was not born under the day star, and will die in the sea, or he will not die in the sea.

Either Fabius was born under the day star, and will not die in the sea, or he was not born under the dogstar.

It is evident that these interpretations are strictly equivalent to the former one.

Let us ascertain, in the form of a conditional proposition, the consequences which flow from the hypothesis, that "Fabius will perish in the sea."

In the equation (2), which expresses the result of the elimination of v from the original equation, we must seek to determine x as a function of y.

We have

$$x = \frac{0}{y} = 0y + \frac{0}{0}(1 - y) \text{ on expansion,}$$

or

$$x = \frac{0}{0}(1 - y);$$

the interpretation of which is,–*If Fabius shall die in the sea, he was not born at the rising of the dogstar.*

These examples serve in some measure to illustrate the connexion which has been established in the previous sections between primary and secondary propositions, a connexion of which the two distinguishing features are identity of process and analogy of interpretation.

6. Ex. 2.–There is a remarkable argument in the second book of the Republic of Plato, the design of which is to prove the immutability of the Divine Nature. It is a very fine example both of the careful induction from familiar instances by which Plato arrives at general principles, and of the clear and connected logic by which he deduces from them the particular inferences which it is his object to establish. The argument is contained in the following dialogue:

"Must not that which departs from its proper form be changed either by itself or by another thing? Necessarily so. Are not things which are in the best state least changed and disturbed, as the body by meats and drinks, and labours, and every species of plant by heats and winds, and such like affections? Is not the healthiest and strongest the least changed? Assuredly. And does not any trouble from without least disturb and change that soul which is strongest and wisest? And as to all made vessels, and furnitures, and garments, according to the same principle, are not those which are well wrought, and in a good condition, least changed by time and other accidents? Even so. And whatever is in a right state, either by nature or by art, or by both these, admits of the smallest change from any other thing. So it seems. But God and things divine are in every sense in the best state. Assuredly. In this way, then, God should least of all bear many forms? Least, indeed, of all. Again, should He transform and change Himself? Manifestly He must do so, if He is changed at all. Changes He then Himself to that which is more good and fair, or to that which is worse and baser? Necessarily to the worse, if he be changed. For never shall we say that God is indigent of beauty or of virtue. You speak most rightly, said I, and

方程的这些形式分别给出如下解释--

或者法比乌斯并非生于昼星之下，且会死于海中，或者他不会死于海中。

或者法比乌斯生于昼星之下，且不会死于海中，或者他并非生于天狼星之下。

显然，这些解释与前一个解释严格等价。

让我们以条件命题的形式确定从"法比乌斯将死于海中"这一假说得出的结论。

在方程(2)中，该方程表达了从原方程中消去v的结果，我们必须设法确定x作为y的函数。

我们有

$$x = \frac{0}{y} = 0y + \frac{0}{0}(1 - y) \text{ on expansion,}$$

或

$$x = \frac{0}{0}(1 - y);$$

其解释为，--若法比乌斯将死于海中，则他并非生于天狼星升起之时。

这些例子在某种程度上说明了前几节中所建立的原始命题与次命题之间的联系，这种联系的两个显著特征是过程的一致性和解释的类比性。

6.例2。--在柏拉图《理想国》第二卷中有一个著名的论证，其目的在于证明神性的不变性。这既是一个极好的例子，展示了柏拉图从熟悉事例出发进行细致归纳以得出一般原则，也展示了他从中推出其目标所要确立的特定推论时那种清晰而连贯的逻辑。该论证包含在以下对话中：

"凡偏离其固有形式的东西，难道不是要么被自身、要么被另一事物所改变吗?必然如此。处于最佳状态的事物难道不是最少被改变和扰乱的么，比如身体被食物饮料、劳作所改变，每一种植物被热和风以及诸如此类的影响所改变?最健康最强壮的难道不是最少被改变的么?当然。来自外部的任何麻烦难道不是最少扰乱和改变那最强壮最智慧的灵魂么?至于一切制成的器皿、家具和衣物，按照同样的原则，那些制作精良、状况良好的，难道不是最少被时间和其它意外所改变的么?正是如此。凡是处于正确状态的东西，无论出于自然、出于技艺，还是兼而有之，都最少接受来自任何其它事物的改变。看来如此。但神和神圣事物在一切意义上都处于最佳状态。当然。那么，按这种方式，神应当最少承受许多形式么?确实最少。再者，他应当转化和改变自身么?显然，如果他终究被改变，他就必然如此。那么他是把自身改变为更好更美的东西，还是改变为更坏更卑下的东西呢?如果他改变，就必然变为更坏的。因为我们绝不会说神缺乏美或德性。你说得极为正确，我说，并且

the matter being so, seems it to you, O Adimantus, that God or man *willingly* makes himself in any sense worse? Impossible, said he. Impossible, then, it is, said I, that a god should wish to change himself; but ever being fairest and best, each of them ever remains absolutely in the same form."

The premises of the above argument are the following:

1st. If the Deity suffers change, He is changed either by Himself or by another.

2nd. If He is in the best state, He is not changed by another.

3rd. The Deity is in the best state.

4th. If the Deity is changed by Himself, He is changed to a worse state.

5th. If He acts willingly, He is not changed to a worse state.

6th. The Deity acts willingly.

Let us express the elements of these premises as follows:

Let x represent the proposition, "The Deity suffers change."

y, He is changed by Himself.

z, He is changed by another.

s, He is in the best state.

t, He is changed to a worse state.

w, He acts willingly.

Then the premises expressed in symbolical language yield, after elimination of the indefinite class symbols v, the following equations:

$$xyz + x(1-y)(1-z) = 0, \tag{1}$$
$$sz = 0, \tag{2}$$
$$s = 1, \tag{3}$$
$$y(1-t) = 0, \tag{4}$$
$$wt = 0, \tag{5}$$
$$w = 1. \tag{6}$$

Retaining x, I shall eliminate in succession z, s, y, t, and w (this being the order in which those symbols occur in the above system), and interpret the successive results.

Eliminating z from (1) and (2), we get

$$xs(1-y) = 0. \tag{7}$$

Eliminating s from (3) and (7),

$$x(1-y) = 0. \tag{8}$$

Eliminating y from (4) and (8),

$$x(1-t) = 0. \tag{9}$$

Eliminating t from (5) and (9),

$$xw = 0. \tag{10}$$

事情既然如此，阿德曼图斯啊，在你看来，神或人会自愿在某种意义上使自己变坏吗?他说，不可能。那么我说，神竟会希望改变自己，这是不可能的；但每一位永远是最美最好的，永远绝对保持同一形式。"

上述论证的前提如下：1. 如果上帝经历变化，祂要么由自身、要么由另一个。

第二，如果祂处于最佳状态，祂就不会被另一个所改变。

3. 神处于最佳状态。

第四，如果神被自己改变，那么他是被改变到更坏的状态。

第五，如果他自愿行动，他就不会陷入更糟的状态。

第六，神性者自愿而行。

让我们将这些前提的元素表达如下：令x表示命题"上帝遭受变化"。

y，祂被自己改变。z，祂被另一个改变。s，祂处于最佳状态。

他变得更糟了。

他自愿行动。

于是，用符号语言表达的前提在消去不定类记号v之后，得出以下方程：

$$xyz + x(1-y)(1-z) = 0, \tag{1}$$
$$sz = 0, \tag{2}$$
$$s = 1, \tag{3}$$
$$y(1-t) = 0, \tag{4}$$
$$wt = 0, \tag{5}$$
$$w = 1. \tag{6}$$

保留x，我将依次消去z、s、y、t和w(这是那些记号在上述系统中出现的顺序)，并解释相继的结果。

从(1)和(2)中消去z，我们得到

$$xs(1-y) = 0. \tag{7}$$

Eliminating s from (3) and (7),

$$x(1-y) = 0. \tag{8}$$

Eliminating y from (4) and (8),

$$x(1-t) = 0. \tag{9}$$

从(5)和(9)中消去t,

$$xw = 0. \tag{10}$$

Eliminating w from (6) and (10),

$$x = 0. \tag{11}$$

These equations, beginning with (8), give the following results:

From (8) we have $x = \frac{0}{0}y$, therefore, *If the Deity suffers change, He is changed by Himself.*

From (9), $x = \frac{0}{0}t$, *If the Deity suffers change, He is changed to a worse state.*

From (10), $x = \frac{0}{0}(1 - w)$. *If the Deity suffers change, He does not act willingly.*

From (11), *The Deity does not suffer change.* This is Plato's result.

Now I have before remarked, that the order of elimination is indifferent. Let us in the present case seek to verify this fact by eliminating the same symbols in a reverse order, beginning with w. The resulting equations are,

$$t = 0, y = 0, x(1 - x) = 0, z = 0, x = 0;$$

yielding the following interpretations:

God is not changed to a worse state. He is not changed by Himself. If He suffers change, He is changed by another. He is not changed by another. He is not changed.

We thus reach by a different route the same conclusion.

Though as an exhibition of the *power* of the method, the above examples are of slight value, they serve as well as more complicated instances would do, to illustrate its nature and character.

7. It may be remarked, as a final instance of analogy between the system of primary and that of secondary propositions, that in the latter system also the fundamental equation,

$$x(1 - x) = 0,$$

admits of interpretation. It expresses the axiom, *A proposition cannot at the same time be true and false.* Let this be compared with the corresponding interpretation (III. 15). Solved under the form

$$x = \frac{0}{1 - x} = \frac{0}{0}x,$$

by development, it furnishes the respective axioms: "A thing is what it is:" "If a proposition is true, it is true:" forms of what has been termed "The principle of identity." Upon the nature and the value of these axioms the most opposite opinions have been entertained. Some have regarded them as the very pith and marrow of philosophy. Locke devoted to them a chapter, headed, "On Trifling Propositions." [1] In both these views there seems to have been a mixture of truth and error. Regarded as supplanting experience, or as furnishing materials for the vain and wordy janglings of the schools, such propositions are worse than trifling. Viewed, on the other hand, as intimately allied with the very laws and conditions of thought, they rise into at least a speculative importance.

[1] Essay on the Human Understanding, Book IV. Chap. viii.

从(6)和(10)中消去w,

$$x = 0. \tag{11}$$

这些方程，从(8)开始，给出以下结果：

由(8)我们有x = 00y,因此，如果上帝遭受变化，则祂被自己改变。

由(9),x = 00t,如果上帝遭受变化，则祂变为更坏状态。

由(10),x = 00(1−w)·如果上帝遭受变化，则祂不行动心甘情愿地。

由(11)可知，神性不变。这是柏拉图的结果。

现在我已在前文指出，消元的顺序无关紧要。在目前情形下，让我们以相反顺序消去同样的记号，从w开始，以验证这一事实。所得方程为，

$$t = 0, y = 0, x(1 - x) = 0, z = 0, x = 0;$$

yielding the following interpretations:

上帝不被变为更坏的状态。祂不被自己改变。如果祂遭受变化，则祂被另一个改变。祂不被另一个改变。祂不遭受变化。

我们由此通过另一条途径得出了相同的结论。

尽管作为对该方法力量的展示，上述例子价值不大，但它们与更复杂的实例一样，足以说明其性质和特点。

7. 作为初级命题体系与次级命题体系之间类比的最后一个实例，可以指出，在后一体系中，基本方程，

$$x(1 - x) = 0,$$

也可以得到解释。它表达了这条公理：一个命题不能同时为真和假。将此与相应的解释(III.15)相比较。以如下形式求解

$$x = \frac{0}{1 - x} = \frac{0}{0}x,$$

通过展开式，它提供了各自的公理："一物即是其所是：""如果一个命题是真的，它就是真的："即所谓"同一性原则"的各种形式。关于这些公理的本性和价值，人们持有最为对立的意见。有些人把它们视为哲学的精髓和骨髓。洛克(Locke)为它们专门写了一章，标题为"论琐屑的命题"。[1]在这两种观点中，似乎都混杂着真理和错误。若被视为取代经验，或为学派空洞无聊的饶舌提供材料，这样的命题比琐屑更糟。另一方面，若被视为与思维本身的规律和条件密切相连，它们至少上升到一种思辨的重要性。

1 《人类理解论》，第四卷，第八章。

Chapter XIII

ANALYSIS OF A PORTION OF DR. SAMUEL CLARKE's "DEMONSTRATION OF THE BEING AND ATTRIBUTES OF GOD," AND OF A PORTION OF THE "ETHICA ORDINE GEOMETRICO DEMONSTRATA" OF SPINOZA.

1. The general order which, in the investigations of the following chapter, I design to pursue, is the following. I shall examine what are the actual premises involved in the demonstrations of some of the general propositions of the above treatises, whether those premises be expressed or implied. By the actual premises I mean whatever propositions are assumed in the course of the argument, without being proved, and are employed as parts of the foundation upon which the final conclusion is built. The premises thus determined, I shall express in the language of symbols, and I shall then deduce from them by the methods developed in the previous chapters of this work, the most important inferences which they involve, in addition to the particular inferences actually drawn by the authors. I shall in some instances modify the premises by the omission of some fact or principle which is contained in them, or by the addition or substitution of some new proposition, and shall determine how by such change the ultimate conclusions are affected. In the pursuit of these objects it will not devolve upon me to inquire, except incidentally, how far the metaphysical principles laid down in these celebrated productions are worthy of confidence, but only to ascertain what conclusions may justly be drawn from given premises; and in doing this, to exemplify the perfect liberty which we possess as concerns both the choice and the order of the elements of the final or concluding propositions, viz., as to determining what elementary propositions are true or false, and what are true or false under given restrictions, or in given combinations.

2. The chief practical difficulty of this inquiry will consist, not in the application of the method to the premises once determined, but in ascertaining what the premises are. In what area regarded as the most rigorous examples of reasoning applied to metaphysical questions, it will occasionally be found that different trains of thought are blended together; that particular but essential

第十三章

对塞缪尔博士部分内容的分析

克拉克的《上帝的存在与属性之证明》，以及斯宾诺莎《以几何次序证明的伦理学》的一部分。

1. 在以下各章的探讨中，我打算遵循的一般次序如下。我将考察上述论著中某些一般命题的证明究竟涉及哪些实际前提，无论这些前提是明确表述的还是隐含的。所谓实际前提，我指的是在论证过程中被假定而未经证明、并被用作最终结论所依据的基础之一部分的那些命题。如此确定的前提，我将用记号的语言加以表述，然后运用本书前面各章所阐明的方法，从这些前提中推出它们所蕴含的最重要的推论，作为对作者们实际引出的那些特定推论的补充。在某些情形下，我将通过删去其中所包含的某一事实或原则，或者通过增加或替换某一新命题，来修改这些前提，并确定这样的改变如何影响最终的结论。在追求这些目标的过程中，我不承担这样的任务--除非附带地--即探究这些著名著作中所奠定的形而上学原则在多大程度上值得信赖，而只在于确定从给定前提中可以正当地引出什么结论；并且在这样做时，例示我们在最终结论或归结命题的诸元素的选择和次序方面所拥有的完全自由，也就是说，在确定哪些基本命题为真或为假，以及哪些在给定限制下或在给定组合中为真或为假方面的自由。

2. 这项探究的主要实际困难，不在于前提一旦确定后方法的应用，而在于确定前提是什么。在被视为应用于形而上学问题的最严格的推理实例中，偶尔会发现不同的思路交织在一起；某些特殊但本质的

parts of the demonstration are given parenthetically, or out of the main course of the argument; that the meaning of a premiss may be in some degree ambiguous; and, not unfrequently, that arguments, viewed by the strict laws of formal reasoning, are incorrect or inconclusive. The difficulty of determining and distinctly exhibiting the true premises of a demonstration may,

in such cases, be very considerable. But it is a difficulty which must be overcome by all who would ascertain whether a particular conclusion is proved or not, whatever form they may be prepared or disposed to give to the ulterior process of reasoning. It is a difficulty, therefore, which is not peculiar to the method of this work, though it manifests itself more distinctly in connexion with this method than with any other. So intimate, indeed, is this connexion, that it is impossible, employing the method of this treatise, to form even a conjecture as to the validity of a conclusion, without a distinct apprehension and exact statement of all the premises upon which it rests. In the more usual course of procedure, nothing is, however, more common than to examine some of the steps of a train of argument, and thence to form a vague general impression of the scope of the whole, without any such preliminary and thorough analysis of the premises which it involves.

The necessity of a rigorous determination of the real premises of a demonstration ought not to be regarded as an evil; especially as, when that task is accomplished, every source doubt or ambiguity is removed. In employing the method of this treatise, the order in which premises are arranged, the mode of connexion which they exhibit, with every similar circumstance may be esteemed a matter of indifference, and the process inference is conducted with a precision which might almost termed mechanical.

3. The "Demonstration of the Being and Attributes of God," consists of a series of propositions or theorems, each of them proved by means of premises resolvable, for the most part, into two distinct classes, viz., facts of observation, such as the existence of a material world, the phenomenon of motion, &c., and hypothetical principles, the authority and universality of which are supposed to be recognised *à priori*. It is, of course, upon the truth of the latter, assuming the correctness of the reasoning, that the validity of the demonstration really depends. But whatever may be thought of its claims in this respect, it is unquestionable that, as an intellectual performance, its merits are very high. Though the trains of argument of which it consists are not in general very clearly arranged, they are almost always specimens of correct Logic, and they exhibit a subtlety of apprehension and a force of reasoning which have seldom been equalled, never perhaps surpassed. We see in them the consummation of those intellectual efforts which were awakened in the realm of metaphysical inquiry, at a period when the dominion of hypothetical principles was less questioned than it now is, and when the rigorous demonstrations of the newly risen school of mathematical physics seemed to have furnished a model for their direction. They appear to me for this reason (not to mention the dignity of the subject of which they treat) to be deserving of high consideration; and I do not deem it a vain or superfluous task to expend upon some of them a careful analysis.

4. The Ethics of Benedict Spinoza is a treatise, the object of which is to

论证部分是以插入语的方式给出的，或者偏离了论证的主线；一个前提的含义可能在某种程度上是模糊的；而且不乏这样的情况：按照形式推理的严格法则来看，论证是不正确的或不能得出结论的。确定并清晰地展示一个论证的真正前提的困难可能，

在此类情形下，可能相当可观。但这是一切想要确定某一特定结论是否得到证明的人都必须克服的困难，无论他们准备或倾向于赋予那进一步的推理过程以何种形式。因此，这是一项并非本书方法所特有的困难，尽管它与此方法的关联比与任何其他方法的关联表现得更为明显。的确，这种关联如此紧密，以至于采用本论著的方法，若不对一个结论所依据的一切前提作出清晰的领会和精确的陈述，甚至不可能对该结论的有效性形成哪怕一种猜测。然而，在更为通常的程序中，没有什么比这更为常见的了：考察一连串论证中的某些步骤，并由此对整体的范围形成一种模糊的一般印象，而全然不对其所涉及的前提作任何这样的预备性的、彻底的分析。

对证明的真正前提作严格确定的必要性不应被视为一种祸害；尤其是因为，当这一任务完成之后，一切怀疑或歧义的根源都被消除了。在采用本论著的方法时，前提排列的次序、它们所呈现的联结方式，以及一切类似的情形，都可以被视为无关紧要的，而推理的过程则以一种几乎可以称为机械的精确性来进行。

3.《上帝的存在与属性的证明》由一系列命题或定理组成，其中每一个都借助前提得到证明，而这些前提大多可分解为两个不同的类别，即观察事实，如物质世界的存在、运动的现象等，以及假设性的原则，其权威性和普遍性被假定为`*a priori*得到承认的。当然，假定推理正确，证明的有效性实际上取决于后者的真理性。但无论人们对其在这方面的主张作何想法，无可置疑的是，作为一种理智的成果，它的价值非常高。尽管构成它的那些论证序列一般安排得并不十分清晰，它们几乎总是正确逻辑的范例，并且展现出一种领会的精微和一种推理的力量，此二者鲜有匹敌，也许从未被超越。我们在其中看到了那些理智努力的极致，这些努力是在形而上学探究的领域中被唤醒的，当时假设性原则的统治所受的质疑比现在少，而新近兴起的数学物理学派的严格证明似乎为其方向提供了一个典范。由于这个理由(更不用说它所论述的主题的尊严)，在我看来它们值得高度考虑；我并不认为对其中某些花费一番仔细的分析是一项徒劳或多余的任务。

4. 本尼迪克特·斯宾诺莎的《伦理学》是一部论著，其目的是

prove the identity of God and the universe, and to establish, upon this doctrine, a system of morals and of philosophy. The analysis of its main argument is extremely difficult, owing not to the complexity of the separate propositions which it involves, but to the use of vague definitions, and of axioms which, through a like defect of clearness, it is perplexing to determine whether we ought to accept or to reject. While the reasoning of Dr. Samuel Clarke is in part verbal, that of Spinoza is so in a much greater degree; and perhaps this is the reason why, to some minds, it has appeared to possess a formal cogency, to which in reality it possesses no just claim. These points will, however, be considered in the proper place.

CLARKE'S DEMONSTRATION.
PROPOSITION I.

5. "*Something has existed from eternity.*"

The proof is as follows:–

"For since something now is, 'tis manifest that something always was. Otherwise the things that now are must have risen out of nothing, absolutely and without cause. Which is a plain contradiction in terms. For to say a thing is produced, and yet that there is no cause at all of that production, is to say that something is effected when it is effected by nothing, that is, at the same time when it is not effected at all. Whatever exists has a cause of its existence, either in the necessity of its own nature, and thus it must have been of itself eternal: or in the will of some other being, and then that other being must, at least in the order of nature and causality, have existed before it."

Let us now proceed to analyze the above demonstration. Its first sentence is resolvable into the following propositions:

1st. Something is.

2nd. If something is, either something always was, or the things that now are must have risen out of nothing.

The next portion of the demonstration consists of a proof that the second of the above alternatives, viz., "The things that now are have risen out of nothing," is impossible, and it may formally be resolved as follows:

3rd. If the things that now are have risen out of nothing, something has been effected, and at the same time that something has been effected by nothing.

4th. If that something has been effected by nothing, it has not been effected at all.

The second portion of this argument appears to be a mere assumption of the point to be proved, or an attempt to make that point clearer by a different verbal statement.

The third and last portion of the demonstration contains a distinct proof of the truth of either the original proposition to be proved, viz., "Something always was," or the point proved in the second part of the demonstration, viz., the untenable nature of the hypothesis, that "the things that now are have risen out of nothing." It is resolvable as follows:–

5th. If something is, either it exists by the necessity of its own nature, or it exists by the will of another being.

证明上帝与宇宙的同一，并在这一定义之上建立一套道德和哲学体系。对其主要论点的分析极为困难，不是由于它所涉及的各个命题的复杂性，而是由于使用了模糊的定义和公理，而这些公理由于同样缺乏清晰性，令人难以确定我们应当接受还是拒绝。塞缪尔·克拉克博士的推理部分地是言辞上的，而斯宾诺莎的推理则在更大得多的程度上是如此；也许这就是为什么在某些人看来，它似乎具有一种形式上的说服力，而实际上它并无正当理由要求这种说服力。然而，这些要点将在适当的地方加以考虑。

克拉克的论证。
命题一。

5. "某物自永恒以来就存在。" 证明如下：--

"因为既然现在有某物存在，那么显然总有某物存在。否则，现在存在的事物必定是从无中产生的，绝对地且无原因地产生的。这在措辞上是明显的矛盾。因为说一个事物被产生，却又说该产生根本没有原因，就等于说某物在无物使其产生时被产生，也就是说，在它根本未被产生的同时被产生。凡存在者，其存在都有原因，或者在其自身本性的必然性中，因而它必定是自存的、永恒的；或者在某个其他存在者的意志中，而那个其他存在者必定至少在自然秩序和因果关系的顺序上先于它存在。"

现在让我们来分析上述论证。其第一句话可以分解为以下命题：

1. 有物存在。

第二，如果某物存在，那么要么某物始终存在，要么现在存在的事物必定是从无中产生的。

论证的下一部分包括一个证明，即上述第二种选择，即"现在存在的事物是从无中产生的"，是不可能的，它可以形式化地分解如下：

第三，如果现在存在的事物是从无中产生的，那么就有某物被产生出来，并且同时有某物是由无产生的。

第四，如果某事物未受任何事物影响，则它未被影响
根本

这一论证的第二部分似乎只是对所要证明的论点的假定，或者是一种通过不同的文字表述使该论点更清楚的尝试。

论证的第三也是最后一部分包含了对以下两者之一为真的明确证明：所要证明的原始命题，即"某物一直存在"，或者论证第二部分所证明的论点，即"现在存在的事物是从无中产生的"这一假说的不可成立性。它可以分解如下：--

第五。如果某物存在，那么它要么凭借自身本性的必然性而存在，要么凭借另一存在者的意志而存在。

6th. If it exists by the necessity of its own nature, something always was.

7th. If it exists by the will of another being, then the proposition, that the things which exist have arisen out of nothing, is false.

The last proposition is not expressed in the same form in the text of Dr. Clarke; but his expressed conclusion of the prior existence of another Being is clearly meant as equivalent to a denial of the proposition that the things which now are have risen out of nothing.

It appears, therefore, that the demonstration consists of two distinct trains of argument: one of those trains comprising what I have designated as the *first* and *second* parts of the demonstration; the other comprising the *first* and *third* parts. Let us consider the latter train.

The premises are:–

1st. Something is.

2nd. If something is, either something always was, or the things that now are have risen out of nothing.

3rd. If something is, either it exists in the necessity of its own nature, or it exists by the will of another being.

4th. If it exists in the necessity of its own nature, something always was.

5th. If it exists by the will of another being, then the hypothesis, that the things which now are have risen out of nothing, is false.

We must now express symbolically the above proposition.

Let

$x =$ Something is.
$y =$ Something always was.
$z =$ The things which now are have risen from nothing.
$p =$ It exists in the necessity of its own nature (i.e. the *something* spoken of above).
$q =$ It exists by the will of another Being.

It must be understood, that by the expression, Let $x =$ "Something is," is meant no more than that x is the representative symbol of that proposition (XI. 7), the equations $x = 1$, $x = 0$, respectively declaring its truth and its falsehood.

The equations of the premises are:–

1st. $x = 1$;
2nd. $x = v[y(1-x) + z(1-y)]$;
3rd. $x = v[p(1-q) + q(1-p)]$;
4th. $p = vy$;
5th. $q = v(1-z)$;

and on eliminating the several indefinite symbols v, we have

$$1 - x = 0; \tag{1}$$
$$x[yz + (1-y)(1-z)] = 0; \tag{2}$$
$$x[pq + (1-p)(1-q)] = 0; \tag{3}$$
$$p(q-y) = 0; \tag{4}$$
$$qz = 0. \tag{5}$$

6. 若某物依其自身本性的必然性而存在，则总有某物存在。

第七，如果它凭另一个存在者的意志而存在，那么"存在的事物是从无中产生的"这一命题就是假的。

最后一个命题在克拉克博士的原文中并非以相同形式表述；但他所表述的关于另一个存在者先前存在的结论，显然意在等同于对"现在存在的事物是从无中产生的"这一命题的否定。

因此可以看出，论证由两条不同的论证线索组成：其中一条线索包含我所标定的论证的第一部分和第二部分；另一条线索包含第一部分和第三部分。让我们考虑后一条线索。

前提是：--第一，有某
物存在。

第二，如果某物存在，那么要么某物始终存在，要么现在存在的事物是从无中产生的。

第三，如果某物存在，那么它要么依据其自身本性的必然性而存在，要么依据另一存在者的意志而存在。

第四，若某物依其自身本性的必然性而存在，则总有某物存在。

第五，如果它凭另一个存在者的意志而存在，那么，现在存在的事物是从无中产生的这一假说就是假的。

我们现在必须用符号来表示上述命题。

让

x = 某物存在。y = 某物一直存在。z = 现在存在的事物是从无中产生的。p = 它因其自身本性的必然性而存在
(即上文所说的某物)。q = 它因另一个存在者的意志而存在。

必须理解，通过"设x = '某物存在'"这一表述，所意味的不过是x是该命题的代表记号(XI.7)，方程x = 1、x = 0分别宣告其真和其假。前提的方程为：--

1st. $x = 1$;
2nd. $x = v[y(1-x) + z(1-y)]$;
3rd. $x = v[p(1-q) + q(1-p)]$;
4th. $p = vy$;
5th. $q = v(1-z)$;
and on eliminating the several indefinite symbols v, we have

$$1 - x = 0; \tag{1}$$
$$x[yz + (1-y)(1-z)] = 0; \tag{2}$$
$$x[pq + (1-p)(1-q)] = 0; \tag{3}$$
$$p(q-y) = 0; \tag{4}$$
$$qz = 0. \tag{5}$$

6. First, I shall examine whether any conclusions are deducible from the above, concerning the truth or falsity of the single propositions represented by the symbols y, z, p, q, viz., of the propositions, "Something always was;" "The things which now are have risen from nothing;" "The something which is exists by the necessity of its own nature;" "The something which is exists by the will of another being."

For this purpose we must separately eliminate all the symbols but y, all these but z, &c. The resulting equation will determine whether any such separate relations exist.

To eliminate x from (1), (2), and (3), it is only necessary to substitute in (2) and (3) the value of x derived from (1). We find as the results,

$$yz + (1-y)(1-z) = 0. \tag{6}$$
$$pq + (1-p)(1-q) = 0. \tag{7}$$

To eliminate p we have from (4) and (7), by addition,

$$p(1-y) + pq + (1-p)(1-q) = 0; \tag{8}$$

whence we find,

$$(1-y)(1-q) = 0. \tag{9}$$

To eliminate q from (5) and (9), we have

$$qz + (1-y)(1-q) = 0;$$

whence we find

$$x(1-y) = 0. \tag{10}$$

There now remain but the two equations (6) and (10), which, on addition, give

$$yz + 1 - y = 0.$$

Eliminating from this equation z, we have

$$1 - y = 0, \text{or, } y = 1. \tag{11}$$

Eliminating from the same equation y, we have

$$z = 0. \tag{12}$$

The interpretation of (11) is
Something always was.
The interpretation of (12) is
The things which are have not risen from nothing.

6. 首先，我将考察从上述内容中是否能推出任何结论，涉及由记号y、z、p、q所代表的单个命题的真或假，即以下命题："某物一直存在";"现在存在的事物是从无中产生的";"存在着的某物因其自身本性的必然性而存在";"存在着的某物因另一个存在者的意志而存在"。

为此，我们必须分别消去除y以外的所有记号，消去除z以外的所有记号，等等。所得方程将判定是否存在任何这样的单独关系。

要从(1)、(2)和(3)中消去x，只需在
(2) 以及(3)由(1)导出的x的值。我们得到如下结果，

$$yz + (1-y)(1-z) = 0. \tag{6}$$
$$pq + (1-p)(1-q) = 0. \tag{7}$$

为消去p，由(4)与(7)相加得，

$$p(1-y) + pq + (1-p)(1-q) = 0; \tag{8}$$

由此我们求得，

$$(1-y)(1-q) = 0. \tag{9}$$

To eliminate q from (5) and (9), we have

$$qz + (1-y)(1-q) = 0;$$

由此我们求得

$$x(1-y) = 0. \tag{10}$$

There now remain but the two equations (6) and (10), which, on addition, give

$$yz + 1 - y = 0.$$

Eliminating from this equation z, we have

$$1 - y = 0, \text{or, } y = 1. \tag{11}$$

Eliminating from the same equation y, we have

$$z = 0. \tag{12}$$

(11)的解释是某物一直存在。 (12)的
解释是

存在的事物并非从虚无中产生。

Next resuming the system (6), (7), with the two equations (4), (5), let us determine the two equations involving p and q respectively.

To eliminate y we have from (4) and (6),

$$p(1-y)+yz+(1-y)(1-z)=(0);$$

whence

$$(p+1-z)z=0, \text{or, } pz=0. \tag{13}$$

To eliminate z from (5) and (13), we have

$$qz+pz=0;$$

whence we get,

$$0=0.$$

There remains then but the equation (7), from which eliminating q, we have $0=0$ for the final equation, in p.

Hence there is no conclusion derivable from the premises affirming the simple truth or falsehood of the proposition, "The something which is exists in the necessity of its own nature." And as, on eliminating p, there is the same result, $0=0$, for the ultimate equation in q, it also follows, that *there is no conclusion deducible from the premises as to the simple truth or falsehood of the proposition, "The something which is exists by the will of another Being."*

Of relations connecting more than one of the propositions represented by the elementary symbols, it is needless to consider any but that which is denoted by the equation (7) connecting p and q, inasmuch as the propositions represented by the remaining symbols are absolutely true or false independently of any connexion of the kind here spoken of. The interpretation of (7), placed under the form

$$p(1-q)+q(1-p)=1, \text{ is,}$$

The something which is, either exists in the necessity of its own nature, or by the will of another being.

I have exhibited the details of the above analysis with a, perhaps, needless fulness and prolixity, because in the examples which will follow, I propose rather to indicate the steps by which results are obtained, than to incur the danger of a wearisome frequency of repetition. The conclusions which have resulted from the above application of the method are easily verified by ordinary reasoning.

The reader will have no difficulty in applying the method to the other train of premises involved in Dr. Clarke's first Proposition, and deducing from them the two first of the conclusions to which the above analysis has led.

PROPOSITION II.

接下来恢复方程组(6)、(7)，连同两个方程(4)、(5)，让我们确定分别涉及p和q的两个方程。

为消去y，我们由(4)和(6)得，

$$p(1-y)+yz+(1-y)(1-z)=(0);$$

由此

$$(p+1-z)z=0, \text{or, } pz=0. \tag{13}$$

To eliminate z from (5) and (13), we have

$$qz+pz=0;$$

由此我们得到，

0 = 0.

那么只剩下方程(7)，从中消去q，我们得到0 = 0作为关于p的最终方程。

因此，从肯定命题"存在着的某物存在于其自身本性的必然性中"的简单真或假的前提中，不能推出任何结论。并且，由于在消去p之后，对于q中的最终方程得到同样的结果0 = 0，所以也可推出，从前提中不能推出关于命题"存在着的某物通过另一个存在者的意志而存在"的简单真或假的任何结论。

在连接由基本记号所代表的多个命题的关系中，除了由方程(7)所表示的连接p和q的关系之外，无需考虑其他关系，因为其余记号所代表的命题是绝对真或绝对假的，与这里所说的那种连接无关。(7)的解释，以如下形式给出

$$p(1-q)+q(1-p)=1, \text{ is,}$$

The something which is, either exists in the necessity of its own nature, or by the will of another being.

我以或许不必要的详尽和冗长展示了上述分析的细节，因为在接下来的例子中，我打算更多地指出获得结果的步骤，而不是冒着令人厌倦的频繁重复的危险。上述方法应用所得出的结论很容易通过普通推理加以验证。

读者将不难把该方法应用于克拉克博士第一命题中所涉及的另-系列前提，并从中推出上述分析所得出的前两个结论。

命题二。

7. *Some one unchangeable and independent Being has existed from eternity.*

The premises from which the above proposition is prove are the following:

1st. Something has always existed.

2nd. If something has always existed, either there has existed some one unchangeable and independent being, or the whole of existing things has been comprehended in a succession of changeable and dependent beings.

3rd. If the universe has consisted of a succession of changeable and dependent beings, either that series has had a cause from without, or it has had a cause from within.

4th. It has not had a cause from without (because it includes, by hypothesis, all things that exist).

5th. It has not had a cause from within (because no part is necessary, and if no part is necessary, the whole cannot be necessary).

Omitting, merely for brevity, the subsidiary proofs contained in the parentheses of the fourth and fifth premiss, we may represent the premises as follows:

Let x = Something has always existed.

y = There has existed some one unchangeable and independent being.

z = There has existed a succession of changeable and dependent beings.

p = That series has had a cause from without.

q = That series has had a cause from within.

Then we have the following system of equations, viz.:

$$\begin{aligned}
\text{1st. } x &= 1; \\
\text{2nd. } x &= v\{y(1-z)+z(1-y)\}; \\
\text{3rd. } z &= v\{p(1-q)+(1-p)q\}; \\
\text{4th. } p &= 0; \\
\text{5th. } q &= 0:
\end{aligned}$$

which, on the separate elimination of the indefinite symbols v, gives

$$l - x = 0; \tag{1}$$
$$x\{yz + (1-y)(1-z)\} = 0; \tag{2}$$
$$z\{pq + (1-p)(1-q)\} = 0; \tag{3}$$
$$p = 0; \tag{4}$$
$$q = 0. \tag{5}$$

The elimination from the above system of x, p, q, and y, conducts to the equation

$$z = 0.$$

7. 某个不可改变且独立的存在者自永恒以来就一直存在。用以证明上述命题的前提如下：第一，某物一直存在。

第二，如果某物一直存在，那么要么存在某个不变且独立的存在者，要么全部存在物都被包含在一个可变且依赖的存在者的序列之中。

第三，如果宇宙由一系列可变且依赖性的存在者构成，那么这一序列要么有一个来自外部的因，要么有一个来自内部的因。

第四，它没有外在的原因(因为根据假设，它包含了，
(一切存在之物)。如果没有任何部分是必要的，
那么整体也不可能是必要的)。

第五。它没有来自内部的原因(因为没有任何部分是必然的，并且，为简洁起见，省略第四和第五前提中括号内所包含的辅助证明，我们可以将前提表示如下：

设= 始终存在某种事物。

y = 存在某个不变且独立的存在者。z = 存在一系列可变且依存的存在者。p = 该系列有一个来自外部的因。

q = 该系列有一个来自内部的因。于是我们得到如下方程组，即：

$$\begin{aligned}
\text{1st. } x &= 1; \\
\text{2nd. } x &= v\{y(1-z)+z(1-y)\}; \\
\text{3rd. } z &= v\{p(1-q)+(1-p)q\}; \\
\text{4th. } p &= 0; \\
\text{5th. } q &= 0:
\end{aligned}$$

将其分别消去不定符号v后，得到

$$l - x = 0; \tag{1}$$
$$x\{yz + (1-y)(1-z)\} = 0; \tag{2}$$
$$z\{pq + (1-p)(1-q)\} = 0; \tag{3}$$
$$p = 0; \tag{4}$$
$$q = 0. \tag{5}$$

从上述系统中消去x、p、q和y，便导致方程

$$z = 0.$$

And the elimination of x, p, q, and z, conducts in a similar manner to the equation

$$y = 1.$$

Of which equations the respective interpretations are:

1st. *The whole of existing things has not been comprehended in a succession of changeable and dependent beings.*

2nd. *There has existed some one unchangeable and independent being.*

The latter of these is the proposition which Dr. Clarke proves. As, by the above analysis, all the propositions represented by the literal symbols x, y, z, p, q, are determined as absolutely true or false, it is needless to inquire into the existence of any further relations connecting those propositions together.

Another proof is given of Prop. II., which for brevity I pass over. It may be observed, that the "impossibility of infinite succession," the proof of which forms a part of Clarke's argument, has commonly been assumed as a fundamental principle of metaphysics, and extended to other questions than that of causation. Aristotle applies it to establish the necessity of first principles of demonstration; [1] the necessity of an end (the good), in human actions, &c. [2] There is, perhaps, no principle more frequently referred to in his writings. By the schoolmen it was similarly applied to prove the impossibility of an infinite subordination of genera and species, and hence the necessary existence of universals. Apparently the impossibility of our forming a definite and complete conception of an infinite series, i.e. of comprehending it as a *whole*, has been confounded with a logical inconsistency, or contradiction in the idea itself.

8. The analysis of the following argument depends upon the theory of Primary Propositions.

Proposition III.

That unchangeable and independent Being must be self-existent.

The premises are:–

1. Every being must either have come into existence out of nothing, or it must have been produced by some external cause, or it must be self-existent.

2. No being has come into existence out of nothing.

3. The unchangeable and independent Being has not been produced by an external cause.

For the symbolical expression of the above, let us assume,

x = Beings which have arisen out of nothing.
y = Beings which have been produced by an external cause.
z = Beings which are self-existent.
w = The unchangeable and independent Being.

[1] Metaphysics, III. 4; Anal. Post. I, 19, *et seq.*
[2] Nic. Ethics, Book I. Cap. II.

And the elimination of x, p, q, and z, conducts in a similar manner to the equation

$$y = 1.$$

这些方程各自的解释是：第一。存在事物的整体并未被理解为一个可变化且相互依赖的存在者的序列。

第二，存在某个不变且独立的存在者。后者即克拉克博士所证明的命题。由于通过上述分析，由字母记号x、y、z、p、q所表示的所有命题都被确定为绝对真或假，因此无需探究是否存在任何进一步的关系将这些命题联系起来。

对于命题II还给出了另一个证明，为简洁起见我略过不提。可以注意到，“无限序列的不可能性”--其证明构成克拉克论证的一部分--通常被假定为形而上学的一条基本原理，并被扩展到因果关系问题之外的其他问题。亚里士多德用它来确立证明的第一原理的必然性；[1]人类行为中目的(善)的必然性，等等。[2]也许没有哪条原理在他的著作中被更频繁地引用。经院哲学家同样用它来证明属和种的无限从属关系的不可能性，并由此证明共相的必然存在。显然，我们无法形成一个关于无限序列的确定而完整的概念，即无法把它作为一个整体来把握，这种不可能性被与观念本身的逻辑不一致或矛盾混为一谈了。

8.以下论证的分析依赖于初阶命题的理论。

命题三。

那个不变且独立的存在者必定是自存的。前提是：--

1.每个存在者要么必定是从无中产生的，要么必定是由某个外部原因产生的，要么必定是自存的。

2. 没有任何存在者是从虚无中产生的。

3.那个不变且独立的存在者不是由外部原因产生的。

为了上述内容的符号表达，让我们假设，

x = 从无中产生的存在者。y = 由外部原因产生的存在者。z = 自存的存在者。w = 那个不变且独立的存在者。

1 形而上学，III.4；后分析篇 I, 19, et seq。
2 尼各马可伦理学，第一卷，第二章。

Then we have

$$x(1-y)(1-z)+y(1-x)(1-z)+z(1-x)(1-y)=l, \quad (1)$$
$$x=0, \quad (2)$$
$$w=v(1-y), \quad (3)$$

from the last of which eliminating v,

$$wy=0. \quad (4)$$

Whenever, as above, the value of a symbol is given as 0 or 1, it is best eliminated by simple substitution. Thus the elimination of x gives

$$y(1-z)+z(1-y) = 1; \quad (5)$$
$$\text{or, } yz+(1-y)(1-z) = 0. \quad (6)$$

Now adding (4) and (6), and eliminating y, we get

$$w(1-z)=0,$$
$$\therefore w=vz;$$

the interpretation of which is,–*The unchangeable and independent being is necessarily self-existing.*

Of (5), in its actual form, the interpretation is,–*Every being has either been produced by an external cause, or it is self-existent.*

9. In Dr. Samuel Clarke's observations on the above proposition occurs a remarkable argument, designed to prove that the material world is not the self-existent being above spoken of. The passage to which I refer is the following:

"If matter be supposed to exist necessarily, then in that necessary existence there is either included the power of gravitation, or not. If not, then in a world merely material, and in which no intelligent being presides, there never could have been any motion; because motion, as has been already shown, and is now granted in the question, is not necessary of itself. But if the power of gravitation be included in the pretended necessary existence of matter: then, it following necessarily that there must be a vacuum (as the incomparable Sir Isaac Newton has abundantly demonstrated that there must, if gravitation be an universal quality or affection of matter), it follows likewise, that matter is not a necessary being. For if a vacuum actually be, then it is plainly more than possible for matter not to be."–(pp. 25, 26).

It will, upon attentive examination, be found that the actual premises involved in the above demonstration are the following:

1st. If matter is a necessary being, either the property of gravitation is necessarily present, or it is necessarily absent.

2nd. If gravitation is necessarily absent, and the world is not subject to any presiding Intelligence, motion does not exist.

于是我们有

$$x(1-y)(1-z)+y(1-x)(1-z)+z(1-x)(1-y)=l, \quad (1)$$
$$x=0, \quad (2)$$
$$w=v(1-y), \quad (3)$$

从最后一个式子中消去 v,

$$wy=0. \quad (4)$$

每当如上所述某个记号的值被给定为0或1时，最好通过简单代入来消去它。于是消去x便得到

$$y(1-z)+z(1-y) = 1; \quad (5)$$
$$\text{or, } yz+(1-y)(1-z) = 0. \quad (6)$$

将(4)与(6)相加，并消去y，我们得到

$$w(1-z)=0,$$
$$\therefore w=vz;$$

其解释是--那个不变且独立的存在者是必然自存。

对于(5)，按其实际形式，其解释是--每个存在者要么是由外部原因产生的，要么是自存的。

9.在塞缪尔·克拉克博士对上述命题的评论中，有一段值得注意的论证，旨在证明物质世界不是上面所说的那个自存的存在者。我所指的那段文字如下：

"如果假定物质必然存在，那么在这种必然存在中，要么包含引力，要么不包含。如果不包含，那么在一个纯粹物质的、没有智能存在者主宰的世界中，就永远不可能有任何运动；因为运动，正如已经表明并在当前问题中已被承认的那样，其本身并非必然。但如果引力的能力被包含在物质所谓的必然存在之中：那么，由此必然推出必定存在真空(正如无与伦比的艾萨克·牛顿爵士已充分证明，如果引力是物质的一种普遍性质或属性，就必定存在真空)，同样也可推出，物质不是必然存在者。因为如果真空确实存在，那么物质不存在显然就不仅仅是可能的了。"--(第25、26页)。

第一，如果物质是必然存在者，那么引力属性要么是必然在场，或者必然不在场。主宰的理智，运动不存在。

第二，如果引力必然不存在，且世界不受任何

3rd. If the property of gravitation is necessarily present, existence of a vacuum is necessary.

4th. If the existence of a vacuum is necessary, matter is not necessary being.

5th. If matter is a necessary being, the world is not subject to a presiding Intelligence.

6th. Motion exists.

Of the above premises the first four are expressed in the demonstration; the fifth is implied in the connexion of its first and second sentences; and the sixth expresses a fact, which the author does not appear to have thought it necessary to state, but which is obviously a part of the ground of his reasoning. Let us represent the elementary propositions in the following manner:

Let x = Matter is a necessary being.
y = Gravitation is necessarily present.
t = Gravitation is necessarily absent.
z = The world is merely material, and not subject to any presiding Intelligence.
w = Motion exists.
v = A vacuum is necessary.

Then the system of premises will be represented by the following equations, in which q is employed as the symbol of time indefinite:

$$\begin{aligned} x &= q\{y(1-t)+(1-y)t\}. \\ tz &= q(1-w). \\ y &= qv. \\ v &= q(1-x). \\ x &= qz. \\ w &= 1. \end{aligned}$$

From which, if we eliminate the symbols q, we have the following system, viz.:

$$x\{yt+(1-y)(1-t)\} = 0. \quad (1)$$
$$tzw = 0. \quad (2)$$
$$y(1-v) = 0. \quad (3)$$
$$vx = 0. \quad (4)$$
$$x(1-z) = 0. \quad (5)$$
$$1-w = 0. \quad (6)$$

Now if from these equations we eliminate w, v, z, y, and t, we obtain the equation

$$x = 0,$$

第三，如果引力的属性必然存在，则
真空是必要的。

第四，如果真空的存在是必然的，那么物质就不是必然的存在。

第五，如果物质是必然存在者，那么世界就不受一个主宰者的支配。
智力。

6. 运动存在。

在上述前提中，前四个在证明中被表达出来；第五个隐含在其第一句和第二句的关联之中；第六个表达了一个事实，作者似乎认为不必陈述该事实，但它显然是其推理依据的一部分。让我们

用以下方式表示基本命题：

设= 物质是必然的存在者。

引力必然存在。

引力必然不存在。

z = 世界仅仅是物质的，不受任何主宰的理智支配。w = 运动存在。

v = 真空是必需的。

那么，前提系统将由以下方程表示，
其中q被用作不定时间的符号：

$$\begin{aligned} x &= q\{y(1-t)+(1-y)t\}. \\ tz &= q(1-w). \\ y &= qv. \\ v &= q(1-x). \\ x &= qz. \\ w &= 1. \end{aligned}$$

由此，如果我们消去记号q，便得到如下系统，
即：

$$x\{yt+(1-y)(1-t)\} = 0. \quad (1)$$
$$tzw = 0. \quad (2)$$
$$y(1-v) = 0. \quad (3)$$
$$vx = 0. \quad (4)$$
$$x(1-z) = 0. \quad (5)$$
$$1-w = 0. \quad (6)$$

Now if from these equations we eliminate w, v, z, y, and t, we obtain the equation

$$x = 0,$$

which expresses the proposition, *Matter is not a necessary being.* This is Dr. Clarke's conclusion. If we endeavour to eliminate any other set of five symbols (except the set v, z, y, t, and x, which would give $w = 1$), we obtain a result of the form $0 = 0$. It hence appears that *there are no other conclusions expressive of the absolute truth or falsehood of any of the elementary propositions designated by single symbols.*

Of conclusions expressed by equations involving two symbols, there exists but the following, viz.:– *If the world is merely material, and not subject to a presiding Intelligence, gravitation is not necessarily absent.* This conclusion is expressed by the equation

$$tz = 0, \text{ whence } z = q(1 - t).$$

If in the above analysis we suppress the concluding premiss, expressing the fact of the existence of motion, and leave the hypothetical principles which are embodied in the remaining premises untouched, some remarkable conclusions follow. To these I shall direct attention in the following chapter.

10. Of the remainder of Dr. Clarke's argument I shall briefly state the substance and connexion, dwelling only on certain portions of it which are of a more complex character than the others, and afford better illustrations of the method of this work.

In Prop. IV. it is shown that the substance or essence of the self-existent being is incomprehensible. The tenor of the reasoning employed is, that we are ignorant of the essential nature of all other things,–much more, then, of the essence of the self-existent being.

In Prop. V. it is contended that "though the substance or essence of the self-existent being is itself absolutely incomprehensible to us, yet many of the essential attributes of his nature are strictly demonstrable, as well as his existence."

In Prop. VI. it is argued that "the self-existent being must of necessity be infinite and omnipresent;" and it is contended that his infinity must be "an infinity of fulness as well as of immensity." The ground upon which the demonstration proceeds is, that an absolute necessity of existence must be independent of time, place, and circumstance, free from limitation, and therefore excluding all imperfection. And hence it is inferred that the self-existent being must be "a most simple, unchangeable, incorruptible being, without parts, figure, motion, or any other such properties as we find in matter."

The premises actually employed may be exhibited as follows:

1. If a finite being is self-existent, it is a contradiction to suppose it not to exist.

2. A finite being may, without contradiction, be absent from one place.

3. That which may without contradiction be absent from one place may without contradiction be absent from all places.

4. That which may without contradiction be absent from all places may without contradiction be supposed not to exist.

它表达了命题物质不是必然存在者。这是克拉克博士的结论。如果我们试图消去任何其他五个记号组成的集合(除了集合v、z、y、t和x，那将给出w = 1)，我们得到形式为0 = 0的结果。由此看来，不存在表达由单个记号所标示的任何基本命题的绝对真或假的其它结论。

在由涉及两个记号的方程所表达的结论中，只存在以下这个，即：--如果世界仅仅是物质的，不受主宰的理智支配，那么引力并非必然不在场。这个结论由以下方程表达

$$tz = 0, \text{ whence } z = q(1 - t).$$

如果在上述分析中我们删去表达运动存在这一事实的结尾前提，并且不去触动其余前提中所体现的假说性原则，那么便会得出一些值得注意的结论。我将在下一章中提请注意这些结论。

10.对于克拉克博士论证的其余部分，我将简要陈述其实质和关联，只详述其中某些比其他部分更为复杂的部分，它们能更好地说明本书的方法。

在命题iv.中表明，自存之物的实体或本质是不可理解的。所运用的推理要旨是：我们对所有其他事物的本质本性一无所知--那么，对自存之物的本质就更是如此了。在命题v.中，有人主张"尽管自存之物的实体或本质本身对我们来说绝对不可理解，但其本性的许多本质属性以及其存在都是可以严格证明的"。在命题vi.中，有人论证"自存之物必然无限且无所不在"；并主张其无限性必定是"充满的无限性以及广袤的无限性"。论证所依据的基础是：存在的绝对必然性必定独立于时间、地点和环境，不受限制，因而排除一切不完美。由此推断，自存之物必定是"一个最单纯、不变、不朽的存在者，没有部分、形状、运动或我们在物质中发现的任何其他此类属性"。

实际使用的前提可展示如下：

1. 如果一个有限的存在者是自存的，那么设想它不存在便是一个矛盾存在。

2. 一个有限的存在者可以不在某一个地方，这并不矛盾。

3.可以无矛盾地从某一地点缺席者，也可以无矛盾地从一切地点缺席。

4.可以无矛盾地从一切地点缺席者，也可以无矛盾地被设想为不存在。

Let us assume

x = Finite beings.
y = Things self-existent.
z = Things which it is a contradiction to suppose not to exist.
w = Things which may be absent without contradiction from one place.
t = Things which without contradiction may be absent from every place.

We have on expressing the above, and eliminating the indefinite symbols,

$$xy(1-z) = 0. \quad (1)$$
$$x(1-w) = 0. \quad (2)$$
$$w(1-t) = 0. \quad (3)$$
$$tz = 0. \quad (4)$$

Eliminating in succession t, w, and z, we get

$$xy = 0,$$
$$\therefore y = \frac{0}{0}(1-x);$$

the interpretation of which is,–*Whatever is self-existent is infinite.*

In Prop. VII. it is argued that the self-existent being must of necessity be One. The order of the proof is, that the self-existent being is "necessarily existent," that "necessity absolute in itself is simple and uniform, and without any possible difference or variety," that all "variety or difference of existence" implies dependence; and hence that "whatever exists necessarily is the one simple essence of the self-existent being."

The conclusion is also made to flow from the following premises:—

1. If there are two or more necessary and independent beings, either of them may be supposed to exist alone.

2. If either may be supposed to exist alone, it is not a contradiction to suppose the other not to exist.

3. If it is not a contradiction to suppose this, there are not two necessary and independent beings.

Let us represent the elementary propositions as follows:–

x = there exist two necessary independent beings.
y = either may be supposed to exist alone.
z = it is not a contradiction to suppose the other not to exist.

We have then, on proceeding as before,

$$x(1-y) = 0. \quad (1)$$
$$y(1-z) = 0. \quad (2)$$
$$zx = 0. \quad (3)$$

让我们假设

x = 有限的存在者。

y = 自存之物。

z = 假设其不存在就会陷入矛盾的事物。w = 可以从一个地方无矛盾地缺席的事物。t = 可以从每个地方无矛盾地缺席的事物。

我们已表述了上述内容，并消去了不定符号，

$$xy(1-z) = 0. \quad (1)$$
$$x(1-w) = 0. \quad (2)$$
$$w(1-t) = 0. \quad (3)$$
$$tz = 0. \quad (4)$$

依次消去t、w和z，我们得到

$$xy = 0,$$
$$\therefore y = \frac{0}{0}(1-x);$$

其解释为--凡是自存者皆为无限。

在命题 vii.中，论证了自存的存在者必然是一。证明的顺序是：自存的存在者是"必然存在的","自身中的绝对必然性是单纯且齐一的，没有任何可能的差异或多样性"，所有"存在的多样性或差异"都意味着依赖；因此"任何必然存在的东西都是自存存在者的一个单纯本质"。

结论也由以下前提推出：--

1. 如果有两个或两个以上必然且独立的存在者，则其中任何一个可以设想其单独存在。

2. 如果二者中任一可被假定为单独存在，那么以下情况并不构成矛盾：假设另一个不存在。

3. 如果假设这一点不是矛盾，那么就没有两个必然的。让我们将基本命题表示如下：--

和独立的存在者。

x = 存在两个必然的独立存在者。y = 两者中的任何一个都可以被假设为单独存在。

z = 假设另一个不存在并不矛盾。

于是，我们按先前的方式继续进行，

$$x(1-y) = 0. \quad (1)$$
$$y(1-z) = 0. \quad (2)$$
$$zx = 0. \quad (3)$$

Eliminating y and z, we have

$$x = 0.$$

Whence, *There do not exist two necessary and independent beings.*

11. To the premises upon which the two previous propositions rest, it is well known that Bishop Butler, who at the time of the publication of the "Demonstration," was a student in a non-conformist academy, made objection in some celebrated letters, which, together with Dr. Clarke's replies to them, are usually appended to editions of the work. The real question at issue is the validity of the principle, that "whatsoever is absolutely necessary at all is absolutely necessary in every part of space, and in every point of duration,"—a principle assumed in Dr. Clarke's reasoning, and explicitly stated in his reply to Butler's first letter. In his second communication Butler says: "I do not conceive that the idea of ubiquity is contained in the idea of self-existence, or *directly follows from it*, any otherwise than as whatever exists must exist *somewhere*." That is to say, necessary existence implies existence in some part of space, but not in every part. It does not appear that Dr. Clarke was ever able to dispose effectually of this objection. The whole of the correspondence is extremely curious and interesting. The objections of Butler are precisely those which would occur to an acute mind impressed with the conviction, that upon the sifting of first principles, rather than upon any mechanical dexterity of reasoning, the successful investigation of truth mainly depends. And the replies of Dr. Clarke, although they cannot be admitted as satisfactory, evince, in a remarkable degree, that peculiar intellectual power which is manifest in the work from which the discussion arose.

12. In Prop. VIII. it is argued that the self-existent and original cause of all things must be an Intelligent Being.

The main argument adduced in support of this proposition is, that as the cause is more excellent than the effect, the self-existent being, as the cause and original of all things, must contain in itself the perfections of all things; and that Intelligence is one of the perfections manifested in a part of the creation. It is further argued that this perfection is not a modification of figure, divisibility, or any of the known properties of matter; for these are not perfections, but *limitations*. To this is added the *à posteriori* argument from the manifestation of design in the frame of the universe.

There is appended, however, a distinct argument for the existence of an intelligent self-existent being, founded upon the phænomenal existence of motion in the universe. I shall briefly exhibit this proof, and shall apply to it the method of the present treatise.

The argument, omitting unimportant explanations, is as follows:–

"'Tis evident there is some such a thing as motion in the world; which either began at some time or other, or was eternal. If it began in time, then the question is granted that the first cause is an intelligent being.... On the contrary, if motion was eternal, either it was eternally caused by some eternal intelligent being, or it must of itself be necessary and self-existent, or else, without any

消去 y 与 z，得

$$x = 0.$$

由此可知，不存在两个必然且独立的存在者。

11. 众所周知，前两个命题所依据的前提，曾遭到巴特勒主教(Bishop Butler)在一些著名的书信中的反对，当时《论证》(Demonstration)一书出版之际，他正在一所非国教徒学院求学；这些书信连同克拉克博士(Dr.Clarke)对它们的答复，通常附于该书的各版本之后。真正的问题在于这一原则的有效性，即"凡在整体上绝对必然者，在空间的每一部分、在时间的每一瞬间亦绝对必然"--这一原则为克拉克博士的推理所假定，并在其答复巴特勒第一封信时明确陈述。在其第二封信中，巴特勒说："我并不设想遍在的观念包含于自存的观念之中，或直接由之推出，除非是就凡存在者必存在于某处而言。"这就是说，必然存在蕴涵着存在于空间的某一部分，而非每一部分。看来克拉克博士始终未能有效地驳倒这一反对意见。整篇通信极为奇特而有趣。巴特勒的反对意见，恰恰是一个敏锐的心灵在深信以下信念时所能想到的：对真理的成功探究，主要取决于对第一原则的审视，而非任何推理的机械技巧。而克拉克博士的答复，尽管不能被认为是令人满意的，却在显著程度上显示出那种独特的心智力量，这种力量在引发这场讨论的那部著作中显而易见。

12. 在命题 viii.中，论证了自存且为万物原始原因的存在者必定是一个有理智的存在者。

为支持这一命题而提出的主要论据是：既然原因比结果更为优越，那么作为万物的原因和本原的自存存在者，其自身必定包含万物的完善性；而理智是在创造的一部分中所显现的完善性之一。进一步论证说，这种完善性不是形状、可分性或任何已知的物质属性的样式；因为这些不是完善性，而是限制。此外还补充了后验论证，即从宇宙构造中设计的显现而来的论证。

然而，附有一个独立的论证，基于宇宙中运动的现象性存在，来证明一个有理性的自存存在者的存在。我将简要展示这一证明，并将本论著的方法应用于它。

论证(略去不重要的解释)如下：--

"世界上显然有运动这样的东西存在；它要么在某个时候开始，要么是永恒的。如果它在时间中开始，那么问题就得到了承认：第一原因是理智的存在者。.....相反，如果运动是永恒的，那么它要么是由某个永恒的有理智的存在者永恒引起的，要么它自身必然是自存的，要么，没有任何

necessity in its own nature, and without any external necessary cause, it must have existed from eternity by an endless successive communication. If motion was eternally caused by some eternal intelligent being, this also is granting the question as to the present dispute. If it was of itself necessary and self-existent, then it follows that it must be a contradiction in terms to suppose any matter to be at rest. And yet, at the same time, because the determination of this self-existent motion must be every way at once, the effect of it would be nothing else but a perpetual rest.... But if it be said that motion, without any necessity in its own nature, and without any external necessary cause, has existed from eternity merely by an endless successive communication, as Spinoza inconsistently enough seems to assert, this I have before shown (in the proof of the second general proposition of this discourse) to be a plain contradiction. It remains, therefore, that motion must of necessity be originally caused by something that is intelligent."

The premises of the above argument may be thus disposed:

1. If motion began in time, the first cause is an intelligent being. 2. If motion has existed from eternity, either it has been eternally caused by some eternal intelligent being, or it is self-existent, or it must have existed by endless successive communication.

3. If motion has been eternally caused by an eternal intelligent being, the first cause is an intelligent being.

4. If it is self-existent, matter is at rest and not at rest.

5. That motion has existed by endless successive communication, and that at the same time it is not self-existent, and has not been eternally caused by some eternal intelligent being, is false.

To express these propositions, let us assume—

x = Motion began in time (and therefore)
$1-x$ = Motion has existed from eternity.
y = The first cause is an intelligent being.
p = Motion has been eternally caused by some eternal intelligent being.
q = Motion is self-existent.
r = Motion has existed by endless successive communication.
s = Matter is at rest.

The equations of the premises then are—

$$x = vy.$$
$$1-x = v\{p(1-q)(1-r) + q(1-p)(1-r) + r(1-p)(1-q)\}.$$
$$p = vy.$$
$$q = vs(1-s) = 0.$$
$$r(1-q)(1-p) = 0.$$

Since, by the fourth equation, $q = 0$, we obtain, on substituting for q its value

在其自身本性中具有必然性，且没有任何外在的必然原因，那么它必定通过一种无尽相继的传递而从永恒就已存在。如果运动是由某个永恒理智的存在者永恒地引起的，这同样是在当前争论中把问题本身当作论据。如果运动是自身必然且自存的，那么由此推出：设想任何物质处于静止状态必是术语上的矛盾。然而同时，由于这种自存运动的确定必定在一切方面同时发生，其结果将不过是永恒的静止.....但如果说，运动在其自身本性中没有任何必然性，也没有任何外在的必然原因，仅仅通过一种无尽相继的传递而从永恒就已存在，正如斯宾诺莎(Spinoza)颇为前后不一地似乎所断言的那样，那么这我已先前表明(在本篇第二一般命题的证明中)是明显的矛盾。因此，剩下的只能是：运动必定必然地由某种有理智的东西原初地引起。"

上述论证的前提可如此排列：

1. 如果运动在时间中开始，第一原因是理智的存在者。2. 如果运动从永恒就存在，那么它要么是由某个永恒的有理智的存在者永恒引起的，要么它是自存的，要么它必定通过无尽相继的传递而存在。

3. 如果运动是由一个永恒的有理智的存在者永恒引起的，那么第一原因是理智的存在者。

4. 如果它是自存的，物质既静止又非静止。

5. 运动通过无尽相继的传递而存在，同时它又不是自存的，并且不是由某个永恒的有理智的存在者永恒引起的，这是假的。

为了表达这些命题，让我们假设--

= 运动始于时间(因而)
1 – x = 运动从永恒以来就存在。
y = 第一原因是理智的存在者。p = 运动是由某个永恒的有理智的存在者永恒引起的。
q = 运动是自存的。

r = 运动通过无尽相继的传递而存在。s = 物质处于静止。

前提的方程于是为--

$$x = vy.$$
$$1-x = v\{p(1-q)(1-r) + q(1-p)(1-r) + r(1-p)(1-q)\}.$$
$$p = vy.$$
$$q = vs(1-s) = 0.$$
$$r(1-q)(1-p) = 0.$$

由于根据第四个方程，q = 0，我们将 q 的值代入后得到

in the remaining equations, the system

$$\begin{aligned} x = vy, \qquad & 1 - x = v\left\{p\left(1-r\right) + r\left(1-p\right)\right\}, \\ p = vy, \qquad & r\left(1-p\right) = 0, \end{aligned}$$

from which eliminating the indefinite symbols v, we have the final reduced system,

$$x\left(1-y\right) = 0, \tag{1}$$
$$\left(1-x\right)\left\{pr + \left(1-p\right)\left(1-r\right)\right\} = 0, \tag{2}$$
$$p\left(1-y\right) = 0. \tag{3}$$
$$r\left(1-p\right) = 0. \tag{4}$$

We shall first seek the value of y, the symbol involved in Dr. Clarke's conclusion. First, eliminating x from (1) and (2), we have

$$(1-y)\{pr + (1-p)(1-r)\} = 0. \tag{5}$$

Next, to eliminate r from (4) and (5), we have

$$\begin{aligned} r(l-p) + (1-y)\{pr + (1-p)(1-r)\} &= 0, \\ \therefore \{1 - p + (1-y)p\} \times (1-y)(1-p) &= 0; \end{aligned}$$

whence

$$(1-y)(1-p) = 0. \tag{6}$$

Lastly, eliminating p from (3) and (6), we have

$$\begin{aligned} 1 - y &= 0, \\ \therefore y &= 1, \end{aligned}$$

which expresses the required conclusion, *The first cause is an intelligent being.*

Let us now examine what other conclusions are deducible from the premises.

If we substitute the value just found for y in the equations (1), (2), (3), (4), they are reduced to the following pair of equations, viz.,

$$(1-x)\{pr + (l-p)(l-r)\} = 0, \quad r(l-p) = 0. \tag{7}$$

Eliminating from these equations x, we have

$$r(1-p) = 0, \text{ whence } r = vp,$$

which expresses the conclusion, *If motion has existed by endless successive communication, it has been eternally caused by an eternal intelligent being.*

Again eliminating, from the given pair, r, we have

$$\begin{aligned} (1-x)(1-p) &= 0, \\ \text{or, } 1 - x &= vp, \end{aligned}$$

在剩余的方程中，该系统

$$\begin{aligned} x = vy, \qquad & 1 - x = v\left\{p\left(1-r\right) + r\left(1-p\right)\right\}, \\ p = vy, \qquad & r\left(1-p\right) = 0, \end{aligned}$$

从中消去不定记号 v，我们得到最终的化简系统，

$$x\left(1-y\right) = 0, \tag{1}$$
$$\left(1-x\right)\left\{pr + \left(1-p\right)\left(1-r\right)\right\} = 0, \tag{2}$$
$$p\left(1-y\right) = 0. \tag{3}$$
$$r\left(1-p\right) = 0. \tag{4}$$

我们将首先寻求 y 的值，这是克拉克博士结论中所涉及的记号。首先，从(1)和(2)中消去 x，我们有

$$(1-y)\{pr + (1-p)(1-r)\} = 0. \tag{5}$$

接下来，为了从(4)和(5)中消去r，我们有

$$\begin{aligned} r(l-p) + (1-y)\{pr + (1-p)(1-r)\} &= 0, \\ \therefore \{1 - p + (1-y)p\} \times (1-y)(1-p) &= 0; \end{aligned}$$

whence

$$(1-y)(1-p) = 0. \tag{6}$$

Lastly, eliminating p from (3) and (6), we have

$$\begin{aligned} 1 - y &= 0, \\ \therefore y &= 1, \end{aligned}$$

它表达了所需的结论，第一原因是理智的现在让我们考察从前提中还能推出什么其他结论。如果我们将刚刚求得的值代入方程(1)、(2)、(3)、(4)中的 y，

存在。

它们被简化为以下这对等式，即，

$$(1-x)\{pr + (l-p)(l-r)\} = 0, \quad r(l-p) = 0. \tag{7}$$

从这些方程中消去x，我们得到

$$r(1-p) = 0, \text{ whence } r = vp,$$

它表达了结论，如果运动通过无尽相继的传递而存在，那么它是由一个永恒的有理智的存在者永恒引起的。

再次从给定的配对中消去r，我们得到

$$\begin{aligned} (1-x)(1-p) &= 0, \\ \text{or, } 1 - x &= vp, \end{aligned}$$

which expresses the conclusion, *If motion has existed from eternity, it has been eternally caused by some eternal intelligent being.*

Lastly, from the same original pair eliminating p, we get

$$(1-x)r = 0,$$

which, solved in the form

$$1 - x = r(1-r),$$

gives the conclusion, *If motion has existed from eternity, it has not existed by an endless successive communication.*

Solved under the form

$$r = vx,$$

the above equation leads to the equivalent conclusion, *If motion exists by an endless successive communication, it began in time.*

13. Now it will appear to the reader that the first and last of the above four conclusions are inconsistent with each other. The two consequences drawn from the hypothesis that motion exists by an endless successive communication, viz., 1st, that it has been eternally caused by an eternal intelligent being; 2ndly, that it began in time,—are plainly at variance. Nevertheless, they are both rigorous deductions from the original premises. The opposition between them is not of a *logical*, but of what is technically termed a *material*, character. This opposition might, however, have been formally stated in the premises. We might have added to them a formal proposition, asserting that "whatever is *externally* caused by an eternal intelligent being, does not begin in time." Had this been done, no such opposition as now appears in our conclusions could have presented itself. Formal logic can only take account of relations which are formally expressed (VI. 16); and it may thus, in particular instances, become necessary to express, in a formal manner, some connexion among the premises which, without actual statement, is involved in the very meaning of the language employed.

To illustrate what has been said, let us add to the equations (2) and (4) the equation

$$px = 0,$$

which expresses the condition above adverted to. We have

$$(1-x)\{pr + (1-p)(1-r)\} + r(1-p) + px = 0. \tag{8}$$

Eliminating p from this, we find simply

$$r = 0,$$

which expresses the proposition, *Motion does not exist by an endless successive communication.* If now we substitute for r its value in (8), we have

$$(1-x)(1-p) + px = 0, \text{ or, } 1 - x = p;$$

它表达了结论，如果运动从永恒就存在，那么它是由某个永恒的有理智的存在者永恒引起的。

最后，从同一原始对中消去p，我们得到

$$(1-x)r = 0,$$

which, solved in the form

$$1 - x = r(1-r),$$

gives the conclusion, *If motion has existed from eternity, it has not existed by an endless successive communication.*

Solved under the form

$$r = vx,$$

上述方程导致等价的结论，如果运动通过无尽相继的传递而存在，那么它在时间中开始。

13. 现在读者会看出，上述四个结论中的第一个和最后一个彼此不一致。从运动通过无尽的相继传递而存在这一假说得出的两个推论，即：第一，它是由一个永恒的理智存在者永恒地引起的；第二，它在时间中开始--显然相互冲突。然而，它们都是从原初前提得出的严格演绎。它们之间的对立不是逻辑的对立，而是技术上所称的实质的对立。然而，这种对立本可以在前提中被形式地陈述出来。我们本可以给它们加上一个形式命题，断言"凡是由一个永恒的理智存在者外在地引起的东西，都不在时间中开始"。假如这样做了，那么我们的结论中现在所出现的那种对立就不会出现。形式逻辑只能考虑被形式地表达出来的关系(VI.16)；因此，在特定事例中，可能有必要以形式的方式表达前提之间的某种联系，而这种联系即使没有实际陈述，也包含在所使用语言的真正含义之中。

To illustrate what has been said, let us add to the equations (2) and (4) the equation

$$px = 0,$$

它表达了上述所提及的条件。我们有

$$(1-x)\{pr + (1-p)(1-r)\} + r(1-p) + px = 0. \tag{8}$$

从中消去p，我们便简单地得到

$$r = 0,$$

它表达了命题，运动不是通过无尽相继的传递而存在。如果现在我们将 r 的值代入(8)，我们有

$$(1-x)(1-p) + px = 0, \text{ or, } 1 - x = p;$$

whence we have the interpretation, *If motion has existed from eternity, it has been eternally caused by an eternal intelligent being;* together with the converse of that proposition.

In Prop. IX. it is argued, that "the self-existent and original cause of all things is not a necessary agent, but a being endued with liberty and choice." The proof is based mainly upon his possession of intelligence, and upon the existence of final causes, implying design and choice. To the objection that the supreme cause operates by necessity for the production of what is best, it is replied, that this is a necessity of fitness and wisdom, and not of nature.

14. In Prop. X. it is argued, that "the self-existent being, the supreme cause of all things, must of necessity have infinite power." The ground of the demonstration is, that as "all the powers of all things are derived from him, nothing can make any difficulty or resistance to the execution of his will." It is defined that the infinite power of the self-existent being does not extend to the "making of a thing which implies a contradiction," or the doing of that "which would imply imperfection (whether natural or moral) in the being to whom such power is ascribed," but that it does extend to the creation of matter, and of an immaterial, cogitative substance, endued with a power of beginning motion, and with a liberty of will or choice. Upon this doctrine of liberty it is contended that we are able to give a satisfactory answer to "that ancient and great question, πόθεν τὸ κακὸν, what is the cause and original of evil?" The argument on this head I shall briefly exhibit,

"All that we call evil is either an evil of imperfection, as the want of certain faculties or excellencies which other creatures have; or natural evil, as pain, death, and the like; or moral evil, as all kinds of vice. The first of these is not properly an evil; for every power, faculty, or perfection, which any creature enjoys, being the free gift of God,... it is plain the want of any certain faculty or perfection in any kind of creatures, which never belonged to their natures is no more an evil to them, than their never having been created or brought into being at all could properly have been called an evil. The second kind of evil, which we call natural evil, is either a necessary consequence of the former, as death to a creature on whose nature immortality was never conferred; and then it is no more properly an evil than the former. Or else it is counterpoised on the whole with as great or greater good, as the afflictions and sufferings of good men, and then also it is not properly an evil; or else, lastly, it is a punishment, and then it is a necessary consequence of the third and last kind of evil, viz., moral evil. And this arises wholly from the abuse of liberty which God gave to His creatures for other purposes, and which it was reasonable and fit to give them for the perfection and order of the whole creation. Only they, contrary to God's intention and command, have abused what was necessary to the perfection of the whole, to the corruption and depravation of themselves. And thus all sorts of evils have entered into the world without any diminution to the infinite goodness of the Creator and Governor thereof."—p. 112.

The main premises of the above argument may be thus stated:

1st. All reputed evil is either evil of imperfection, or natural evil, or moral evil.

由此我们得到解释，如果运动从永恒就存在，那么它是由一个永恒的有理智的存在者永恒引起的；连同该命题的逆命题。

在命题 ix.中，论证了"万物的自存且原始的原因不是一个必然的动因，而是一个被赋予自由和选择的存在者"。证明主要基于他拥有理智，以及基于目的因的存在，这暗示着设计和选择。对于最高原因必然地运作以产生最好的东西这一反对意见，回答是：这是一种适宜性和智慧的必然性，而不是本性的必然性。

14. 在命题x中论证说，"自存的存在者，万物的最高原因，必定必然具有无限的能力。"论证的依据是：既然"万物的一切能力都从他派生"，那么没有任何东西能对他的意志的执行造成任何困难或抵抗。文中界定说，自存存在者的无限能力并不扩展到"制造一个蕴涵矛盾的东西"，也不扩展到做那"将蕴涵归于被赋予此种能力的存在者的不完善(无论是自然的还是道德的)之事"，但它确实扩展到创造物质，以及创造一种非物质的、能思的实体，该实体被赋予开始运动的能力以及意志或选择的自由。依据这种自由学说，文中主张我们能够对"那个古老而伟大的问题，πόθεν τὸ κακόν，恶的原因和本原是什么?"给出令人满意的回答。关于这一点的论证，我将简要展示，

"我们所称的一切恶，要么是缺陷之恶，例如缺乏其他造物所具有的某些能力或优点；要么是自然之恶，例如痛苦、死亡之类；要么是道德之恶，例如各种恶行。第一种严格说来并非恶；因为任何造物所享有的每一种力量、能力或优点，都是上帝的自由赐予，..... 显然，任何一类造物若缺乏某种从未属于其本性的能力或优点，这对它们来说并非恶，正如它们从未被创造或产生出来，严格说来也不能被称为恶一样。第二种恶，我们称之为自然之恶，要么是前一种恶的必然结果，例如死亡之于一个其本性从未被赋予不死的造物；那么它严格说来并不比前一种更是恶。要么它在整体上被同等或更大的善所抵消，例如好人的苦难与痛苦，那么它严格说来也不是恶；要么最后，它是一种惩罚，那么它就是第三种亦即最后一种恶，即道德之恶的必然结果。而这种恶完全源于对自由的滥用，上帝赋予其造物自由是为了其他目的，并且为了整个造物的完善与秩序，赋予他们自由是合理而适宜的。只是他们违背上帝的意图和命令，滥用了本为整个造物之完善所必需的东西，以致自身败坏堕落。于是，各种恶进入了世界，而造物主与主宰者的无限善性并未因此减损。"--第112页。

上述论证的主要前提可陈述如下：第一，一切被认为的恶要么是缺陷之恶，要么是自然之恶，要么是道德之恶
恶。

2nd. Evil of imperfection is not absolute evil.

3rd. Natural evil is either a consequence of evil of imperfection, or it is compensated with greater good, or it is a consequence of moral evil.

4th. That which is either a consequence of evil of imperfection, or is compensated with greater good, is not absolute evil.

5th. All absolute evils are included in reputed evils.

To express these premises let us assume—

$w =$ reputed evil.
$x =$ evil of imperfection.
$y =$ natural evil.
$z =$ moral evil.
$p =$ consequence of evil of imperfection.
$q =$ compensated with greater good.
$r =$ consequence of moral evil.
$t =$ absolute evil.

Then, regarding the premises as Primary Propositions, of which all the predicates are particular, and the conjunctions *either, or*, as absolutely disjunctive, we have the following equations:

$$w = v\{x(1-y)(1-q) + y(1-x)(1-z) + z(1-x)(1-y)\}$$
$$x = v(1-t).$$
$$y = v\{p(1-q)(1-r) + q(1-p)(1-r) + r(1-p)(1-q)\}$$
$$p(l-q) + q(l-p) = v(1-t).$$
$$t = vw\,.$$

From which, if we separately eliminate the symbol v, we have

$$w\{1 - x(1-y)(1-z) - y(1-x)(1-z) - z(1-x)(1-y)\} = 0, \quad (1)$$
$$xt = 0, \quad (2)$$
$$y\{1 - p(1-q)(1-r) - q(1-p)(1-r) - r(1-p)(1-q)\} = 0, \quad (3)$$
$$\{p(1-q) + q(1-p)\}\,t = 0, \quad (4)$$
$$t(1-w) = 0. \quad (5)$$

Let it be required, first, to find what conclusion the premises warrant us in forming respecting absolute evils, as concerns their dependence upon moral evils, and the consequences of moral evils.

For this purpose we must determine t in terms of z and r.

The symbols w, x, y, p, q must therefore be eliminated. The process is easy, as any set of the equations is reducible to a single equation by addition.

Eliminating w from (1) and (5), we have

$$t\{1 - x(1-y)(1-z) - y(1-x)(1-z) - z(1-x)(1-y)\} = 0. \quad (6)$$

第二，不完美之恶并非绝对的恶。

第三，自然的恶要么是不完善之恶的结果，要么以更大的善得到补偿，要么是道德之恶的结果。

第四，凡或者是缺陷之恶的结果，或者以更大的善得到补偿者，都不是绝对的恶。

第五。一切绝对的恶都包含在所谓的恶之中。为表达这些前提，让我们假定--

w = 被认为是恶的。
x = 不完美之恶。
y = 自然恶。
z = 道德恶。
p = 不完美之恶的后果。
q = 以更大的善来补偿。
r = 道德恶果的后果。
t = 绝对的恶。

然后，将这些前提视为所有谓词都是特称的原始命题，并将连词要么，要么视为绝对析取的，我们得到以下方程：

$$w = v\{x(1-y)(1-q) + y(1-x)(1-z) + z(1-x)(1-y)\}$$
$$x = v(1-t).$$
$$y = v\{p(1-q)(1-r) + q(1-p)(1-r) + r(1-p)(1-q)\}$$
$$p(l-q) + q(l-p) = v(1-t).$$
$$t = vw\,.$$

由此，如果我们分别消去符号 v，则得到

$$w\{1 - x(1-y)(1-z) - y(1-x)(1-z) - z(1-x)(1-y)\} = 0, \quad (1)$$
$$xt = 0, \quad (2)$$
$$y\{1 - p(1-q)(1-r) - q(1-p)(1-r) - r(1-p)(1-q)\} = 0, \quad (3)$$
$$\{p(1-q) + q(1-p)\}\,t = 0, \quad (4)$$
$$t(1-w) = 0. \quad (5)$$

首先，要求找出前提允许我们就绝对的恶、就其依赖于道德之恶以及道德之恶的后果而言，形成什么结论。

为此，我们必须用 z 和 r 来确定 t。

因此必须消去记号 w、x、y、p、q。这个过程很容易，因为任何一组方程都可以通过相加化为一个方程。

从(1)和(5)中消去w，我们得到

$$t\{1 - x(1-y)(1-z) - y(1-x)(1-z) - z(1-x)(1-y)\} = 0. \quad (6)$$

The elimination of p from (3) and (4) gives

$$yqr + yqt + yt(1-r)(1-q) = 0. \tag{7}$$

The elimination of q from this gives

$$yt(1-r) = 0. \tag{8}$$

The elimination of x between (2) and (6) gives

$$t\{yz + (1-y)(1-z)\} = 0. \tag{9}$$

The elimination of y from (8) and (9) gives

$$t(1-z)(1-r) = 0.$$

This is the only relation existing between the elements t, z, and r. We hence get

$$\begin{aligned} t &= \frac{0}{(1-z)(1-r)} \\ &= \frac{0}{0}zr + \frac{0}{0}z(1-r) + \frac{0}{0}(1-z)r + 0(1-z)(1-r) \\ &= \frac{0}{0}z + \frac{0}{0}(1-z)r; \end{aligned}$$

the interpretation of which is, *Absolute evil is either moral evil, or it is, if not moral evil, a consequence of moral evil.*

Any of the results obtained in the process of the above solution furnish us with interpretations. Thus from (8) we might deduce

$$\begin{aligned} t = \frac{0}{y(1-r)} = \frac{0}{0}yr + \frac{0}{0}(1-y)r + \frac{0}{0}(1-y)(1-r) \\ = \frac{0}{0}yr + \frac{0}{0}(1-y); \end{aligned}$$

whence, *Absolute evils are either natural evils, which are the consequences of moral evils, or they are not natural evils at all.*

A variety of other conclusions may be deduced from the given equations in reply to questions which may be arbitrarily proposed. Of such I shall give a few examples, without exhibiting the intermediate processes of solution.

Quest. 1.—Can any relation be deduced from the premises connecting the following elements, viz.: absolute evils, consequences of evils of imperfection, evils compensated with greater good?

Ans.—*No relation exists.* If we eliminate all the symbols but z, p, q, the result is $0 = 0$.

Quest. 2.—Is any relation implied between absolute evils, evils of imperfection, and consequences of evils of imperfection.

从(3)和(4)中消去p，得到

$$yqr + yqt + yt(1-r)(1-q) = 0. \tag{7}$$

由此消去q，便得到

$$yt(1-r) = 0. \tag{8}$$

从(2)和(6)中消去x，得到

$$t\{yz + (1-y)(1-z)\} = 0. \tag{9}$$

从(8)和(9)中消去y，得到

$$t(1-z)(1-r) = 0.$$

这是元素t、z和r之间存在的唯一关系。我们因此

$$\begin{aligned} t &= \frac{0}{(1-z)(1-r)} \\ &= \frac{0}{0}zr + \frac{0}{0}z(1-r) + \frac{0}{0}(1-z)r + 0(1-z)(1-r) \\ &= \frac{0}{0}z + \frac{0}{0}(1-z)r; \end{aligned}$$

其解释是，绝对的恶要么是道德上的恶，要么，如果不是道德上的恶，就是道德上的恶的后果。在上述解决过程中获得的任何结果都为我们提供了各种解释。因此，从(8)我们可以推导出

$$\begin{aligned} t = \frac{0}{y(1-r)} = \frac{0}{0}yr + \frac{0}{0}(1-y)r + \frac{0}{0}(1-y)(1-r) \\ = \frac{0}{0}yr + \frac{0}{0}(1-y); \end{aligned}$$

由此，绝对的恶要么是自然之恶，即道德之恶的结果，要么根本不是自然之恶。从给定的方程还可以推出各种其他结论，以回答可以任意提出的问题。我将举出几个这样的例子，而不展示求解的中间过程。问题1--能否从前提中推出任何关系，把以下元素联系起来，即：绝对的恶、缺陷之恶的结果、以更大善补偿的恶?答--不存在任何关系。如果我们消去除z、p、q之外的所有记号，结果就是0 = 0。问题2--绝对的恶、缺陷之恶与缺陷之恶的结果之间是否蕴含任何关系。

Ans.—The final relation between x, t, and p is

$$xt + pt = 0;$$

whence

$$t = \frac{0}{p+x} = \frac{0}{0}(1-p)(1-x).$$

Therefore, *Absolute evils are neither evils of imperfection, nor consequences of evils of imperfection.* Quest. 3. — Required the relation of natural evils to evils of imperfection and evils compensated with greater good.

We find

$$pqy = 0,$$
$$\therefore y = \frac{0}{pq} = \frac{0}{0}p(1-q) + \frac{0}{0}(1-p).$$

Therefore, *Natural evils are either consequences of evils of imperfection which are not compensated with greater good, or they are not consequences of evils of imperfection at all.*

Quest. 4. — In what relation do those natural evils which are not moral evils stand to absolute evils and the consequences of moral evils?

If $y(1-z) = s$, we find, after elimination,

$$ts(1-r) = 0;$$
$$\therefore s = \frac{0}{t(1-r)} = \frac{0}{0}tr + \frac{0}{0}(1-t).$$

Therefore, *Natural evils, which are not moral evils, are either absolute evils, which are the consequences of moral evils, or they are not absolute evils at all.*

The following conclusions have been deduced in a similar manner. The subject of each conclusion will show of what particular things a description was required, and the predicate will show what elements it was designed to involve: —

Absolute evils, which are not consequences of moral evils, are moral and not natural evils.

Absolute evils which are not moral evils are natural evils, which are the consequences of moral evils.

Natural evils which are not consequences of moral evils are not absolute evils.

Lastly, let us seek a description of evils which are not absolute, expressed in terms of natural and moral evils.

We obtain as the final equation,

$$1 - t = yz + \frac{0}{0}y(1-z) + \frac{0}{0}(1-y)z + (1-y)(1-z).$$

The direct interpretation of this equation is a necessary truth, but the reverse interpretation is remarkable. *Evils which are both natural and moral, and evils which are neither natural nor moral, are not absolute evils.*

答：x、t、p 的最终关系为

$$xt + pt = 0;$$

由此

$$t = \frac{0}{p+x} = \frac{0}{0}(1-p)(1-x).$$

因此，绝对的恶既不是不完美的恶，也不是不完美的恶的结果。问题3。--要求确定自然的恶与不完美的恶以及以更大善补偿的恶之间的关系。

我们发现

$$pqy = 0,$$
$$\therefore y = \frac{0}{pq} = \frac{0}{0}p(1-q) + \frac{0}{0}(1-p).$$

因此，自然的恶要么是未以更大善补偿的不完美的恶的结果，要么它们根本不是不完美的恶的结果。

问题4。--那些不是道德的恶的自然的恶与绝对的恶以及道德的恶的结果处于什么关系？

如果y(1 – z) = s，我们经过消元后发现，

$$ts(1-r) = 0;$$
$$\therefore s = \frac{0}{t(1-r)} = \frac{0}{0}tr + \frac{0}{0}(1-t).$$

因此，不是道德的恶的自然的恶，要么是绝对的恶，即道德的恶的结果，要么它们根本不是绝对的恶。

以下结论是以类似方式推导出来的。每个结论的主词将表明需要描述的是哪些特殊事物，而谓词将表明它旨在包含哪些元素：

-

不是道德的恶的结果的绝对的恶，是道德的恶而非自然的恶。

不是道德的恶的绝对的恶，是自然的恶，即道德的恶的结果。

不是道德的恶的结果的自然的恶，不是绝对的恶。最后，让我们寻求一种用自然的恶和道德的恶来表达的非绝对的恶的描述。

我们得到最终方程：

$$1 - t = yz + \frac{0}{0}y(1-z) + \frac{0}{0}(1-y)z + (1-y)(1-z).$$

这个方程的直接解释是一个必然真理，但反向解释则值得注意。既是自然的又是道德的恶，以及既不是自然的也不是道德的恶，不是绝对的恶。

This conclusion, though it may not express a truth, is certainly involved in the given premises, as *formally* stated.

15. Let us take from the same argument a somewhat fuller system of premises, and let us in those premises suppose that the particles, *either, or,* are not absolutely disjunctive, so that in the meaning of the expression, "either evil of imperfection, or natural evil, or moral evil," we include whatever possesses one or more of these qualities.

Let the premises be —

1. All evil (w) is either evil of imperfection (x), or natural evil (y), or moral evil (z).
2. Evil of imperfection (x) is not absolute evil (t).
3. Natural evil (y) is either a consequence of evil of imperfection (p), or it is compensated with greater good (q), or it is a consequence of moral evil (r).
4. Whatever is a consequence of evil of imperfection (p) is not absolute evil (t).
5. Whatever is compensated with greater good (q) is not absolute evil (t).
6. Moral evil (z) is a consequence of the abuse of liberty (u).
7. That which is a consequence of moral evil (r) is a consequence of the abuse of liberty (u).
8. Absolute evils are included in reputed evils.

The premises expressed in the usual way give, after the elimination of the indefinite symbols v, the following equations:

$$w(1-x)(1-y)(1-z) = 0, \quad (1)$$
$$xt = 0, \quad (2)$$
$$y(1-p)(1-q)(1-r) = 0, \quad (3)$$
$$pt = 0, \quad (4)$$
$$qt = 0, \quad (5)$$
$$z(1-u) = 0, \quad (6)$$
$$r(1-u) = 0, \quad (7)$$
$$t(1-w) = 0. \quad (8)$$

Each of these equations satisfies the condition $V(1-V) = 0$.

The following results are easily deduced —

Natural evil is either absolute evil, which is a consequence of moral evil, or it is not absolute evil at all.

All evils are either absolute evils, which are consequences of the abuse of liberty, or they are not absolute evils.

Natural evils are either evils of imperfection, which are not absolute evils, or they are not evils of imperfection at all.

Absolute evils are either natural evils, which are consequences of the abuse of liberty, or they are not natural evils, and at the same time not evils of imperfection.

这个结论，尽管可能并未表达一个真理，却无疑蕴含于所给前提，如形式上所述。

15.让我们从同一论证中取一个更为完整的前提体系，并在那些前提中假定，*either*、*or*并非绝对不相容，以至于在"要么不完美的恶，要么自然的恶，要么道德的恶"这一表达式的含义中，我们包括任何具有这些性质之一或更多的对象。

Let the premises be —

1. All evil (w) is either evil of imperfection (x), or natural evil (y), or moral evil (z).
2. Evil of imperfection (x) is not absolute evil (t).
3. Natural evil (y) is either a consequence of evil of imperfection (p), or it is compensated with greater good (q), or it is a consequence of moral evil (r).
4. Whatever is a consequence of evil of imperfection (p) is not absolute evil (t).
5. Whatever is compensated with greater good (q) is not absolute evil (t).
6. Moral evil (z) is a consequence of the abuse of liberty (u).
7. That which is a consequence of moral evil (r) is a consequence of the abuse of liberty (u).
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The premises expressed in the usual way give, after the elimination of the indefinite symbols v, the following equations:

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$$pt = 0, \quad (4)$$
$$qt = 0, \quad (5)$$
$$z(1-u) = 0, \quad (6)$$
$$r(1-u) = 0, \quad (7)$$
$$t(1-w) = 0. \quad (8)$$

这些方程中的每一个都满足条件V(1 – V) = 0。以下结果容易推导出来--

自然的恶要么是绝对的恶，即道德的恶的结果，要么一切恶要么是绝对的恶，即滥用自由的结果，自然的恶要么是不完美的恶，即非绝对的恶，要么绝对的恶要么是自然的恶，即滥用自由的结果，要么它们不是自然的恶，同时也不是不完美的恶。
自由，或者它们并非绝对的恶。

它们根本不是不完善的恶。

Consequences of the abuse of liberty include all natural evils which are absolute evils, and are not evils of imperfection, with an indefinite remainder of natural evils which are not absolute, and of evils which are not natural.

16. These examples will suffice for illustration. The reader can easily supply others if they are needed. We proceed now to examine the most essential portions of the demonstration of Spinoza.

DEFINITIONS.

1. By a *cause of itself (causa sui)*, I understand that of which the essence involves existence, or that of which the nature cannot be conceived except as existing.

2. That thing is said to be finite or bounded in its own kind *(in suo genere finita)* which may be bounded by another thing of the same kind; e. g. Body is said to be finite, because we can always conceive of another body greater than a given one. So thought is bounded by other thought. But body is not bounded by thought, nor thought by body.

3. By substance, I understand that which is in itself *(in se)*, and is conceived by itself *(per se concipitur)*, i.e., that whose conception does not require to be formed from the conception of another thing.

4. By attribute, I understand that which the intellect perceives in substance, as constituting its very essence.

5. By mode, I understand the affections of substance, or that which is in another thing, by which thing also it is conceived.

6. By God, I understand the Being absolutely infinite, that is the substance consisting of infinite attributes, each of which expresses an eternal and infinite essence.

Explanation.—I say absolutely infinite, not infinite in its own kind. For to whatever is only infinite in its own kind we may deny the possession of (some) infinite attributes. But when a thing is absolutely infinite, whatsoever expresses essence and involves no negation belongs to its essence.

7. That thing is termed *free*, which exists by the sole necessity of its own nature, and is determined to action by itself alone; *necessary*, or rather constrained, which is determined by another thing to existence and action, in a certain and determinate manner.

8. By eternity, I understand existence itself, in so far as it is conceived necessarily to follow from the sole definition of the eternal thing.

Explanation.—For such existence, as an eternal truth, is conceived as the essence of the thing, and therefore cannot be explained by mere duration or time, though the latter should be conceived as without beginning and without end.

AXIOMS.

1. All things which exist are either in themselves *in se* or in another thing.

2. That which cannot be conceived by another thing ought to be conceived by itself.

滥用自由的结果包括一切是绝对的恶且不是不完美的恶的自然的恶，以及一个不确定的剩余，即非绝对的自然的恶和非自然的恶。

16.这些例子足以说明。读者如果需要，可以很容易地补充其他例子。我们现在来考察斯宾诺莎论证中最本质的部分。

定义。

1.所谓自因*(causa sui)*，我理解为其本质包含存在的东西，或其本性除非作为存在者否则不能被设想的东西。

2.那种在其自身种类中有限或有界的东西*(in suo genere finita)*，被称为可以由同种类的另一事物所限制；例如，物体被称为有限的，因为我们总能设想一个比给定物体更大的另一物体。思想也是如此，受其他思想限制。但物体不受思想限制，思想也不受物体限制。

3.所谓实体，我理解为在自身内*(in se)*并通过自身而被设想*(per se concipitur)*的东西，即其概念不需要借助另一事物的概念来形成的东西。

4.所谓属性，我理解为理智在实体中所感知到的、构成其实质的东西。

5.所谓样式，我理解为实体的分殊，或在他物内并通过该他物而被设想的东西。

6.所谓上帝，我理解为绝对无限的存在者，即由无限属性构成的实体，其中每一属性都表达永恒无限的本质。

解释。--我说绝对无限，而非在其自身种类中无限。因为对于仅在其自身种类中无限的东西，我们可以否认其具有(某些)无限属性。但当一事物是绝对无限时，凡表达本质而不包含否定的，都属于其本质。

7.那种仅由其自身本性的必然性而存在、仅由自身而决定行动的东西，被称为自由的；那种由另一事物以某种确定的方式决定其存在和行动的东西，被称为必然的，或毋宁说被强制的。

8.所谓永恒，我理解为存在本身，就它被设想为必然从永恒之物的唯一定义推出而言。

解释。--因为这样的存在作为永恒真理被设想为事物的本质，因此不能用单纯的绵延或时间来解释，尽管后者应被设想为无始无终。

公理。

1. 一切存在的事物，要么在自身内*in se*，要么在他物内。
2.不能通过他物而被设想的东西，应当通过自身而被设想。

3. From a given determinate cause the effect necessarily follows, and, contrariwise, if no determinate cause be granted, it is impossible that an effect should follow.

4. The knowledge of the effect depends upon, and involves, the knowledge of the cause.

5. Things which have nothing in common cannot be understood by means of each other; or the conception of the one does not involve the conception of the other.

6. A true idea ought to agree with its own object. (*Idea vera debet cum suo ideato convenire.*)

7. Whatever can be conceived as non-existing does not involve existence in its essence. Other definitions are implied, and other axioms are virtually assumed, in some of the demonstrations. Thus, in Prop. I., "Substance is prior in nature to its affections," the proof of which consists in a mere reference to Defs. 3 and 5, there seems to be an assumption of the following axiom, viz., "That by which a thing is conceived is prior in nature to the thing conceived." Again, in the demonstration of Prop. V. the converse of this axiom is assumed to be true. Many other examples of the same kind occur. It is impossible, therefore, by the mere processes of Logic, to deduce the whole of the conclusions of the first book of the Ethics from the axioms and definitions which are prefixed to it, and which are given above. In the brief analysis which will follow, I shall endeavour to present in their proper order what appear to me to be the real premises, whether formally stated or implied, and shall show in what manner they involve the conclusions to which Spinoza was led.

17. I conceive, then, that in the course of his demonstration, Spinoza effects several parallel divisions of the universe of possible existence, as,

1st. Into things which are in themselves, x, and things which are in some other thing, x'; whence, as these classes of thing together make up the universe, we have

$$x + x' = 1; \text{ (Ax. I.)}$$
$$\text{or, } x = 1 - x'.$$

2nd. Into things which are conceived by themselves, y, and things which are conceived through some other thing,y'; whence

$$y = 1 - y'. \text{ (Ax. II)}$$

3rd. Into substance, z, and modes, z'; whence

$$z = 1 - z'. \text{ (Def. III. V.)}$$

4th. Into things free, f, and things necessary,f'; whence

$$f = 1 - f'. \text{ (Def. VII.)}$$

5th. Into things which are causes and self-existent, e, and things caused by some other thing, e'; whence

$$e = 1 - e'. \text{ (Def. I. Ax. VII.)}$$

3.从给定的确定原因，结果必然随之产生；反之，如果没有给定确定的原因，结果就不可能随之产生。

4. 关于结果的知识依赖于并包含着关于原因的知识。

5. 没有任何共同之处的事物不能通过彼此而被理解；或者，其一的概念不包含另一的概念。

6. 真观念应当与其自身的对象相符合。(*Idea vera debet cum suo ideato convenire.*)

7.凡能被设想为不存在的东西，其本质都不包含存在。在一些证明中，还隐含了其他定义，并实际上假定了其他公理。例如，在命题I"实体在自然上先于其属性"中，其证明仅诉诸定义3和定义5，似乎假定了以下公理，即："借以设想一物的东西在自然上先于被设想之物。"又如，在命题V的证明中，又假定这条公理的逆命题为真。同类例子还有很多。因此，仅凭逻辑过程，不可能从《伦理学》第一卷前面所列并已在上文给出的公理和定义推出该卷的全部结论。在接下来的简要分析中，我将努力按适当顺序呈现在我看来是真正前提的东西，无论它们是正式陈述的还是隐含的，并说明它们以何种方式蕴含了斯宾诺莎所得出的结论。

17. 那么我认为，在其论证过程中，斯宾诺莎对可能存在的宇宙作出了若干平行的划分，即，

第一，划分为自在之物，x，以及在某个他物之中的事物，x′；由此，由于这些类的事物共同构成了宇宙，我们有

$$x + x' = 1; \text{ (Ax. I.)}$$
$$\text{or, } x = 1 - x'.$$

2nd. Into things which are conceived by themselves, y, and things which are conceived through some other thing,y'; whence

$$y = 1 - y'. \text{ (Ax. II)}$$

3rd. Into substance, z, and modes, z'; whence

$$z = 1 - z'. \text{ (Def. III. V.)}$$

4th. Into things free, f, and things necessary,f'; whence

$$f = 1 - f'. \text{ (Def. VII.)}$$

5th. Into things which are causes and self-existent, e, and things caused by some other thing, e'; whence

$$e = 1 - e'. \text{ (Def. I. Ax. VII.)}$$

And his reasoning proceeds upon the expressed or assumed principle, that these divisions are not only parallel, but equivalent. Thus in Def. III., Substance is made equivalent with that which is conceived by itself; whence

$$z = y.$$

Again, Ax. IV., as it is actually applied by Spinoza, establishes the identity of cause with that by which a thing is conceived; whence

$$y = e.$$

Again, in Def. VII., things free are identified with things self-existent; whence

$$f = e.$$

Lastly, in Def. V mode is made identical with that which is in another thing; whence $z' = x'$, and therefore,

$$z = x.$$

All these results may be collected together into the following series of equations, viz.:

$$x = y = z = f = e = 1 - x' = 1 - y' = 1 - f' = 1 - z' = 1 - e'.$$

And any two members of this series connected together by the sign of equality express a conclusion, whether drawn by Spinoza or not, which is a legitimate consequence of his system. Thus the equation

$$z = 1 - e',$$

expresses the sixth proposition of his system, viz., One substance cannot be produced by another. Similarly the equation

$$z = e,$$

expresses his seventh proposition, viz., "It pertains to the nature of substance to exist." This train of deduction it is unnecessary to pursue. Spinoza applies it chiefly to the deduction according to his views of the properties of the Divine Nature, having first endeavoured to prove that the only substance is God. In the steps of this process, there appear to me to exist some fallacies, dependent chiefly upon the ambiguous use of words, to which it will be necessary here to direct attention. 18. In Prop. V. it is endeavoured to show, that "There cannot exist two or more substances of the same nature or attribute." The proof is virtually as follows: If there are more substances than one, they are distinguished either by attributes or modes; if by attributes, then there is only one substance of the same attribute; if by modes, then, laying aside these as non-essential, there remains no *real* ground of distinction. Hence there exists but one substance of the same attribute. The assumptions here involved are inconsistent with those which are found in other parts of the treatise. Thus substance, Def. IV.,

而他的推理依据一个明示的或假定的原则进行，即这些划分不仅是平行的，而且是等价的。因此，在界说三中，实体被等同于由自身而被设想的东西；由此

$$z = y.$$

再者，公理四，正如斯宾诺莎实际应用的那样，确立了原因与事物借以被设想的东西的同一性；由此

$$y = e.$$

再者，在界说七中，自由的事物被等同于自存的事物；由此

$$f = e.$$

Lastly, in Def. V mode is made identical with that which is in another thing; whence $z' = x'$, and therefore,

$$z = x.$$

所有这些结果可以汇集为以下一系列方程，即：

$$x = y = z = f = e = 1 - x' = 1 - y' = 1 - f' = 1 - z' = 1 - e'.$$

而这一系列中任意两个以等号连接的项表达一个结论，无论是否为斯宾诺莎所引出，该结论都是其体系的合法推论。因此，方程

$$z = 1 - e',$$

表达其体系的第六个命题，即：一个实体不能由另一个实体产生。类似地，方程

$$z = e,$$

表述了他的第七个命题，即："存在属于实体的本性。"这一演绎链条无须继续追踪。斯宾诺莎主要将它用于按照他的观点对神性诸属性的演绎，此前他已首先力图证明唯一的实体是上帝。在这一过程的步骤中，我认为存在一些谬误，主要取决于词语的歧义使用，这里有必要予以注意。18. 在命题v中，力图表明"不能存在两个或更多具有相同本性或属性的实体"。其证明实际上如下：如果有不止一个实体，它们要么通过属性、要么通过样式来区分；如果通过属性区分，那么具有同一属性的实体只有一个；如果通过样式区分，那么把这些非本质的东西撇开，就不剩下任何实在的区分根据。因此，具有同一属性的实体只有一个。这里所涉及的假定与论著其他部分中的假定不一致。例如，实体，定义iv,

is apprehended by the intellect through the means of attribute. By Def. VI. it may have many attributes. One substance may, therefore, *conceivably* be distinguished from another by a difference in some of its attributes, while others remain the same.

In Prop. VIII. it is attempted to show that, All substance is necessarily infinite. The proof is as follows. There exists but one substance, of one attribute, Prop. V.; and it pertains to its nature to exist, Prop. VII. It will, therefore, be of its nature to exist either as finite or infinite. But not as finite, for, by Def. II. it would require to be bounded by another substance of the same nature, which also ought to exist *necessarily*, Prop. VII. Therefore, there would be two substances of the same attribute, which is absurd, Prop. V. Substance, therefore, is infinite.

In this demonstration the word "finite" is confounded with the expression, "Finite in its own kind," Def. II. It is thus assumed that nothing can be finite, unless it is bounded by another thing of the same kind. This is not consistent with the ordinary meaning of the term. Spinoza's use of the term finite tends to make space the only form of substance, and all existing things but affections of space, and this, I think, is really one of the ultimate foundations of his system.

The first scholium applied to the above Proposition is remarkable. I give it in the original words: "Quum finitum esse revera sit ex parte negatio, et infinitum absoluta affirmatio existentiae alicujus naturae, sequitur ergo ex sola Prop. VII. omnem substantiam debere esse infinitam." Now this is in reality an assertion of the principle affirmed by Clarke, and controverted by

Butler (XIII. 11), that necessary existence implies existence in every part of space. Probably this principle will be found to lie at the basis of every attempt to demonstrate, *à priori*, the existence of an Infinite Being.

From the general properties of substance above stated, and the definition of God as the substance consisting of infinite attributes, the peculiar doctrines of Spinoza relating to the Divine Nature necessarily follow. As substance is self-existent, free, causal in its very nature, the thing in which other things are, and by which they are conceived; the same properties are also asserted of the Deity. He is self-existent, Prop. XI.; indivisible, Prop. XIII.; the only substance, Prop. XIV.; the Being in which all things are, and by which all things are conceived, Prop. XV.; free, Prop. XVII.; the immanent cause of all things, Prop. XVIII. The proof that God is the only substance is drawn from Def. VI., which is interpreted into a declaration that "God is the Being absolutely infinite, of whom no attribute which expresses the essence of substance can be denied." Every conceivable attribute being thus assigned by definition to Him, and it being determined in Prop. V. that there cannot exist two substances of the same attribute, it follows that God is the only substance.

Though the "Ethics" of Spinoza, like a large portion of his other writings, is presented in the geometrical form, it does not afford a good praxis for the symbolical method of this work. Of course every train of reasoning admits, when its ultimate premises are truly determined, of being treated by that method; but in the present instance, such treatment scarcely differs, except in the use of letters for words, from the processes employed in the original demonstrations.

被理智通过属性的手段所领会。根据界说六，它可能具有许多属性。因此，一个实体可以设想地通过其某些属性的差异而与另一个实体相区别，而其他属性则保持不变。

在命题viii中，试图表明：一切实体都必然是无限的。证明如下。只有一个实体，具有一种属性，命题v；而存在属于其本性，命题vii。因此，按其本性，它要么作为有限而存在，要么作为无限而存在。但不是作为有限而存在，因为按照定义ii，它将需要由另一个具有相同本性的实体所限制，而那个实体也应当必然存在，命题vii。因此，将会有两个具有相同属性的实体，这是荒谬的，命题v。所以，实体是无限的。

在这一论证中，"有限的"一词与界说二中的表述"在其自身种类中有限"相混淆。于是便假定，没有任何事物可以是有限的，除非它被另一个同种类的事物所限制。这与该术语的通常含义不一致。斯宾诺莎对有限一词的使用倾向于使空间成为实体的唯一形式，而一切存在的事物不过是空间的情状，而我认为这实际上是他体系的终极基础之一。

应用于上述命题的第一个附释值得注意。我照录原文如下："Quum finitum esse revera sit ex parte negatio, et infinitum absoluta affirmatio existentiae alicujus naturae, sequitur ergo ex sola Prop.vii.omnem substantiam debere esse infinitam." 那么这实际上是对克拉克所主张、而被

巴特勒(XIII.11)所反驳的那一原则的断言，即必然存在蕴含着在空间每一部分中的存在。很可能这一原则将被发现是每一次`*a priori*证明一个无限存在者之存在的尝试的基础。

由上述关于实体的普遍属性以及将上帝定义为由无限属性构成的实体，斯宾诺莎关于神性的独特学说必然随之而来。正如实体是自因的、自由的、在其本性中就是原因性的，是其他事物存在于其中、并通过它而被设想的东西；同样的属性也被断言为上帝所具有。他是自因的，命题十一；不可分的，命题十三；唯一的实体，命题十四；万物存在于其中、万物通过它而被设想的存在者，命题十五；自由的，命题十七；万物的内在原因，命题十八。上帝是唯一实体的证明取自定义六，该定义被解释为一项声明："上帝是绝对无限的存在者，凡表达实体本质的属性都不能被否定于他。"每一个可设想的属性都由此通过定义被归于他，而命题五又确定了不可能存在两个具有同一属性的实体，因此上帝是唯一的实体。

尽管斯宾诺莎的《伦理学》，如同他其他著作的很大一部分一样，以几何学形式呈现，但它并不为本书的符号方法提供一个良好的实践范例。当然，每一推理链条，当其终极前提被真正确定时，都容许用该方法来处理；但在目前的情形下，这种处理除了用字母代替词语之外，与原始论证中所采用的过程几乎没有差别。

Reasoning which consists so largely of a play upon terms defined as equivalent, is not often met with; and it is rather on account of the interest attaching to the subject, than of the merits of the demonstrations, highly as by some they are esteemed, that I have devoted a few pages here to their exposition.

19. It is not possible, I think, to rise from the perusal of the arguments of Clarke and Spinoza without a deep conviction of the futility of all endeavours to establish, entirely *à priori*, the existence of an Infinite Being, His attributes, and His relation to the universe. The fundamental principle of all such speculations, viz., that whatever we can clearly conceive, must exist, fails to accomplish its end, even when its truth is admitted. For how shall the finite comprehend the infinite? Yet must the possibility of such conception be granted, and in something more than the sense of a mere withdrawal of the limits of phaenomenal existence, before any solid ground can be established for the knowledge, *à priori*, of things infinite and eternal. Spinoza's affirmation of the reality of such knowledge is plain and explicit: "Mens humana adaequatum habet cognitionem aeternae et infinitae essentiae Dei" (Prop. XLVII., Part 2nd). Let this be compared with Prop. XXXIV., Part 2nd: "Omnis idea quae in nobis est absoluta sive adaequata et perfecta, vera est;" and with Axiom VI., Part 1st, "Idea vera debet cum suo ideato convenire." Moreover, this species of knowledge is made the essential constituent of all other knowledge: "De natura rationis est res sub quadam aeternitatis specie percipere" (Prop. XLIV., Cor. II., Part 2nd). Were it said, that there is a tendency in the human mind to rise in contemplation from the particular towards the universal, from the finite towards the infinite, from the transient towards the eternal; and that this tendency suggests to us, with high probability, the existence of more than sense perceives or understanding comprehends; the statement might be accepted as true for at least a large number of minds. There is, however, a class of speculations, the character of which must be explained in part by reference to other causes,—impatience of probable or limited knowledge, so often all that we can really attain to; a desire for absolute certainty where intimations sufficient to mark out before us the path of duty, but not to satisfy the demands of the speculative intellect, have alone been granted to us; perhaps, too, dissatisfaction with the present scene of things. With the undue predominance of these motives, the more sober procedure of analogy and probable induction falls into neglect. Yet the latter is, beyond all question, the course most adapted to our present condition. To infer the existence of an intelligent cause from the teeming evidences of surrounding design, to rise to the conception of a moral Governor of the world, from the study of the constitution and the moral provisions of our own nature;– these, though but the feeble steps of an understanding limited in its faculties and its materials of knowledge, are of more avail than the ambitious attempt to arrive at a certainty unattainable on the ground of natural religion. And as these were the most ancient, so are they still the most solid foundations, Revelation being set apart, of the belief that the course of this world is not abandoned to chance and inexorable fate.

如此大量地由对被定义为等价的术语的玩弄所构成的推理并不常见；而我在此用几页篇幅来阐述它们，与其说是由于这些论证的价值--尽管有些人高度推崇它们--不如说是由于该主题本身所引起的兴趣。

19. 我认为，人们在阅读克拉克和斯宾诺莎的论证之后，不可能不深深确信：一切试图完全`*a priori*'地确立一个无限存在者、其属性及其与宇宙的关系的努力，都是徒劳的。所有这类思辨的基本原则，即凡我们能清晰概念到的，必定存在，即使其真理性被承认，也未能达到其目的。因为有限者如何能理解无限者呢?然而，必须承认这种概念的可能性，并且是在比仅仅撤除现象存在的界限更多的意义上，才能为关于无限和永恒之物的`*a priori*'知识确立任何坚实的基础。斯宾诺莎对这种知识的实在性的肯定明白而明确："Mens humana adaequatum habet cognitionem aeternae et infinitae essentiae Dei"(第二部分，命题四十七)。将此与第二部分命题三十四相比较："Omnis idea quae in nobis est absoluta sive adaequata et perfecta, vera est;"并与第一部分公理六相比较："Idea vera debet cum suo ideato convenire."此外，这种知识被规定为一切其他知识的本质构成要素："De natura rationis est res sub quadam aeternitatis specie percipere"(第二部分，命题四十四，推论二)。倘若有人说，人类心灵中有一种倾向，在沉思中从特称上升到全称，从有限上升到无限，从短暂上升到永恒；并且这种倾向以很高的概率向我们暗示，存在着比感官所感知或知性所理解更多的东西；这一陈述至少对大多数心灵来说可以被接受为真。然而，有一类思辨，其性质必须部分地参照其他原因来解释--对或然或有局限的知识的不耐烦，而这往往是我们真正所能达到的全部；对绝对确定性的渴望，而给予我们的只有足以在我们面前标出义务之路、却不足以满足思辨理智要求的暗示；也许还有对当前事物景象的不满。由于这些动机的过度主导，类比和或然归纳的更为审慎的程序陷入被忽视。然而，后者无疑是最适合我们当前状况的途径。从周围设计的丰富证据推断一个理智原因的存在，从对我们自身本性的构造和道德规定的考察上升到世界道德治理者的概念--这些虽然只是一个在其能力和知识材料上受限的知性的微弱步伐，却比在自然宗教的立场上企图达到不可企及的确定性的雄心勃勃的尝试更有用。并且，正如这些是最古老的，它们也仍然是(启示除外)关于这个世界进程并非被抛弃给偶然和无情的命运的信念的最坚实的基础。

Chapter XIV

EXAMPLE OF THE ANALYSIS OF A SYSTEM OF EQUATIONS BY THE METHOD OF REDUCTION TO A SINGLE EQUIVALENT EQUATION $V = 0$, WHEREIN V SATISFIES THE CONDITION $V(1 - V) = 0$.

1. Let us take the remarkable system of premises employed in the previous Chapter, to prove that "Matter is not a necessary being;" and suppressing the 6th premiss, viz., Motion exists,—examine some of the consequences which flow from the remaining premises. This is in reality to accept as true Dr. Clarke's hypothetical principles; but to suppose ourselves ignorant of the fact of the existence of motion. Instances may occur in which such a selection of a portion of the premises of an argument may lead to interesting consequences, though it is with other views that the present example has been resumed. The premises actually employed will be—

1. If matter is a necessary being, either the property of gravitation is necessarily present, or it is necessarily absent.

2. If gravitation is necessarily absent, and the world is not subject to any presiding intelligence, motion does not exist.

3. If gravitation is necessarily present, a vacuum is necessary.

4. If a vacuum is necessary, matter is not a necessary being.

5. If matter is a necessary being, the world is not subject to a presiding intelligence.

If, as before, we represent the elementary propositions by the following notation, viz.:

x = Matter is a necessary being.
y = Gravitation is necessarily present.
w = Motion exists.
t = Gravitation is necessarily absent.
z = The world is merely material, and not subject to a presiding intelligence.
v = A vacuum is necessary.

第十四章

用化归为单一等价方程的方法分析方程组的示例 V = 0 ，其中满足条件 V(1 – V) = 0 。

1. 让我们采用前一章中用来证明"物质不是必然存在者"的那套著名前提，并删去第6个前提，即运动存在，--考察从其余前提中得出的某些结论。这实际上是接受克拉克博士的假设原则为真；但设想我们不知道运动存在这一事实。可能会出现这样的情况：这样选取一个论证的部分前提可能导致有趣的结论，尽管重新审视当前这个例子的用意另有其他。实际采用的前提将是--

1. 如果物质是一个必然存在者，那么引力属性要么必然存在，要么必然不存在。

2. 如果引力必然不存在，并且世界不受任何主宰理智的支配，那么运动不存在。

3. 如果引力必然存在，那么真空就是必要的。

4. 如果真空是必要的，那么物质就不是一个必然的存在者。

5. 如果物质是一个必然存在者，那么世界不受主宰理智的支配。

如果像之前一样，我们用以下记号表示这些基本命题，即：

x = 物质是必然存在者。
引力必然存在。
w = 运动存在。
引力必然不存在。
z = 世界仅仅是物质的，且不受主宰理智的支配。v = 真空是必然的。

We shall on expression of the premises and elimination of the indefinite class symbols (q), obtain the following system of equations:

$$\begin{aligned} xyt + x\bar{y}\bar{t} &= 0, \\ tzw &= 0, \\ y\bar{v} &= 0, \\ vx &= 0, \\ x\bar{z} &= 0; \end{aligned}$$

in which for brevity $\bar{y}$ stands for $1 - y$, $\bar{t}$ for $1 - t$, and so on; whence, also, $1 - \bar{t} = t$, $1 - \bar{y} = y$, &c.

As the first members of these equations involve only positive terms, we can form a single equation by adding them together (VIII. Prop. 2), viz.:

$$xyt + x\bar{y}\bar{t} + y\bar{v} + vx + x\bar{z} + tzw = 0,$$

and it remains to reduce the first member so as to cause it to satisfy the condition $V(1 - V) = 0$.

For this purpose we will first obtain its development with reference to the symbols x and y. The result is—

$$\begin{aligned} (t + \bar{v} + v + \bar{z} + tzw)xy + (\bar{t} + v + \bar{z} + tzw)x\bar{y} \\ +(\bar{v} + tzw)\bar{x}y + tzw\bar{x}\bar{y} = 0. \end{aligned}$$

And our object will be accomplished by reducing the four coefficients of the development to equivalent forms, themselves satisfying the condition required.

Now the first coefficient is, since $v + \bar{v} = 1$,

$$1 + t + \bar{z} + tzw,$$

which reduces to unity (IX. Prop. 1).

The second coefficient is

$$\bar{t} + v + \bar{z} + tzw;$$

and its reduced form (X. 3) is

$$\bar{t} + tv + t\bar{v}\bar{z} + t\bar{v}zw.$$

The third coefficient, $\bar{v} + tzw$, reduces by the same method to $\bar{v} + tzwv$; and the last coefficient tzw needs no reduction. Hence the development becomes

$$xy + (\bar{t} + tv + t\bar{v}\bar{z} + t\bar{v}zw)\, x\bar{y} + (\bar{v} + tzwv)\, \bar{x}y + tzw\bar{x}\bar{y} = 0; \qquad (1)$$

and this is the form of reduction sought.

2. Now according to the principle asserted in Prop. III., Chap. X., the whole relation connecting any particular set of the symbols in the above equation may be deduced by developing that equation with reference to the particular symbols

我们将对前提进行表达，并消去不定类记号(q)，得到如下方程组：

$$\begin{aligned} xyt + x\bar{y}\bar{t} &= 0, \\ tzw &= 0, \\ y\bar{v} &= 0, \\ vx &= 0, \\ x\bar{z} &= 0; \end{aligned}$$

其中为简洁起见， y 代表 1 – y, ¯t 代表 1 – t，依此类推；由此也可得 1 – t = t,1 – ¯y = y，等等。

由于这些方程的第一项只涉及正项，我们可以将它们相加构成一个单一方程(VIII.命题2)，即：

$$xyt + x\bar{y}\bar{t} + y\bar{v} + vx + x\bar{z} + tzw = 0,$$

剩下的就是化简第一项，使其满足条件 V(1 – V)= 0。

为此，我们首先获得它关于记号 x 和 y 的展开式。结果为--

$$\begin{aligned} (t + \bar{v} + v + \bar{z} + tzw)xy + (\bar{t} + v + \bar{z} + tzw)x\bar{y} \\ +(\bar{v} + tzw)\bar{x}y + tzw\bar{x}\bar{y} = 0. \end{aligned}$$

我们的目标将通过把展开式的四个系数化为等价的、自身满足所需条件的形式来实现。

现在第一个系数是，因为 v + ¯v = 1,

$$1 + t + \bar{z} + tzw,$$

这归结为统一(IX.命题1)。

第二个系数是

$$\bar{t} + v + \bar{z} + tzw;$$

及其约化形式(X.3)为

$$\bar{t} + tv + t\bar{v}\bar{z} + t\bar{v}zw.$$

第三个系数 v + tzw 用同样的方法化为 v + tzwv；最后一个系数 tzw 无需化简。因此展开式变为

$$xy + (\bar{t} + tv + t\bar{v}\bar{z} + t\bar{v}zw)\, x\bar{y} + (\bar{v} + tzwv)\, \bar{x}y + tzw\bar{x}\bar{y} = 0; \qquad (1)$$

而这正是所要寻求的化简形式。

2. 现在，根据第x章命题iii.中所断言的原理，上述方程中任何特定一组记号之间的整个关系，可以通过关于该特定记号展开该方程来推出，

in question, and retaining in the result only those constituents whose coefficients are unity. Thus, if x and y are the symbols chosen, we are immediately conducted to the equation

$$xy = 0,$$

whence we have

$$y = \frac{0}{0}(1 - x),$$

with the interpretation, *If gravitation is necessarily present, matter is not a necessary being.*

Let us next seek the relation between x and w. Developing (1) with respect to those symbols, we get

$$(y + \bar{t}\bar{y} + tv\bar{y} + t\bar{v}\bar{z}\bar{y} + t\bar{v}z\bar{y})\, xw + (y + \bar{t}\bar{y} + tv\bar{y} + t\bar{v}\bar{z}\bar{y})\, x\bar{w} + (\bar{v}y + tzvy + tz\bar{y})\, \bar{x}w + \bar{v}yx\bar{w} = 0.$$

The coefficient of xw, and it alone, reduces to unity. For $t\bar{v}\bar{z}\bar{y} + t\bar{v}z\bar{y} = t\bar{v}\bar{y}$, and $tv\bar{y} + t\bar{v}\bar{y} = t\bar{y}$, and $\bar{t}\bar{y} + t\bar{y} = \bar{y}$, and lastly, $y + \bar{y} = 1$. This is always the mode in which such reductions take place. Hence we get

$$xw = 0,$$
$$\therefore w = \frac{0}{0}(1 - x),$$

of which the interpretation is, *If motion exists, matter is not a necessary being.*

If, in like manner, we develop (1) with respect to x and z, we get the equation

$$x\bar{z} = 0,$$
$$\therefore x = \frac{0}{0}z,$$

with the interpretation, *If matter is a necessary being, the world is merely material, and without a presiding intelligence.*

This, indeed, is only the fifth premiss reproduced, but it shows that there is no other relation connecting the two elements which it involves.

If we seek the whole relation connecting the elements x, w, and y, we find, on developing (1) with reference to those symbols, and proceeding as before,

$$xy + xw\bar{y} = 0.$$

Suppose it required to determine hence the consequences of the hypothesis, "Motion does not exist," relatively to the questions of the necessity of matter, and the necessary presence of gravitation. We find

$$w = \frac{-xy}{x\bar{y}},$$
$$\therefore 1 - w = \frac{x}{x\bar{y}} = \frac{1}{0}xy + x\bar{y} + \frac{0}{0}\bar{x};$$
$$\text{or, } 1 - w = x\bar{y} + \frac{0}{0}\bar{x}, \text{ with } xy = 0.$$

并在结果中只保留系数为1的那些成分。因此，如果 x 和 y 是所选的记号，我们便立即得到方程

$$xy = 0,$$

由此我们得到

$$y = \breve{0}(1 - x),$$

with the interpretation, *If gravitation is necessarily present, matter is not a necessary being.*

Let us next seek the relation between x and w. Developing (1) with respect to those symbols, we get

$$(y + \bar{t}\bar{y} + tv\bar{y} + t\bar{v}\bar{z}\bar{y} + t\bar{v}z\bar{y})\, xw + (y + \bar{t}\bar{y} + tv\bar{y} + t\bar{v}\bar{z}\bar{y})\, x\bar{w} + (\bar{v}y + tzvy + tz\bar{y})\, \bar{x}w + \bar{v}yx\bar{w} = 0.$$

The coefficient of xw, and it alone, reduces to unity. For $t\bar{v}\bar{z}\bar{y} + t\bar{v}z\bar{y} = t\bar{v}\bar{y}$, and $tv\bar{y} + t\bar{v}\bar{y} = t\bar{y}$, and $\bar{t}\bar{y} + t\bar{y} = \bar{y}$, and lastly, $y + \bar{y} = 1$. This is always the mode in which such reductions take place. Hence we get

$$xw = 0,$$
$$\therefore w = \frac{0}{0}(1 - x),$$

其解释为，如果运动存在，则物质不是必然存在者。

如果以同样的方式，我们关于 x 和 z 展开(1)，便得到方程

$$x\bar{z} = 0,$$
$$\therefore x = \frac{0}{0}z,$$

其解释为，如果物质是必然存在者，则世界仅是物质的，且没有主宰的智慧。

这实际上只是第五个前提的再现，但它表明，在它所涉及的两个元素之间不存在其他关系。

如果我们寻求连接元素 x、w 和 y 的整个关系，我们关于这些记号展开(1)并如前所述进行，便发现

$$xy + xw\bar{y} = 0.$$

假设要求由此确定假说"运动不存在"相对于物质必然性和引力必然存在这两个问题的推论。我们发现

$$w = \frac{-xy}{x\bar{y}},$$
$$\therefore 1 - w = \frac{x}{x\bar{y}} = \frac{1}{0}xy + x\bar{y} + \frac{0}{0}\bar{x};$$
$$\text{or, } 1 - w = x\bar{y} + \frac{0}{0}\bar{x}, \text{ with } xy = 0.$$

The direct interpretation of the first equation is, *If motion does not exist, either matter is a necessary being, and gravitation is not necessarily present, or matter is not a necessary being.*

The reverse interpretation is, *If matter is a necessary being, and gravitation not necessary, motion does not exist.*

In exactly the same mode, if we sought the full relation between x, z, and w, we should find

$$xzw + x\bar{z} = 0.$$

From this we may deduce

$$z = x\bar{w} + \frac{0}{0}\bar{x}, \text{ with } xw = 0.$$

Therefore, *If the world is merely material, and not subject to any presiding intelligence, either matter is a necessary being, and motion does not exist, or matter is not a necessary being.*

Also, *reversely, If matter is a necessary being, and there is no such thing as motion, the world is merely material.*

3. We might, of course, extend the same method to the determination of the consequences of any complex hypothesis u, such as, "The world is merely material, and without any presiding intelligence (z), but motion exists" (w), with reference to any other elements of doubt or speculation involved in the original premises, such as, "Matter is a necessary being" (x), "Gravitation is a necessary quality of matter," (y). We should, for this purpose, connect with the general equation (1) a new equation,

$$u = wz,$$

reduce the system thus formed to a single equation, $V = 0$, in which V satisfies the condition $V(1 - V) = 0$, and proceed as above to determine the relation between u, x, and y, and finally u as a developed function of x and y. But it is very much better to adopt the methods of Chapters VIII. and IX. I shall here simply indicate a few results, with the leading steps of their deduction, and leave their verification to the reader's choice.

In the problem last mentioned we find, as the relation connecting x, y, w, and z,

$$xw + x\bar{w}y + x\bar{w}\bar{y}\bar{z} = 0.$$

And if we write $u = xy$, and then eliminate the symbols x and y by the general problem, Chap. IX., we find

$$xu + xy\bar{u} = 0,$$

whence

$$u = \frac{1}{0}xy + 0x\bar{y} + \frac{0}{0}\bar{x};$$

wherefore

$$wz = \frac{0}{0}\bar{x} \text{ with } xy = 0.$$

第一个方程的直接解释是，如果运动不存在，则要么物质是必然存在者且引力不必然存在，要么物质不是必然存在者。

The reverse interpretation is, *If matter is a necessary being, and gravitation not necessary, motion does not exist.*

In exactly the same mode, if we sought the full relation between x, z, and w, we should find

$$xzw + x\bar{z} = 0.$$

由此我们可以推出

$$z = x\bar{w} + \frac{0}{0}\bar{x}, \text{ with } xw = 0.$$

因此，如果世界仅是物质的，且不受任何主宰智慧支配，则要么物质是必然存在者且运动不存在，要么物质不是必然存在者。

同样，反过来，如果物质是必然存在者，且不存在运动这种东西，则世界仅是物质的。

3. 我们当然可以将同样的方法推广到确定任何复杂假说 u 的推论，例如“世界仅仅是物质的，并且没有任何主宰的智慧(z)，但运动存在”(w)，并参照原前提中涉及的任何其他怀疑或推测的元素，例如“物质是必然的存在者”(x)、“引力是物质的一种必然性质”(y)。为此，我们应当将一个新方程与一般方程(1)联系起来，

$$u = wz,$$

将如此形成的系统归结为一个单一方程 V = 0，其中 V 满足条件 V(1 – V) = 0，并如上所述去确定 u、x 和 y 之间的关系，最后将 u 作为 x 和 y 的展开函数。但采用第八章和第九章的方法要好得多。我在此仅指出若干结果及其演绎的主要步骤，而将其验证留给读者自行选择。

In the problem last mentioned we find, as the relation connecting x, y, w, and z,

$$xw + x\bar{w}y + x\bar{w}\bar{y}\bar{z} = 0.$$

And if we write $u = xy$, and then eliminate the symbols x and y by the general problem, Chap. IX., we find

$$xu + xy\bar{u} = 0,$$

whence

$$u = \frac{1}{0}xy + 0x\bar{y} + \frac{0}{0}\bar{x};$$

wherefore

$$wz = \frac{0}{0}\bar{x} \text{ with } xy = 0.$$

Hence, *If the world is merely material, and without a presiding intelligence, and at the same time motion exists, matter is not a necessary being.*

Now it has before been shown that *if motion exists, matter is not a necessary being*, so that the above conclusion tells us even less than we had before ascertained to be (inferentially) true. Nevertheless, that conclusion is the proper and complete answer to the question which was proposed, which was, to determine simply the consequences of a certain complex hypothesis. 4. It would thus be easy, even from the limited system of premises before us, to deduce a great variety of additional inferences, involving, in the conditions which are given, any proposed combinations of the elementary propositions. If the condition is one which is inconsistent with the premises, the fact will be indicated by the form of the solution. The value which the method will assign to the combination of symbols expressive of the proposed condition will be 0. If, on the other hand, the fulfilment of the condition in question imposes no restriction upon the propositions among which relation is sought, so that every combination of those propositions is equally possible,—the fact will also be indicated by the form of the solution. Examples of each of these cases are subjoined.

If in the ordinary way we seek the consequences which would flow from the condition that *matter is a necessary being*, and at the same time that *motion exists*, as affecting the Propositions, *The world is merely material, and without a presiding intelligence*, and, *Gravitation is necessarily present*, we shall obtain the equation

$$xw = 0,$$

which indicates that the condition proposed is inconsistent with the premises, and therefore cannot be fulfilled.

If we seek the consequences which would flow from the condition that *Matter is not a necessary being*, and at the same time that *Motion does exist*, with reference to the same elements as above, viz., *the absence of a presiding intelligence*, and the *necessity of gravitation*,–we obtain the following result,

$$(1-x)w = \frac{0}{0}yz + \frac{0}{0}y(1-z) + \frac{0}{0}(1-y)z + \frac{0}{0}(1-y)(1-z),$$

which might literally be interpreted as follows:

If matter is not a necessary being, and motion exists, then either the world is merely material and without a presiding intelligence, and gravitation is necessary, or one of these two results follows without the other, or they both fail of being true. Wherefore of the four possible combinations, of which some one is true of necessity, and of which of necessity one only can be true, it is affirmed that any one may be true. Such a result is a truism— a mere *necessary* truth. Still it contains the only answer which can be given to the question proposed.

I do not deem it necessary to vindicate against the charge of laborious trifling these applications. It may be requisite to enter with some fulness into details useless in themselves, in order to establish confidence in general principles and methods. When this end shall have been accomplished in the subject of the

因此，如果世界仅是物质的，且没有主宰的智慧，同时运动存在，则物质不是必然存在者。

前面已经表明，如果运动存在，物质就不是必然的存在者，因此上述结论告诉我们的甚至少于我们此前已(从推论上)确定为真的东西。然而，该结论是对所提问题的恰当而完整的回答，该问题仅仅是确定某一复杂假说的推论。4. 因此，即使从我们面前有限的 premise 系统出发，也很容易演绎出大量额外的推论，在给定的条件中涉及基本命题的任何被提出的组合。如果该条件与前提不一致，这一事实将由解的形式表明。该方法将赋予表达所提条件的记号组合的值将为 0。另一方面，如果所讨论条件的满足对所寻求关系的那些命题不施加任何限制，以致这些命题的每一种组合都同等可能，--这一事实也将由解的形式表明。下面附上每种情况的例子。

如果我们以通常的方式寻求由物质是必然存在者且同时运动存在这一条件所产生的推论，就其对命题世界仅是物质的，且没有主宰的智慧以及引力必然存在的影响而言，我们将得到方程

$$xw = 0,$$

这表明所提出的条件与前提不一致，因此无法满足。

如果我们寻求由物质不是必然存在者且同时运动确实存在这一条件所产生的推论，就上述同样的元素而言，即没有主宰的智慧和引力的必然性,--我们得到如下结果，

$$(1-x)w = \frac{0}{0}yz + \frac{0}{0}y(1-z) + \frac{0}{0}(1-y)z + \frac{0}{0}(1-y)(1-z),$$

这可以字面解释如下：

如果物质不是必然的存在者，并且运动存在，那么要么世界仅仅是物质的且没有主宰的智慧，并且引力是必然的，要么这两个结果之一成立而另一个不成立，要么两者都不为真。因此，在四种可能的组合中--其中某一种必然为真，且必然只有一种能为真--据称任何一种都可能为真。这样的结果是一种自明之理--一种纯粹必然的真理。然而它包含着对所提问题所能给出的唯一回答。

我认为没有必要为这些应用辩护以反驳劳而无功的琐碎之讥。为了确立对一般原理和方法的信心，或许有必要较为充分地进入本身无用的细节。当这一目的在本探究的主题中

present inquiry, let all that has contributed to its attainment, but has afterwards been found superfluous, be forgotten.

得以实现时，让一切曾有助于其实现、但后来被发现为多余的东西被遗忘吧。

Chapter XV

THE ARISTOTELIAN LOGIC AND ITS MODERN EXTENSIONS, EXAMINED BY THE METHOD OF THIS TREATISE.

1. The logical system of Aristotle, modified in its details, but unchanged in its essential features, occupies so important a place in academical education, that some account of its nature, and some brief discussion of the leading problems which it presents, seem to be called for in the present work. It is, I trust, in no narrow or harshly critical spirit that I approach this task. My object, indeed, is not to institute any direct comparison between the time-honoured system of the schools and that of the present treatise; but, setting truth above all other considerations, to endeavour to exhibit the real nature of the ancient doctrine, and to remove one or two prevailing misapprehensions respecting its extent and sufficiency.

That which may be regarded as essential in the spirit and procedure of the Aristotelian, and of all cognate systems of Logic, is the attempted classification of the allowable forms of inference, and the distinct reference of those forms, collectively or individually, to some general principle of an axiomatic nature, such as the "dictum of Aristotle:" Whatsoever is affirmed or denied of the genus may in the same sense be affirmed or denied of any species included under that genus. Concerning such general principles it may, I think, be observed, that they either state directly, but in an abstract form, the argument which they are supposed to elucidate, and, so stating that argument, affirm its validity; or involve in their expression technical terms which, after definition, conduct us again to the same point, viz., the abstract statement of the supposed allowable forms of inference. The idea of classification is thus a pervading element in those systems. Furthermore, they exhibit Logic as resolvable into two great branches, the one of which is occupied with the treatment of categorical, the other with that of hypothetical or conditional propositions. The distinction is nearly identical with that of primary and secondary propositions in the present work. The discussion of the theory of categorical propositions is, in all the ordinary treatises of Logic, much more full and elaborate than that of hypothetical propositions, and is occupied partly with ancient scholastic distinctions, partly with the canons of deductive inference. To the latter application only is it

第十五章

亚里士多德逻辑及其现代扩展，以本论著的方法加以考察。

1. 亚里士多德的逻辑体系，其细节虽有修改，但本质特征未变，在学院教育中占据如此重要的地位，以至于在本著作中似乎有必要对其性质作一些说明，并对其所呈现的主要问题作一些简要讨论。我确信，我着手这项任务时并无狭隘或苛刻批评的精神。的确，我的目的不是要在由来已久的经院体系与本论著的体系之间作任何直接比较；而是将真理置于一切其他考虑之上，努力展示古代学说的真正性质，并消除一两个关于其范围与充分性的流行误解。

在亚里士多德逻辑学以及所有同类逻辑体系的精髓与程序中被视为本质性的东西，是对可允许的推论形式进行分类的尝试，以及将这些形式(无论是集合地还是个别地)明确地关联到某种公理性质的一般原则，例如"亚里士多德的箴言"：凡对属有所肯定或否定的，也可以在同一意义上对包含在该属下的任何种有所肯定或否定。关于这类一般原则，我认为可以观察到，它们要么直接地、但以抽象的形式陈述了它们被认为要阐明的论证，并在这样陈述该论证时肯定了其有效性；要么在其表达中包含了专业术语，这些术语在定义之后又将我们引向同一点，即对所设想的可允许的推论形式的抽象陈述。因此，分类的观念是那些体系中一个普遍渗透的元素。此外，它们将逻辑学展示为可分解为两大分支，其中一个分支致力于处理直言命题，另一个分支致力于处理假言或条件命题。这一区分与本书中关于第一性命题和第二性命题的区分几乎相同。在所有普通的逻辑学论著中，关于直言命题理论的讨论远比关于假言命题的讨论更为充分和精细，并且部分地涉及古代经院式的区分，部分地涉及演绎推论的准则。只有对后一种应用才是

necessary to direct attention here.

2. Categorical propositions are classed under the four following heads, viz.:

		TYPE
1st.	Universal affirmative Propositions:	All Y's are X's.
2nd.	Universal negative "	No Y's are X's.
3rd.	Particular affirmative "	Some Y's are X's.
4th.	Particular negative "	Some Y's are not X's.

To these forms, four others have recently been added, so as to constitute in the whole eight forms (see the next article) susceptible, however, of reduction to six, and subject to relations which have been discussed with great fulness and ability by Professor De Morgan, in his Formal Logic. A scheme somewhat different from the above has been given to the world by Sir W. Hamilton, and is made the basis of a method of syllogistic inference, which is spoken of with very high respect by authorities on the subject of Logic.[1]

The processes of Formal Logic, in relation to the above system of propositions, are described as of two kinds, viz., "Conversion" and "Syllogism." By Conversion is meant the expression of any proposition of the above kind in an equivalent form, but with a reversed order of terms. By Syllogism is meant the deduction from two such propositions having a common term, whether subject or predicate, of some third proposition inferentially involved in the two, and forming the "conclusion." It is maintained by most writers on Logic, that these processes, and according to some, the single process of Syllogism, furnish the universal types of reasoning, and that it is the business of the mind, in any train of demonstration, to conform itself, whether consciously or unconsciously, to the particular models of the processes which have been classified in the writings of logicians.

3. The course which I design to pursue is to show how these processes of Syllogism and Conversion may be conducted in the most general manner upon the principles of the present treatise, and, viewing them thus in relation to a system of Logic, the foundations of which, it is conceived, have been laid in the ultimate laws of thought, to seek to determine their true place and essential character.

The expressions of the eight fundamental types of proposition in the language of symbols are as follows:

$$
\begin{array}{lll}
1. & \text{All } Y\text{'s are } X\text{'s}, & y = vx. \\
2. & \text{No } Y\text{'s are } X\text{'s}, & y = v(1-x). \\
3. & \text{Some } Y\text{'s are } X\text{'s}, & vy = vx. \\
4. & \text{Some } Y\text{'s are not-}X\text{'s}, & vy = v(1-x). \\
5. & \text{All not-}Y\text{'s are } X\text{'s}, & 1-y = vx. \\
6. & \text{No not-}Y\text{'s are } X\text{'s}, & 1-y = v(l-x). \\
7. & \text{Some not-}Y\text{'s are } X\text{'s}, & v(l-y) = vx. \\
8. & \text{Some not-}Y\text{'s are not-}X\text{'s}, & v(1-y) = v(1-x).
\end{array}
\qquad (1)
$$

[1] Thomson's Outlines of the Laws of Thought, p. 177.

此处有必要提请注意。

2. 直言命题分为以下四类，即：

		TYPE
第一。	全称肯定命题：	All Y's are X's.
第二。	全称否定 ?	No Y's are X's.
第三。	特称肯定 ?	Some Y's are X's.
第4。	特称否定 ?	Some Y's are not X's.

最近又增添了四种形式，从而总共构成八种形式(见下一篇文章)，然而这些形式可以归约为六种，并且受制于一些关系，这些关系已由德摩根教授在其《形式逻辑》中作了极为充分和出色的讨论。W.汉密尔顿爵士向世人提出了一个与上述略有不同的方案，并使之成为三段论推论方法的基础，该方法受到逻辑学领域权威们的高度推崇。[1]

形式逻辑相对于上述命题系统的过程被描述为两种，即"换位"和"三段论"。所谓换位，是指以上述种类的任何命题用一个等价的形式来表达，但项的顺序相反。所谓三段论，是指从两个具有共同项(无论是主词还是谓词)的此类命题中，推出某种在推论上蕴含于这两个命题之中的第三命题，并形成"结论"。大多数逻辑学作者主张，这些过程--并且根据某些人的观点，单一的三段论过程--提供了推理的普遍类型，而心灵在任何论证序列中的职责，就是使自身--无论有意识还是无意识地--符合逻辑学家著作中所分类的那些过程的特定模型。

3. 我所打算采取的路线是表明，三段论和换位这些过程如何能够按照本论著的原理以最一般的方式进行，并且，如此从与一个逻辑体系的关系来看待它们--该体系的基础被认为已奠定于思维的终极规律之中--以寻求确定它们的真正位置和本质特征。

八种基本命题类型在语言中的表达式
符号的规则如下：

$$
\begin{array}{lll}
1. & \text{All } Y\text{'s are } X\text{'s}, & y = vx. \\
2. & \text{No } Y\text{'s are } X\text{'s}, & y = v(1-x). \\
3. & \text{Some } Y\text{'s are } X\text{'s}, & vy = vx. \\
4. & \text{Some } Y\text{'s are not-}X\text{'s}, & vy = v(1-x). \\
5. & \text{All not-}Y\text{'s are } X\text{'s}, & 1-y = vx. \\
6. & \text{No not-}Y\text{'s are } X\text{'s}, & 1-y = v(l-x). \\
7. & \text{Some not-}Y\text{'s are } X\text{'s}, & v(l-y) = vx. \\
8. & \text{Some not-}Y\text{'s are not-}X\text{'s}, & v(1-y) = v(1-x).
\end{array}
\qquad (1)
$$

[1] Thomson's Outlines of the Laws of Thought, p. 177.

In referring to these forms, it will be convenient to apply, in a sense shortly to be explained, the epithets of logical quantity, "universal" and "particular," and of quality, "affirmative" and "negative," to the terms of propositions, and not to the propositions themselves. We shall thus consider the term "All Y's," as universal-affirmative; the term "Y's," or "Some Y's," as particular-affirmative; the term "All not-Y's," as universal-negative; the term "Some not-Y's," as particular-negative. The expression "No Y's," is not properly a *term* of a proposition, for the true meaning of the proposition, "No Y's are X's," is "All Y's are not-X's." The subject of that proposition is, therefore, universal-affirmative, the predicate particular-negative. That there is a real distinction between the conceptions of "men" and "not men" is manifest. This distinction is all that I contemplate when applying as above the designations of affirmative and negative, without, however, insisting upon the etymological propriety of the application to the terms of propositions. The designations positive anil privative would have been more appropriate, but the former term is already employed in a fixed sense in other parts of this work.

4. From the symbolical forms above given the laws of conversion immediately follow. Thus from the equation

$$y = vx,$$

representing the proposition, "All Y's are X's," we deduce, on eliminating v,

$$y(1-x) = 0,$$

which gives by solution with reference to $1-x$,

$$1 - x = \frac{0}{0}(1-y);$$

the interpretation of which is,

All not-X's are not-Y's.

This is an example of what is called "negative conversion." In like manner, the equation

$$y = v(1-x),$$

representing the proposition, "No Y's are X's," gives

$$x = \frac{0}{0}(1-y),$$

the interpretation of which is, "No X's are Y's." This is an example of what is termed simple conversion; though it is in reality of the same kind as the conversion exhibited in the previous example. All the examples of conversion which have been noticed by logicians are either of the above kind, or of that which consists in the mere transposition of the terms of a proposition, without altering their quality, as when we change

$vy = vx$, representing, Some Y's are X's,

在指称这些形式时，将逻辑的量--"全称"和"特称"--以及质--"肯定"和"否定"--这些称谓，在稍后将解释的意义上，应用于命题的项而非命题本身，将是方便的。因此我们将把"所有 Y "这一项视为全称肯定；把" Y "或"有些 Y "这一项视为特称肯定；把"所有非 Y "这一项视为全称否定；把"有些非 Y "这一项视为特称否定。"没有 Y "这一表达式严格说来不是命题的项，因为命题"没有 Y 是 X "的真正含义是"所有 Y 都不是 X "。因此，该命题的主词是全称肯定的，谓词是特称否定的。"人"与"非人"这两个概念之间存在真实的区别，这是显而易见的。当我如上所述应用肯定和否定的称谓时，我所考虑的正是这一区别，然而并不坚持这种应用在词源上的恰当性。肯定的和privative的(剥夺的)这两个称谓本会更合适，但前者已在本著作的其他部分以固定的意义使用。

4. 从上面给出的符号形式，换位的规律立即随之而来。因此从方程

$$y = vx,$$

它表示命题"所有 Y 都是 X"，我们消去 v 后推出，

$$y(1-x) = 0,$$

其解参照 1 – x 给出，

$$1 - x = \frac{0}{0}(1-y);$$

其解释为，

所有非- X 都不是非- Y。

This is an example of what is called "negative conversion." In like manner, the equation

$$y = v(1-x),$$

representing the proposition, "No Y's are X's," gives

$$x = \frac{0}{0}(1-y),$$

其解释是，"没有 X 是 Y"。这是所谓简单换位的一个例子；尽管它实际上与前一个例子中所展示的换位属于同一种类。逻辑学家所注意到的所有换位例子，要么属于上述种类，要么属于仅仅调换命题各项而不改变其性质的那种，例如当我们把

$vy = vx$, representing, Some Y's are X's,

into

$$vx = vy, \text{ representing, Some } X\text{'s are } Y\text{'s;}$$

or they involve a combination of those processes with some auxiliary process of limitation, as when from the equation

$$y = vx, \text{ representing, All } Y\text{'s are } X\text{'s,}$$

we deduce on multiplication by v,

$$vy = vx, \text{ representing, Some } Y\text{'s are } X\text{'s,}$$

and hence

$$vx = vy, \text{ representing, Some } X\text{'s are } Y\text{'s.}$$

In this example, the process of limitation precedes that of transposition.

From these instances it is seen that conversion is a particular application of a much more general process in Logic, of which many examples have been given in this work. That process has for its object the determination of any element in any proposition, however complex, as a logical function of the remaining elements. Instead of confining our attention to the subject and predicate, regarded as simple terms, we can take any element or any combination of elements entering into either of them; make that element, or that combination, the "subject" of a new proposition; and determine what its predicate shall be, in accordance with the data afforded to us. It may be remarked, that even the simple forms of propositions enumerated above afford some ground for the application of such a method, beyond what the received laws of conversion appear to recognise. Thus the equation

$$y = vx, \text{ representing, All } Y\text{'s are } X\text{'s,}$$

gives us, in addition to the proposition before deduced, the three following:

1st. $y(1-x) = 0$. There are no Y's that are not-X's.

2nd. $1 - y = \frac{0}{0}x + (1-x)$. Things that are not-$Y$'s include all things that are not-X's, and an indefinite remainder of things that are X's.

3rd. $x = y + \frac{0}{0}(1-y)$. Things that are X's include all things that are Y's, and an indefinite remainder of things that are not-Y's.

变为

$$vx = vy, \text{ representing, Some } X\text{'s are } Y\text{'s;}$$

或者它们涉及那些过程与某种辅助的限制过程的结合，正如当从方程

$$y = vx, \text{ representing, All } Y\text{'s are } X\text{'s,}$$

我们通过乘以v来推导，

$$vy = vx, \text{ representing, Some } Y\text{'s are } X\text{'s,}$$

因此

$$vx = vy, \text{ representing, Some } X\text{'s are } Y\text{'s.}$$

在这个例子中，限制的过程先于换位的过程。

从这些实例可以看出，换位是逻辑学中一个更为普遍的过程的特殊应用，本著作中已给出了该过程的许多例子。该过程的目标是将任何命题中的任何元素--无论多么复杂--确定为其余元素的逻辑函数。我们不必将注意力局限于被视为简单项的主词和谓词，而是可以取进入其中任一者的任何元素或元素的任何组合；使该元素或该组合成为新命题的"主词"；并根据提供给我们的数据确定其谓词应当是什么。可以指出，即使上面所列举的命题的简单形式，也为这样一种方法的应用提供了一定的依据，超出了公认的换位规律似乎所承认的范围。因此，方程

$$y = vx, \text{ representing, All } Y\text{'s are } X\text{'s,}$$

除前面已推出的命题外，还给我们以下三个命题：

第一。 $y(1-x) = 0$. 没有不是 X 的 Y。

第二，1 – y = 0 0 x+(1–x). 不是-Y 的事物包括所有不是-X 的事物，以及是 X 的事物的一种不确定的剩余。

第三。 $x = y + \frac{0}{0}(1-y)$. 属于 X 的事物包括所有事物是 Y 的事物，以及不是-Y 的事物的一种不确定的剩余。

These conclusions, it is true, merely place the given proposition in other and equivalent forms,–but such and no more is the office of the received mode of "negative conversion."

Furthermore, these processes of conversion are not elementary, but they are combinations of processes more simple than they, more immediately dependent upon the ultimate laws and axioms which govern the use of the symbolical instrument of reasoning. This remark is equally applicable to the case of Syllogism, which we proceed next to consider.

5. The nature of syllogism is best seen in the particular instance. Suppose that we have the propositions,

All X's are Y's,
All Y's are Z's.

From these we may deduce the conclusion,

All X's are Z's.

This is a syllogistic inference. The terms X and Z are called the extremes, and Y is called the middle term. The function of the syllogism generally may now be defined. Given two propositions of the kind whose species are tabulated in (1), and involving one middle or common term Y, which is connected in one of the propositions with an extreme X, in the other with an extreme Z; required the relation connecting the extremes X and Z. The term Y may appear in its affirmative form, as, All Y's, Some Y's; or in its negative form, as, All not-Y's, Some not-Y's; in either proposition, without regard to the particular form which it assumes in the other.

Nothing is easier than in particular instances to resolve the Syllogism by the method of this treatise. Its resolution is, indeed, a particular application of the process for the reduction of systems of propositions. Taking the examples above given, we have,

$$x = vy,$$
$$y = v'z;$$

whence by substitution,

$$x = vv'z,$$

which is interpreted into

All X's are Z's.

Or, proceeding rigorously in accordance with the method developed in (VIII.7), we deduce

$$x(1-y) = 0, \qquad y(1-z) = 0.$$

Adding these equations, and eliminating y, we have

$$x(1-z) = 0;$$

诚然，这些结论仅仅把给定的命题置入其他等价的 form 中，--但被接受的"否定换质"方式的职责不过如此，别无其他。

此外，这些换质过程并非初级的，而是比它们更为简单的过程的组合，这些过程更直接地依赖于支配推理的符号工具之使用的那些终极规律和公理。这一评论同样适用于三段论的情形，我们接下来就考虑它。

5. 三段论的本性在具体实例中看得最清楚。假设我们有这些命题，

All X's are Y's,
All Y's are Z's.

由这些我们可以推出结论，

All X's are Z's.

这是一个三段论推论。项X和Z称为端项，Y称为中项。现在可以一般地定义三段论的功能。给定两个命题，其种类已在(1)中列表，并涉及一个中项或共同项Y，它在一个命题中与一个端项X相连，在另一个命题中与一个端项Z相连；要求给出连接端项X和Z的关系。项Y可以以其肯定形式出现，如"所有Y","有些Y"；或以其否定形式出现，如"所有非Y","有些非Y"；在任一命题中，而不考虑它在另一命题中所采取的特殊形式。

在具体实例中，用本论著的方法来分解三段论，没有比这更容易的了。它的分解实际上是把命题系统化归的过程的一种具体应用。取上面给出的例子，我们有，

$$x = vy,$$
$$y = v'z;$$

由此通过代换，

$$x = vv'z,$$

它被解释为

All X's are Z's.

Or, proceeding rigorously in accordance with the method developed in (VIII.7), we deduce

$$x(1-y) = 0, \qquad y(1-z) = 0.$$

Adding these equations, and eliminating y, we have

$$x(1-z) = 0;$$

whence $x = \frac{0}{0}z$, or, All X's are Z's.
And in the same way may any other case be treated.

6. Quitting, however, the consideration of special examples, let us examine the general forms to which all syllogism may be reduced.

Proposition I.
To deduce the general rules of Syllogism.

By the general rules of Syllogism, I here mean the rules applicable to premises admitting of every variety both of quantity and of quality in their subjects and predicates, except the combination of two universal terms in the same proposition. The admissible forms of propositions are therefore those of which a tabular view is given in (1).

Let X and Y be the elements or things entering into the first premiss, Z and Y those involved in the second. Two cases, fundamentally different in character, will then present themselves. The terms involving Y will either be of *like* or of *unlike quality*, those terms being regarded as of like quality when they both speak of "Y's," or both of "Not-Y's," as of unlike quality when one of them speaks of "Y's," and the other of "Not-Y's." Any pair of premises, in which the former condition is satisfied, may be represented by the equations

$$vx = v'y, \tag{1}$$
$$wz = w'y; \tag{2}$$

for we can employ the symbol y to represent either "All Y's," or "All not-Y's," since the interpretation of the symbol is purely conventional. If we employ y in the sense of "All not-Y's," then $1 - y$ will represent "All Y's," and no other change will be introduced. An equal freedom is permitted with respect to the symbols x and z, so that the equations (1) and (2) may, by properly assigning the interpretations of x, y, and z, be made to represent all varieties in the combination of premises dependent upon the *quality* of the respective terms. Again, by assuming proper interpretations to the symbols v, v', w, w', in those equations, all varieties with reference to *quantity* may also be represented. Thus, if we take $v = 1$, and represent by v' a class indefinite, the equation (1) will represent a universal proposition according to the ordinary sense of that term, i. e., a proposition with universal subject and particular predicate. We may, in fact, give to subject and predicate in either premiss whatever *quantities* (using this term in the scholastic sense) we please, except that by hypothesis, they must not both be universal. The system (1), (2), represents, therefore, with perfect generality, the possible combinations of premises which have like middle terms.

7. That our analysis may be as general as the equations to which it is applied, let us, by the method of this work, eliminate y from (1) and (2), and seek the expressions for x, $1 - x$, and vx, in terms of z and of the symbols v, v', w, w'. The above will include all the possible forms of the subject of the conclusion. The form $v(1 - x)$ is excluded, inasmuch as we cannot from the

由此 x = 00z,或者，所有X的都是Z的。
而任何其他情形也可依同样方式处理。
6. 然而，放下对特殊例子的考虑，让我们来考察一切三段论都可化归到的一般形式。

命题一。
推导三段论的一般规则。

我在这里所谓三段论的一般规则，是指适用于其主词和谓词在量与质两方面都容许各种变化的前提的规则，但同一命题中两个全称项的结合除外。因此，可容许的命题形式就是(1)中给出表格视图的那些形式。

设X和Y为进入第一个前提的元素或事物，Z和Y为第二个前提所涉及的。于是会出现两种性质根本不同的情况。涉及Y的项将要么具有相同的性质，要么具有不同的性质，当它们都谈论"Y"或都谈论"非Y"时，这些项被视为具有相同性质；当其中一个谈论"Y"而另一个谈论"非Y"时，则被视为具有不同性质。任何一对满足前一种条件的前提，都可以用方程

$$vx = v'y, \tag{1}$$
$$wz = w'y; \tag{2}$$

来表示，因为我们可以用记号y来表示"所有Y"或"所有非Y"，因为该记号的解释纯粹是约定的。如果我们用y表示"所有非Y"，那么1−y将表示"所有Y"，并且不会引入其他变化。对于记号x和z也允许同样的自由，因此通过适当指定x、y和z的解释，方程(1)和(2)可以表示依赖于各项性质的前提组合的所有变体。再者，通过在这些方程中赋予记号v、v′、w、w′以适当的解释，所有关于量的变体也可以被表示。因此，如果我们取v=1，并用v′表示一个不确定的类，那么方程(1)将按照该词的通常意义表示一个全称命题，即一个具有全称主词和特称谓词的命题。事实上，我们可以赋予任一前提中的主词和谓词以我们想要的任何量(在此使用该词的经院哲学意义)，除了根据假设它们不能同时为全称。因此，系统(1)、(2)完全一般地表示了具有相同中项的前提的可能组合。

7.为了使我们的分析像它所应用的方程一样一般，让我们用本工作的方法从(1)和(2)中消去y，并寻求x、1−x和vx用z以及记号v、v′、w、w′表示的表达式。上述内容将包括结论主词的所有可能形式。形式v(1−x)被排除，因为我们不能从

interpretation vx = Some X's, given in the premises, interpret $v(1-x)$ as Some not-X's. The symbol v, when used in the sense of "some," applies to that term only with which it is connected in the premises.

The results of the analysis are as follows:

$$x = \big[vv'ww' + \frac{0}{0}\{vv'(1-w)(1-w') + ww'(1-v)(1-v') + (1-v)(1-w)\}\big]z + \frac{0}{0}\{vv'(1-w') + 1 - v\}(1-z), \quad \text{(I.)}$$

$$1-x = \big[v(1-v')\{ww' + (1-w)(1-w')\} + v(1-w)w' + \frac{0}{0}\{vv'(1-w)(1-w') + ww'(1-v)(1-v') + (1-v)(1-w)\}\big]z + \big[v(1-w)w' + \frac{0}{0}\{vv'(1-w') + 1 - v\}\big](1-z), \quad \text{(II.)}$$

$$vx = \{vv'ww' + \frac{0}{0}vv'(1-w)(1-w')\}z + \frac{0}{0}(1-w')(1-z). \quad \text{(III.)}$$

Each of these expressions involves in its second member two terms, of one of which z is a factor, of the other $1-z$. But syllogistic inference does not, as a matter of form, admit of contrary classes in its conclusion, as of Z's and not-Z's together.

We must, therefore, in order to determine the rules of that species of inference, ascertain under what conditions the second members of any of our equations are reducible to a single term.

The simplest form is (III.), and it is reducible to a single term if $w' = 1$. The equation then becomes

$$vx = vv'wz, \tag{3}$$

the first member is identical with the extreme in the first premiss; the second is of the same quantity and quality as the extreme in the second premiss. For since $w' = 1$, the second member of (2), involving the middle term y, is universal; therefore, by the hypothesis, the first member is particular, and therefore, the second member of (3), involving the same symbol w in its coefficient, is particular also. Hence we deduce the following law.

Condition of Inference.—One middle term, at least, universal.

Rule of Inference.—Equate the extremes.

From an analysis of the equations (I.) and (II.), it will further appear, that the above is the only condition of syllogistic inference when the middle terms are of like quality. Thus the second member of (I.) reduces to a single term, if $w' = 1$ and $v = 1$; and the second member of (II.) reduces to a single term, if $w' = 1$, $v = 1$, $w = 1$. In each of these cases, it is necessary that $w' = 1$, the solely sufficient condition before assigned.

解释 vx = 前提中给出的某些 X 的事物，把 v(1 – x) 解释为某些不是-X 的事物。记号 v 当在"某些"的意义上使用时，只适用于它在前提中与之相连接的那个项。分析的结果如下：

$$x = \big[vv'ww' + \frac{0}{0}\{vv'(1-w)(1-w') + ww'(1-v)(1-v') + (1-v)(1-w)\}\big]z + \frac{0}{0}\{vv'(1-w') + 1 - v\}(1-z), \quad \text{(I.)}$$

$$1-x = \big[v(1-v')\{ww' + (1-w)(1-w')\} + v(1-w)w' + \frac{0}{0}\{vv'(1-w)(1-w') + ww'(1-v)(1-v') + (1-v)(1-w)\}\big]z + \big[v(1-w)w' + \frac{0}{0}\{vv'(1-w') + 1 - v\}\big](1-z), \quad \text{(II.)}$$

$$vx = \{vv'ww' + \frac{0}{0}vv'(1-w)(1-w')\}z + \frac{0}{0}(1-w')(1-z). \quad \text{(III.)}$$

这些表达式中每一个都在其第二成员中涉及两个项，其中之一以 z 为因子，另一以 1 – z 为因子。但三段论推论在形式上并不容许其结论中出现对立类，如 Z 的事物与不是-Z 的事物并存。

因此，为了确定那种推论方式的规则，我们必须查明：在什么条件下，我们任一方程的第二成员可化归为单一项。

The simplest form is (III.), and it is reducible to a single term if $w' = 1$. The equation then becomes

$$vx = vv'wz, \tag{3}$$

第一成员与第一前提中的端项相同；第二成员在量与质上都与第二前提中的端项相同。因为既然 w ′ = 1，则(2)的第二成员，由于含有中词 y，是全称的；因此，按假说，第一成员是特称的，所以(3)的第二成员，由于在其系数中含有同一记号 w，也是特称的。由此我们推出以下规律。

推论的条件。--至少有一个中词是全称的。推论的规则。--使两端项相等。

从对方程(I.)和(II.)的分析中还将进一步看出，当中间项具有相同性质时，上述是三段论推论的唯一条件。因此，如果w′=1且v=1,(I.)的第二项就化为单项；如果w′=1、v=1、w=1,(II.)的第二项就化为单项。在每一种情况下，都必须有w′=1，即前面所指定的唯一充分条件。

Consider, secondly, the case in which the middle terms are of unlike quality. The premises may then be represented under the forms

$$vx = v'y, \tag{4}$$
$$wz = w'(l-y); \tag{5}$$

and if, as before, we eliminate y, and determine the expressions of x, $1-x$, and vx, we get

$$x = \big[vv'(l-w)w' + \frac{0}{0}\{ww'(1-v) + (1-v)(1-v')(1-w) + v'(1-w)(1-w')\}\big] + \big[vv'w' + \frac{0}{0}\{(1-v)(1-v') + v'(l-w')\}\big](1-z). \quad \text{(IV.)}$$

$$1-x = \big[ww'v + v(1-v')(1-w) + \frac{0}{0}\{ww'(1-v) + (1-v)(1-v')(1-w) + v'(1-w)(1-w')\}\big]z + \big[v(1-v') + \frac{0}{0}\{v'(1-w') + (1-v)(1-v')\}\big](1-z). \quad \text{(V.)}$$

$$vx = \{vv'(1-w)w' + \frac{0}{0}vv'(1-w)(1-w')\}z + \{vv'w' + \frac{0}{0}vv'(1-w')\}(1-z). \quad \text{(VI.)}$$

Now the second member of (VI.) reduces to a single term relatively to z, if $w = 1$, giving

$$vx = \{vv'w' + \frac{0}{0}vv'(1-w')\}(1-z);$$

the second member of which is opposite, both in quantity and quality, to the corresponding extreme, wz, in the second premiss. For since $w = 1$, wz is universal. But the factor vv' indicates that the term to which it is attached is particular, since by hypothesis v and v' are not both equal to 1. Hence we deduce the following law of inference in the case of like middle terms:

First Condition of Inference.—*At least one universal extreme.*

Rule of Inference.—*Change the quantity and quality of that extreme, and equate the result to the other extreme.*

Moreover, the second member of (V.) reduces to a single term if $v' = 1$, $w' = 1$; it then gives

$$1-x = \{vw + \frac{0}{0}(1-v)w\}z.$$

Now since $v' = 1$, $w' = 1$, the middle terms of the premises are both universal, therefore the extremes vx, wz, are particular. But in the conclusion the extreme

其次，考虑中词性质不同的情形。这时前提可表示为以下形式

$$vx = v'y, \tag{4}$$
$$wz = w'(l-y); \tag{5}$$

并且，如果像以前一样消去 y，并确定 x、1 – x 和 vx 的表达式，我们得到

$$x = \big[vv'(l-w)w' + \frac{0}{0}\{ww'(1-v) + (1-v)(1-v')(1-w) + v'(1-w)(1-w')\}\big] + \big[vv'w' + \frac{0}{0}\{(1-v)(1-v') + v'(l-w')\}\big](1-z). \quad \text{(IV.)}$$

$$1-x = \big[ww'v + v(1-v')(1-w) + \frac{0}{0}\{ww'(1-v) + (1-v)(1-v')(1-w) + v'(1-w)(1-w')\}\big]z + \big[v(1-v') + \frac{0}{0}\{v'(1-w') + (1-v)(1-v')\}\big](1-z). \quad \text{(V.)}$$

$$vx = \{vv'(1-w)w' + \frac{0}{0}vv'(1-w)(1-w')\}z + \{vv'w' + \frac{0}{0}vv'(1-w')\}(1-z). \quad \text{(VI.)}$$

现在，(VI.) 的第二成员相对于 z 化归为单一项，如果 w = 1，则给出

$$vx = \{vv'w' + \frac{0}{0}vv'(1-w')\}(1-z);$$

其第二成员在量与质两方面都与第二前提中相应的端项 wz 相反。因为既然 w = 1,wz 是全称的。但因子 vv′表明它所附着的项是特称的，因为按假说 v 和 v′并非都等于 1。由此我们在中词相同的情形下推出以下推论规律：

First Condition of Inference.—*At least one universal extreme.*

Rule of Inference.—*Change the quantity and quality of that extreme, and equate the result to the other extreme.*

Moreover, the second member of (V.) reduces to a single term if $v' = 1$, $w' = 1$; it then gives

$$1-x = \{vw + \frac{0}{0}(1-v)w\}z.$$

现在既然 v′ = 1,w′ = 1，前提的中词都是全称的，因此端项 vx、wz 是特称的。但在结论中，端项

involving x is opposite, both in quantity and quality, to the extreme vx in the first premiss, while the extreme involving z agrees both in quantity and quality with the corresponding extreme wz in the second premiss. Hence the following general law:

SECOND CONDITION OF INFERENCE.—*Two universal middle terms.*

RULE OF INFERENCE.—*Change the quantity and quality of either extreme, and equate the result to the other extreme unchanged.*

There are in the case of unlike middle terms no other conditions or rules of syllogistic inference than the above. Thus the equation (IV.), though reducible to the form of a syllogistic conclusion, when $w = 1$ and $v = 1$, does not thereby establish a new condition of inference; since, by what has preceded, the single condition $v = 1$, or $w = 1$, would suffice.

8. The following examples will sufficiently illustrate the general rules of syllogism above given:

1. All Y's are X's.
All Z's are Y's.

This belongs to Case 1. All Y's is the universal middle term. The extremes equated give as the conclusion

All Z's are X's;

the universal term, All Z's, becoming the subject; the particular term (some) X's, the predicate.

2. All X's are Y's.
No Z's are Y's.

The proper expression of these premises is

All X's are Y's.
All Z's are not-Y's.

They belong to Case 2, and satisfy the first condition of inference. The middle term, Y's, in the first premiss, is particular-affirmative; that in the second premiss, not-Y's, particular-negative. If we take All Z's as the universal extreme, and change its quantity and quality according to the rule, we obtain the term Some not-Z's, and this equated with the other extreme, All X's, gives,

All X's are not-Z's, i. e., No X's are Z's.

If we commence with the other universal extreme, and proceed similarly, we obtain the equivalent result,

No Z's are X's.

含有 x 的那个在量与质两方面都与第一前提中的端项 vx 相反，而含有 z 的那个端项在量与质两方面都与第二前提中相应的端项 wz 一致。由此得出以下一般规律：

推理的第二个条件。- 两个全称中项。
推论的规则。--改变任一端项的量与质，并使结果与另一未变的端项相等。

在中词不同的情形下，除上述之外没有其他三段论推论的条件或规则。因此，方程(IV.) 虽然在 w = 1 且 v = 1 时可化归为三段论结论的形式，却并不由此确立一个新的推论条件；因为根据前面所述，单是条件 v = 1，或 w = 1，就已足够。

8. 以下例子将充分说明上面给出的一般三段论规则：

1. All Y's are X's.
All Z's are Y's.

这属于情形 1。所有 Y 的事物是全称中词。使端项相等便给出结论

All Z's are X's;

全称项“所有 Z 的事物”成为主词；特称项“(某些) X 的事物”成为谓词。

2. All X's are Y's.
No Z's are Y's.

这些前提的恰当表达是

All X's are Y's.
所有Z都不是Y。

它们属于情形 2，并满足推论的第一条件。第一前提中的中词 Y 的事物是特称肯定的；第二前提中的中词不是-Y 的事物是特称否定的。如果我们取“所有 Z 的事物”作为全称端项，并按规则改变其量与质，我们得到项“某些不是-Z 的事物”，再使之与另一端项“所有 X 的事物”相等，便给出，

所有X都不是Z，即没有X是Z。

如果我们从另一个全称端项开始，并同样进行，我们便得到等价的结果，

No Z's are X's.

3. All Y's are X's.

All not-Y's are Z's.

Here also the middle terms are unlike in quality. The premises therefore belong to Case 2, and there being two universal middle terms, the second condition of inference is satisfied. If by the rule we change the quantity and quality of the first extreme, (some) X's, we obtain All not-X's, which, equated with the other extreme, gives

All not-X's are Z's.

The reverse order of procedure would give the equivalent result,

All not-Z's are X's.

The conclusions of the two last examples would not be recognised as valid in the scholastic system of Logic, which virtually requires that the subject of a proposition should be affirmative. They are, however, perfectly legitimate in themselves, and the rules by which they are determined form undoubtedly the most general canons of syllogistic inference. The process of investigation by which they are deduced will probably appear to be of needless complexity; and it is certain that they might have been obtained with greater facility, and without the aid of any symbolical instrument whatever. It was, however, my object to conduct the investigation in the most general manner, and by an analysis thoroughly exhaustive. With this end in view, the brevity or prolixity of the method employed is a matter of indifference. Indeed the analysis is not properly that of the syllogism, but of a much more general combination of propositions; for we are permitted to assign to the symbols v, v', w, w', any class-interpretations that we please. To illustrate this remark, I will apply the solution (I.) to the following imaginary case:

Suppose that a number of pieces of cloth striped with different colours were submitted to inspection, and that the two following observations were made upon them:

1st. That every piece striped with white and green was also striped with black and yellow, and *vice versâ*.

2nd. That every piece striped with red and orange was also striped with blue and yellow, and *vice versâ*. Suppose it then required to determine how the pieces marked with green stood affected with reference to the colours white, black, red, orange, and blue.

Here if we assume v = white, x = green, v' = black, y = yellow, w = red, z = orange, w' = blue, the expression of our premises will be

$$vx = v'y,$$
$$wz = w'y,$$

agreeing with the system (1) (2). The equation (I.) then leads to the following conclusion:

3. All Y's are X's.

所有非Y都是Z。

这里中项在性质上也不相同。因此前提属于情形2，并且由于有两个全称中项，推论的第二条件得到满足。如果我们按规则改变第一个端项的量与性质，(某些) X 的，我们得到所有非- X 的，将其与另一个端项等同，得到

所有非- X 的都是 Z 的。

若将程序倒序进行，也会得到等价的结果，

所有非- Z 的都是 X。

最后两个例子的结论在经院逻辑学体系中不会被认作有效，因为该体系实际上要求命题的主词应当是肯定的。然而，这些结论本身是完全正当的，而据以确定它们的规则无疑构成了三段论推论最普遍的准则。用以推导出这些结论的探究过程或许会显得不必要地复杂；而且可以肯定，它们本可以更容易地获得，无需任何符号工具的帮助。然而，我的目标是以最普遍的方式进行探究，并通过一种彻底穷尽的分析。鉴于这一目的，所采用方法的简洁或冗长是无关紧要的。事实上，这一分析并非恰当意义上的三段论分析，而是远为普遍的命题组合的分析；因为我们被允许对记号 v、v′、w、w′赋予我们任意选择的类解释。为说明这一评述，我将把解(I.)应用于以下虚构的情形：

假设若干块带有不同颜色条纹的布被提交检验，并且对它们作出了以下两个观察：

第一，凡带有白色和绿色条纹的，也都带有黑色和黄色条纹，反之亦然。

第二，每一块带有红色和橙色条纹的布片，也都带有蓝色和黄色条纹，反之亦然。假设接下来需要确定带有绿色标记的布片相对于白色、黑色、红色、橙色和蓝色这些颜色处于何种关系。

这里如果我们假定 v = 白色，x = 绿色，v′ = 黑色，y = 黄色，w = 红色，z = 橙色，w′ = 蓝色，我们前提的表达式将是

$$vx = v'y,$$
$$wz = w'y,$$

与系统(1)(2)一致。方程(I.) 然后导致以下结论：

Pieces striped with green are either striped with orange, white, black, red, and blue, together, all pieces possessing which character are included in those striped with green; or they are striped with orange, white, and black, but not with red or blue; or they are striped with orange, red, and blue, but not with white or black; or they are striped with orange, but not with white or red; or they are striped with white and black, but not with blue or orange; or they are striped neither with white nor orange.

Considering the nature of this conclusion, neither the symbolical expression (I.) by which it is conveyed, nor the analysis by which that expression is deduced, can be considered as needlessly complex.

9. The form in which the doctrine of syllogism has been presented in this chapter affords ground for an important observation. We have seen that in each of its two great divisions the entire discussion is reducible, so far, at least, as concerns the determination of rules and methods, to the analysis of a pair of equations, viz., of the system (1), (2), when the premises have like middle terms, and of the system (4), (5), when the middle terms are unlike. Moreover, that analysis has been actually conducted by a method founded upon certain general laws deduced immediately from the constitution of language, Chap. II. confirmed by the study of the operations of the human mind, Chap. III., and proved to be applicable to the analysis of all systems of equations whatever, by which propositions, or combinations of propositions, can be represented, Chap. VIII. Here, then, we have the means of definitely resolving the question, whether syllogism is indeed the fundamental type of reasoning,—whether the study of its laws is co-extensive with the study of deductive logic. For if it be so, some indication of the fact must be given in the systems of equations upon the analysis of which we have been engaged. It cannot be conceived that syllogism should be the one essential process of reasoning, and yet the manifestation of that process present nothing indicative of this high quality of pre-eminence. No sign, however, appears that the discussion of all systems of equations expressing propositions is involved in that of the particular system examined in this chapter. And yet writers on Logic have been all but unanimous in their assertion, not merely of the supremacy, but of the universal sufficiency of syllogistic inference in deductive reasoning. The language of Archbishop Whately, always clear and definite, and on the subject of Logic entitled to peculiar attention, is very express on this point. "For Logic," he says, "which is, as it were, the Grammar of Reasoning, does not bring forward the regular Syllogism as a *distinct mode of argumentation*, designed to be *substituted* for any other mode; but as the form to which *all* correct reasoning may be ultimately reduced."[2] And Mr. Mill, in a chapter of his System of Logic, entitled, "Of Ratiocination or Syllogism," having enumerated the ordinary forms of syllogism, observes, "All valid ratiocination, all reasoning by which from general propositions previously admitted, other propositions, equally or less general, are inferred, may be exhibited in some of the above forms." And again: "We are therefore at liberty, in conformity with the general opinion of logicians, to consider the two elementary forms of the first

[2]Elements of Logic, p. 13, ninth edition.

带有绿色条纹的棋子，或者同时带有橙色、白色、黑色、红色和蓝色条纹，凡具有该性质的一切棋子都包含在带有绿色条纹的棋子之中；或者它们带有橙色、白色和黑色条纹，但不带红色或蓝色条纹；或者它们带有橙色、红色和蓝色条纹，但不带白色或黑色条纹；或者它们带有橙色条纹，但不带白色或红色条纹；或者它们带有白色和黑色条纹，但不带蓝色或橙色条纹；或者它们既不带白色条纹也不带橙色条纹。

考虑到这个结论的性质，无论是传达它的符号表达式(I.)，还是推导出该表达式的分析，都不能被认为是多余的复杂。

9. 本章呈现三段论学说的形式，为一项重要的观察提供了依据。我们已经看到，在其两大划分的每一部分中，整个讨论至少就规则和方法的确定而言，都可归结为一对方程的分析，即当诸前提具有相同的中项时为系统(1)、(2)的分析，而当诸中项不同时为系统(4)、(5)的分析。此外，这一分析实际上是通过一种方法进行的，该方法建立在某些直接由语言的构造推演而出的一般规律之上(第ii章)，经对人类心灵活动的研究所确认(第iii章)，并被证明适用于分析一切方程系统--凡命题或命题的组合得以由之表示者(第viii章)。于是，我们在此拥有了明确解决这一问题的手段：三段论是否确实是推理的基本类型--对其规律的研究是否与演绎逻辑的研究外延相同。因为倘若如此，这一事实的某种迹象必定会在我们所从事分析的那些方程系统中显示出来。不可设想三段论竟是推理的唯一本质过程，而该过程的表现却丝毫不显示这种卓越的至高性质。然而，没有任何迹象表明，讨论一切表达命题的方程系统都包含于本章所考察的特定系统的讨论之中。然而逻辑学著作家们几乎一致地断言三段论推论在演绎推理中不仅具有至高地位，而且具有普遍充分性。沃特利大主教的言辞总是清晰而明确，在逻辑学问题上尤应予以特别注意，在这一点上说得非常肯定。"因为逻辑学，"他说，"可以说就是推理的语法，它提出规范的三段论，并非作为一种独特的论证方式，旨在取代任何其他方式；而是作为一切正确推理最终都可归结于其下的形式。"[2]而穆勒先生在其《逻辑体系》中题为"论推理或三段论"的一章里，列举了三段论的通常形式之后评论道："一切有效的推理，一切由先前承认的一般命题推出其他同等或较少一般性的命题的推理，都可以在上述某些形式中展示出来。"又说："因此，我们不妨遵照逻辑学家们的一般意见，把第一格的两种基本形式

2《逻辑学原理》，第13页，第九版。

figure as the universal types of all correct ratiocination." In accordance with these views it has been contended that the science of Logic enjoys an immunity from those conditions of imperfection and of progress to which all other sciences are subject;[3] and its origin from the travail of one mighty mind of old has, by a somewhat daring metaphor, been compared to the mythological birth of Pallas.

As Syllogism is a species of elimination, the question before us manifestly resolves itself into the two following ones:—1st. Whether all elimination is reducible to Syllogism; 2ndly. Whether deductive reasoning can with propriety be regarded as consisting only of elimination. I believe, upon careful examination, the true answer to the former question to be, that it is always theoretically possible so to resolve and combine propositions that elimination may subsequently be effected by the syllogistic canons, but that the process of reduction would in many instances be constrained and unnatural, and would involve operations which are not syllogistic. To the second question I reply, that reasoning cannot, except by an arbitrary restriction of its meaning, be confined to the process of elimination. No definition can suffice which makes it less than the aggregate of the methods which are founded upon the laws of thought, as exercised upon propositions; and among those methods, the process of elimination, eminently important as it is, occupies only a place.

Much of the error, as I cannot but regard it, which prevails respecting the nature of the Syllogism and the extent of its office, seems to be founded in a disposition to regard all those truths in Logic as *primary* which possess the character of simplicity and intuitive certainty, without inquiring into the relation which they sustain to other truths in the Science, or to general methods in the Art, of Reasoning. Aristotle's *dictum de omni et nullo* is a self-evident principle, but it is not found among those *ultimate* laws of the reasoning faculty to which all other laws, however plain and self-evident, admit of being traced, and from which they may in strictest order of scientific evolution be deduced. For though of every science the fundamental truths are usually the most simple of apprehension, yet is not that simplicity the *criterion* by which their title to be regarded as fundamental must be judged. This must be sought for in the nature and extent of the structure which they are capable of supporting. Taking this view, Leibnitz appears to me to have judged correctly when he assigned to the "principle of contradiction" a fundamental place in Logic;[4] for we have seen the consequences of that law of thought of which it is the axiomatic expression (III. 15). But enough has been said upon the nature of deductive inference and upon its constitutive elements. The subject of induction may probably receive some attention in another part of this work.

10. It has been remarked in this chapter that the ordinary treatment of hypothetical, is much more defective than that of categorical, propositions. What is commonly termed the hypothetical syllogism appears, indeed, to be no syllogism at all.

Let the argument—

[3]Introduction to Kant's "Logik."

[4]Nouveaux Essais sur l'entendement humain. Liv. IV. cap. 2. Theodicée Pt. I. sec. 44.

被视为一切正确推理的普遍形式。"根据这些观点，有人主张逻辑科学享有免于所有其他科学所服从的不完善与进步条件的特权；[3]而其起源于古代一位伟大心智的艰辛劳作，则通过一个颇为大胆的比喻，被比作帕拉斯的 mythological 诞生。

既然三段论是消元的一种，那么摆在我们面前的问题显然归结为以下两个：第一，是否一切消元都可还原为三段论；第二，演绎推理是否可以恰当地被视为仅仅由消元构成。我相信，经过仔细考察，对前一个问题的真正答案是：在理论上总是可能如此分解和组合命题，使得消元随后能够按照三段论的准则来实现，但在许多情况下，这种还原过程将是勉强的、不自然的，并且会涉及并非三段论的操作。对第二个问题，我的回答是：除非任意限制推理一词的含义，否则推理不能被局限于消元过程。任何定义，若使它小于那些建立在思维规律之上、作用于命题的方法之总和，便不能成立；而在这些方法之中，消元过程尽管极为重要，却只占一席之地。

关于三段论的性质及其职司范围，流行着许多错误--我不得不如此看待--其中大部分似乎源于一种倾向：把逻辑中所有具有简单性和直觉确定性特征的真理都视为基本真理，而不探究它们与科学中其他真理、或与推理技艺中一般方法的关系。亚里士多德的*dictum de omni et nullo*是一条自明原则，但它并不属于那些终极的推理能力规律--所有其他规律，无论多么明白自明，都可追溯到这些规律，并可按照最严格的科学演进次序从它们演绎出来。因为，尽管每一门科学的基本真理通常最易于领会，然而这种简单性并不是判断它们是否有资格被视为基本真理的标准。这一标准必须从它们所能支撑的结构之性质与范围中去寻找。持此观点，莱布尼茨在我看来判断正确，当他将"矛盾原则"赋予逻辑中的基本地位时；[4]因为我们已经看到那条思维规律的种种后果，而矛盾原则正是它的公理式表达(III.15)。但关于演绎推论的性质及其构成元素，已经说得够多了。归纳这一主题或许将在本书另一部分受到一些关注。

10. 本章已指出，对假言命题的通常处理远比对直言命题的处理更为不完善。通常所谓的假言三段论，实际上似乎根本不是三段论。

设论证--

3 康德《逻辑学》导论

4 《人类理解新论》。第iv卷，第2章。《神正论》第一部分，第44节。

If A is B, C is D,
But A is B,
Therefore C is D,

be put in the form—

If the proposition X is true, Y is true,
But X is true,
Therefore Y is true;

wherein by X is meant the proposition A is B, and by Y, the proposition C is D. It is then seen that the premises contain only two terms or elements, while a syllogism essentially involves three. The following would be a genuine hypothetical syllogism:

If X is true, Y is true;
If Y is true, Z is true;
$\therefore$ If X is true, Z is true.

After the discussion of secondary propositions in a former part of this work, it is evident that the forms of hypothetical syllogism must present, in every respect, an exact counterpart to those of categorical syllogism. *Particular* Propositions, such as, "Sometimes if X is true, Y is true," may be introduced, and the conditions and rules of inference deduced in this chapter for categorical syllogisms may, without abatement, be interpreted to meet the corresponding cases in hypotheticals.

11. To what final conclusions are we then led respecting the nature and extent of the scholastic logic? I think to the following: that it is not a science, but a collection of scientific truths, too incomplete to form a system of themselves, and not sufficiently fundamental to serve as the foundation upon which a perfect system may rest. It does not, however, follow, that because the logic of the schools has been invested with attributes to which it has no just claim, it is therefore undeserving of regard. A system which has been associated with the very growth of language, which has left its stamp upon the greatest questions and the most famous demonstrations of philosophy, cannot be altogether unworthy of attention. Memory, too, and usage, it must be admitted, have much to do with the intellectual processes; and there are certain of the canons of the ancient logic which have become almost inwoven in the very texture of thought in cultured minds. But whether the mnemonic forms, in which the particular rules of conversion and syllogism have been exhibited, possess any real utility,—whether the very skill which they are supposed to impart might not, with greater advantage to the mental powers, be acquired by the unassisted efforts of a mind left to its own resources,—are questions which it might still be not unprofitable to examine. As concerns the particular results deduced in this chapter, it is to be observed, that they are solely designed to aid the inquiry concerning the nature of the ordinary or scholastic logic, and its relation to a more perfect theory of deductive reasoning.

If A is B, C is D,
But A is B,
Therefore C is D,

be put in the form—

If the proposition X is true, Y is true,
But X is true,
Therefore Y is true;

其中 X 表示命题 A 是 B，而 Y 表示命题 C 是 D。于是可以看出，前提只包含两个词项或元素，而三段论本质上涉及三个。下面将是一个真正的假言三段论：

If X is true, Y is true;
If Y is true, Z is true;
$\therefore$ If X is true, Z is true.

在本书前一部分讨论了次等命题之后，显而易见，假言三段论的形式在各方面都必定与直言三段论的形式精确对应。特称命题，例如"有时若X为真，则Y为真"，可以被引入，而本章为直言三段论所推得的推论条件和规则，可以毫无减损地被解释以适用于假言三段论中的相应情形。

11. 那么，关于经院逻辑的性质与范围，我们最终得出什么结论呢?我认为如下：它不是一门科学，而是一些科学真理的汇集，过于不完整，不足以自身构成一个体系，也不够基本，不足以充当一个完美体系得以立足的基础。然而，并不能因此就说，既然经院逻辑被赋予了它并无正当权利拥有的属性，它便不值得重视。一个与语言的发展本身相伴、在哲学最重大的问题和最著名的论证上留下印记的体系，不可能完全不值得注意。还必须承认，记忆和习惯与理智过程大有关系；古代逻辑的某些规则几乎已经织入有教养心灵的思维纹理之中。但是，那些用以展示换位和三段论特定规则的助记形式是否具有任何真正的效用--它们被认为传授的那种技能，是否不能以对心智能力更大的益处，由一个任其自身资源的心智在无辅助的努力中获得--这些问题仍然值得考察，并非无益。至于本章所推得的特定结果，应当指出，它们仅仅旨在辅助关于普通逻辑或经院逻辑之性质、及其与更完善的演绎推理理论之关系的探究。

Chapter XVI

ON THE THEORY OF PROBABILITIES

1. Before the expiration of another year just two centuries will have rolled away since Pascal solved the first known question in the theory of Probabilities, and laid, in its solution, the foundations of a science possessing no common share of the attraction which belongs to the more abstract of mathematical speculations. The problem which the Chevalier de Méré, a reputed gamester, proposed to the recluse of Port Royal (not yet withdrawn from the interests of science[1] by the more distracting contemplation of the "greatness and the misery of man"), was the first of a long series of problems, destined to call into existence new methods in mathematical analysis, and to render valuable service in the practical concerns of life. Nor does the interest of the subject centre merely in its mathematical connexion, or its associations of utility. The attention is repaid which is devoted to the theory of Probabilities as an independent object of speculation,—to the fundamental modes in which it has been conceived,—to the great secondary principles which, as in the contemporaneous science of Mechanics, have gradually been annexed to it,—and, lastly, to the estimate of the measure of perfection which has been actually attained. I speak here of that perfection which consists in unity of conception and harmony of processes. Some of these points it is designed very briefly to consider in the present chapter.

2. A distinguished writer[2] has thus stated the fundamental definitions of the science:

"The probability of an event is the reason we have to believe that it has taken place, or that it will take place."

"The measure of the probability of an event is the ratio of the number of cases favourable to that event, to the total number of cases favourable or contrary, and all equally possible" (equally likely to happen).

From these definitions it follows that the word *probability*, in its mathematical acceptation, has reference to the state of our knowledge of the circumstances under which an event may happen or fail. With the degree of information

[1] See in particular a letter addressed by Pascal to Fermat, who had solicited his attention to a mathematical problem (Port Royal, par M. de Sainte Beuve); also various passages in the collection of Fragments published by M. Prosper Faugère.

[2] Poisson, Recherches sur la Probabilitè des Jugemens.

第十六章

论概率理论

1. 再过不到一年，就将满整整两个世纪，自帕斯卡解决了概率论中第一个已知问题，并在其解答中奠定了一门科学的基础--这门科学所拥有的吸引力，绝不亚于较为抽象的数学思辨所具有的吸引力。骑士德·梅雷--一个据说好赌的人--向波尔·罗亚尔的隐士(此人尚未因对"人的伟大与悲惨"的更令人分心的沉思而退出对科学的关注[1])提出的那个问题，是一长串问题中的第一个，这些问题注定要催生数学分析中的新方法，并在生活的实际事务中提供宝贵的服务。这一主题的趣味也并不仅仅在于其数学关联或实用联系。投入概率论研究的注意力会得到回报--无论是将其作为思辨的独立对象，还是考察它被构想的基本方式，考察那些如同在同时代的力学中那样逐渐被附加于它的重大次级原则，最后，还是评估实际已达到的完善程度。我在这里所说的完善，是指概念的统一与过程的和诸所构成的完善。本章打算非常简要地考察其中若干要点。

2. 一位杰出的作者[2]这样陈述了这门科学的基本定义：

"一个事件的概率，就是我们相信它已经发生或将会发生的理由。"

"一个事件的概率的度量，是有利于该事件的情况数目与有利或不利且全部同等可能(同等可能发生)的情况总数之比。"

从这些定义可以推出，概率一词在其数学含义上，关涉到我们关于一个事件可能发生或不发生所处情况的知识状态。随着信息程度的

1 特别参见帕斯卡致费马的一封信，费马曾提请其注意一个数学问题(《波尔罗亚尔》，圣伯夫先生著)；另见各处段落

M.Prosper Faugère 出版的《残篇集》。

2 泊松，《关于判决概率的研究》。

which we possess concerning the circumstances of an event, the reason we have to think that it will occur, or, to use a single term, our *expectation* of it, will vary. Probability is expectation founded upon partial knowledge. A perfect acquaintance with *all* the circumstances affecting the occurrence of an event would change expectation into certainty, and leave neither room nor demand for a theory of probabilities.

3. Though our expectation of an event grows stronger with the increase of the ratio of the number of the known cases favourable to its occurrence to the whole number of equally possible cases, favourable or unfavourable, it would be unphilosophical to affirm that the strength of that expectation, viewed as an emotion of the mind, is capable of being referred to any numerical standard. The man of sanguine temperament builds high hopes where the timid despair, and the irresolute are lost in doubt. As subjects of scientific inquiry, there is some analogy between *opinion* and *sensation*. The thermometer and the carefully prepared photographic plate indicate, not the intensity of the sensations of heat and light, but certain physical circumstances which accompany the production of those sensations. So also the theory of probabilities contemplates the numerical measure of the circumstances upon which expectation is founded; and this object embraces the whole range of its legitimate applications. The rules which we employ in life-assurance, and in the other statistical applications of the theory of probabilities, are altogether independent of the *mental* phænomena of expectation. They are founded upon the assumption that the future will bear a resemblance to the past; that under the same circumstances the same event will tend to recur with a definite numerical frequency; not upon any attempt to submit to calculation the strength of human hopes and fears.

Now experience actually testifies that events of a given species do, under given circumstances, tend to recur with definite frequency, whether their true causes be known to us or unknown. Of course this tendency is, in general, only manifested when the area of observation is sufficiently large. The judicial records of a great nation, its registries of births and deaths, in relation to age and sex, &c., present a remarkable uniformity from year to year. In a given language, or family of languages, the same sounds, and successions of sounds, and, if it be a written language, the same characters and successions of characters recur with determinate frequency. The key to the rude Ogham inscriptions, found in various parts of Ireland, and in which no distinction of words could at first be traced, was, by a strict application of this principle, recovered.[3]The same method, it is understood, has been applied [4] to the deciphering of the cuneiform records recently disentombed from the ruins of Nineveh by the enterprise of Mr. Layard.

4. Let us endeavour from the above statements and definitions to form a conception of the legitimate object of the theory of Probabilities.

[3]The discovery is due to the Rev. Charles Graves, Professor of Mathematics in the University of Dublin.– *Vide* Proceedings of the Royal Irish Academy, Feb. 14, 1848. Professor Graves informs me that he has verified the principle by constructing sequence tables for all the European languages.

[4]By the learned Orientalist, Dr. Edward Hincks.

我们关于一个事件所处情况的了解，我们据以认为它将会发生的理由，或者用一个词来说，我们对它的期望，将会变化。概率是建立在部分知识之上的期望。对影响一个事件发生的全部情况的完善认识，将把期望变为确定性，既不留有也不要求概率论。

3. 尽管我们对某一事件的预期，随着有利于其发生的已知事例数目与全部同等可能事例(无论有利或不利)数目之比增大而增强，但若断言这种预期强度--被视为心灵的一种情感--能够被归附于任何数值标准，那将是不合哲学的。性情乐观的人在怯懦者绝望之处构筑高昂的希望，而优柔寡断者则迷失于疑虑之中。作为科学探究的对象，意见与感觉之间存在着某种类比。温度计和精心制备的照相底片所指示的，并非热与光的感觉强度，而是伴随那些感觉之产生而出现的某些物理境况。概率论同样也考虑预期所依据的境况的数值度量；而这一对象涵盖了其正当应用的全部范围。我们在人寿保险以及概率论的其他统计应用中所使用的规则，完全独立于预期的心理现象。它们建立在如下假设之上：未来将与过去相似；在相同境况下，同一事件将以确定的数值频率趋于重现；而非建立在任何试图将人类希望与恐惧的强度交付计算的尝试之上。

如今经验确实证明，某一给定种类的事件在给定境况下，无论其真正原因为我们所知或未知，都趋于以确定的频率重现。当然，这种趋势一般只有在观察范围足够大时才得以显现。一个伟大民族的司法记录、其按年龄与性别等登记的出生与死亡登记册，年复一年呈现出显著的齐一性。在某一给定语言或语系中，相同的音及其连续、以及(若为书面语言)相同的字符及其连续，以确定的频率重现。爱尔兰各地所发现的粗陋的欧甘铭文--其中起初无法辨识出任何词的区分--其解读之钥正是通过严格应用这一原理而得以恢复的。[3]据了解，同样的方法已被应用于[4]解读最近由莱亚德先生的进取精神从尼尼微废墟中发掘出的楔形文字记录。

4. 让我们从上述陈述和定义出发，努力形成关于概率论合法对象的概念。

3 这一发现归功于都柏林大学数学教授查尔斯·格雷夫斯牧师(Rev.Charles Graves)。--参见《爱尔兰皇家科学院院刊》，1848年2月14日。格雷夫斯教授告知我，他已通过为所有欧洲语言构造序列表验证了这一原理。

4 由博学的东方学家爱德华·欣克斯博士。

Probability, it has been said, consists in the expectation founded upon a particular kind of knowledge, viz., the knowledge of the relative frequency of occurrence of events. Hence the probabilities of events, or of combinations of events, whether deduced from a knowledge of the particular constitution of things under which they happen, or derived from the long-continued observation of a past series of their occurrences and failures, constitute, in all cases, our data. The probability of some connected event, or combination of events, constitutes the corresponding *quæsitum*, or object sought. Now in the most general, yet strict meaning of the term "event," every combination of events constitutes also an event. The simultaneous occurrence of two or more events, or the occurrence of an event under given conditions, or in any conceivable connexion with other events, is still an event. Using the term in this liberty of application, the object of the theory of probabilities might be thus defined. Given the probabilities of any events, of whatever kind, to find the probability of some other event connected with them.

5. Events may be distinguished as simple or compound, the latter term being applied to such events as consist in a combination of simple events (I. 13). In this manner we might define it as the practical end of the theory under consideration to determine the probability of some event, simple or compound, from the given probabilities of other events, simple or compound, with which, by the terms of its definition, it stands connected.

Thus if it is known from the constitution of a die that there is a probability, measured by the fraction $\frac{1}{6}$, that the result of any particular throw will be an ace, and if it is required to determine the probability that there shall occur one ace, and only one, in two successive throws, we may state the problem in the order of its *data* and its *quæsitum*, as follows:

First Datum.—Probability of the event that the first throw will give an ace $= \frac{1}{6}$.

Second Datum.—Probability of the event that the second throw will give an ace $= \frac{1}{6}$.

Quæsitum.—Probability of the event that either the first throw will give an ace, and the second not an ace; or the first will not give an ace, and the second will give one.

Here the two data are the probabilities of simple events defined as the first throw giving an ace, and the second throw giving an ace. The quæsitum is the probability of a compound event,—a certain disjunctive combination of the simple events involved or implied in the data. Probably it will generally happen, when the numerical conditions of a problem are capable of being deduced, as above, from the constitution of things under which they exist, that the data will be the probabilities of *simple* events, and the *quæsitum* the probability of a *compound* event dependent upon the said simple events. Such is the case with a class of problems which has occupied perhaps an undue share of the attention of those who have studied the theory of probabilities, viz., games of chance and skill, in the former of which some physical circumstance, as the

有人说，概率在于建立在一种特定知识之上的预期，即关于事件发生的相对频率的知识。因此，事件的概率、或事件组合的概率，无论是由关于事件发生所依据的事物特定构造的知识推得，还是由对其过去一系列发生与未发生的长期观察导出，在一切情况下都构成我们的数据。某一相关联事件、或事件组合的概率，构成相应的所求项，即所寻求的对象。如今，在"事件"一词最一般却又严格的意义上，每一事件组合也构成一个事件。两个或多个事件的同时发生，或某一事件在给定条件下、或在与其他事件的任何可设想联系中的发生，仍然是一个事件。在这一应用的自由度上使用该词，概率论的对象或可如此定义：给定任何事件的概率，无论其种类如何，求与它们相关联的某一其他事件的概率。

5. 事件可以分为简单事件或复合事件，后一词用于指由简单事件组合而成的事件(I.13)。以这种方式，我们可以将其定义为：所考察理论的实践目的，是从其他事件--简单或复合--的给定概率出发，确定某个事件--简单或复合--的概率，而按照其定义的条件，该事件与那些其他事件处于联系之中。

因此，如果从骰子的构造已知存在一个概率，以分数$\frac{1}{6}$，即任何特定投掷的结果将是一个幺点，并且如果要求确定在连续两次投掷中出现一个幺点且仅出现一个幺点的概率，我们可以按照其数据和所求的顺序陈述该问题如下：

第一数据。--第一次投掷将给出幺点这一事件的概率 $= \frac{1}{6}$.

第二数据。--第二次投掷将给出幺点这一事件的概率 $= \frac{1}{6}$.

所求。--如下事件的概率：或者第一次投掷给出幺点而第二次不给幺点；或者第一次不给幺点而第二次给出幺点。

这里两个数据是简单事件的概率，即第一次投掷出现幺点、第二次投掷出现幺点。所求项是一个复合事件的概率--即数据中所涉及或隐含的简单事件的某种析取组合。很可能通常会出现这样的情况：当一个问题的数值条件能够如上所述从其存在所依据的事物构造中推得时，数据将是简单事件的概率，而所求项则是依赖于所述简单事件的一个复合事件的概率。研究概率论的人们或许过度关注的一类问题即属此情形，即机遇与技巧的游戏；在前者中，某种物理境况，如

constitution of a die, determines the probability of each possible step of the game, its issue being some definite combination of those steps; while in the latter, the relative dexterity of the players, supposed to be known *à priori*, equally determines the same element. But where, as in statistical problems, the elements of our knowledge are drawn, not from the study of the constitution of things, but from the registered observations of Nature or of human society, there is no reason why the data which such observations afford should be the probabilities of simple events. On the contrary, the occurrence of events or conditions in marked combinations (indicative of some secret connexion of a causal character) suggests to us the propriety of making such concurrences, profitable for future instruction by a numerical record of their frequency. Now the data which observations of this kind afford are the probabilities of compound events. The solution, by some general method, of problems in which such data are involved, is thus not only essential to the perfect development of the theory of probabilities, but also a perhaps necessary condition of its application to a large and practically important class of inquiries.

6. Before we proceed to estimate to what extent known methods may be applied to the solution of problems such as the above, it will be advantageous to notice, that there is another form under which all questions in the theory of probabilities may be viewed; and this form consists in substituting for *events* the propositions which assert that those events have occurred, or will occur; and viewing the element of numerical probability as having reference to the *truth* of those *propositions*, not to the *occurrence* of the *events* concerning which they make assertion. Thus, instead of considering the numerical fraction p as expressing the probability of the occurrence of an event E, let it be viewed as representing the probability of the truth of the proposition X, which asserts that the event E will occur. Similarly, instead of any probability, q, being considered as referring to some compound event, such as the concurrence of the events E and F, let it represent the probability of the truth of the proposition which asserts that E and F will jointly occur; and in like manner, let the transformation be made from disjunctive and hypothetical combinations of events to disjunctive and conditional propositions. Though the new application thus assigned to probability is a necessary concomitant of the old one, its adoption will be attended with a practical advantage drawn from the circumstance that we have already discussed the theory of propositions, have defined their principal varieties, and established methods for determining, in every case, the amount and character of their mutual dependence. Upon this, or upon some equivalent basis, any general theory of probabilities must rest. I do not say that other considerations may not in certain cases of applied theory be requisite. The data may prove insufficient for definite solution, and this defect it may be thought necessary to *supply* by hypothesis. Or, where the statement of large numbers is involved, difficulties may arise *after the solution*, from this source, for which special methods of treatment are required. But in every instance, some form of the general problem as above stated (Art. 4) is involved, and in the discussion of that problem the proper and peculiar work of the theory consists. I desire it to be observed, that to this object the

骰子的构造，决定了游戏每一可能步骤的概率，其结果则是那些步骤的某种确定组合；而在后者中，游戏者的相对灵巧--被假定为先验已知--同样决定了同一元素。但是，当如在统计问题中那样，我们的知识元素并非取自对事物构造的研究，而是取自对自然或人类社会的记录观察时，则没有理由认为这类观察所提供的数据应当是简单事件的概率。相反，事件或境况以显著组合的方式发生(暗示某种因果性的秘密联系)，向我们提示：应当通过对其频率的数值记录，使这类同时发生之事有益于未来的指导。如此，这类观察所提供的数据便是复合事件的概率。因此，通过某种一般方法求解涉及此类数据的问题，不仅对于概率论的完善展开是本质性的，而且对于将其应用于一大类具有实际重要性的探究而言，或许也是必要的条件。

6. 在我们着手估计已知方法可在多大程度上应用于解决上述这类问题之前，先注意到以下一点将是有益的：概率论中的所有问题都可以从另一种形式来看待；这种形式在于用断言那些事件已经发生或将会发生的命题，来替换事件；并把数值概率的元素视为关涉那些命题的真理，而非关涉它们所断言的那些事件的发生。于是，与其把数值分数 p 看作表达事件 E 发生的概率，不如把它看作表示命题 X 为真的概率，该命题断言事件 E 将会发生。同样，与其把任何概率 q 看作关涉某个复合事件，例如事件 E 与 F 的共现，不如让它表示断言 E 与 F 将共同发生的命题为真的概率；并以同样方式，把事件的析取组合与假言组合转换为析取命题与条件命题。尽管这样赋予概率的新应用是旧应用的必要伴随物，但采纳它将会带来一种实际的好处，这来自以下情况：我们已经讨论了命题的理论，定义了它们的主要种类，并建立了在每一场合确定其相互依赖的程度与性质的方法。任何一般的概率理论都必须立足于这一点，或立足于某种等价的基础之上。我并不是说，在应用理论的某些场合中，其他考虑可能并非必需。数据可能被证明不足以给出确定的解答，而人们可能认为有必要通过假说来补充这一缺陷。或者，在涉及大数陈述的地方，在解答之后可能由此产生困难，需要特殊的处理方法。但在每一实例中，都涉及如上所述(第4条)的一般问题的某种形式，而理论的正当且特有的工作就在于对该问题的讨论。我希望人们注意到，正是为了这一目标，

investigations of the following chapters are mainly devoted. It is not intended to enter, except incidentally, upon questions involving supplementary hypotheses, because it is of primary importance, even with reference to such questions (I. 17), that a general method, founded upon a solid and sufficient basis of theory, be first established.

7. The following is a summary, chiefly taken from Laplace, of the principles which have been applied to the solution of questions of probability. They are consequences of its fundamental definitions already stated, and may be regarded as indicating the degree in which it has been found possible to render those definitions available.

PRINCIPLE 1st. If p be the probability of the occurrence of any event, $1-p$ will be the probability of its non-occurrence.

2nd. The probability of the concurrence of two independent events is the product of the probabilities of those events.

3rd. The probability of the concurrence of two dependent events is equal to the product of the probability of one of them by the probability that if that event occur, the other will happen also.

4th. The probability that if an event, E, take place, an event, F, will also take place, is equal to the probability of the concurrence of the events E and F, divided by the probability of the occurrence of E.

5th. The probability of the occurrence of one or the other of two events which cannot concur is equal to the sum of their separate probabilities.

6th. If an observed event can only result from some one of n different causes which are *à priori* equally probable, the probability of any one of the causes is a fraction whose numerator is the probability of the event, on the hypothesis of the existence of that cause, and whose denominator is the sum of the similar probabilities relative to all the causes.

7th. The probability of a future event is the sum of the products formed by multiplying the probability of each cause by the probability that if that cause exist, the said future event will take place.

8. Respecting the extent and the relative sufficiency of these principles, the following observations may be made.

1st. It is always possible, by the due combination of these principles, to express the probability of a compound event, dependent in any manner upon independent simple events whose distinct probabilities are given. A very large proportion of the problems which have been actually solved are of this kind, and the difficulty attending their solution has not arisen from the insufficiency of the indications furnished by the theory of probabilities, but from the need of an analysis which should render those indications available when functions of large numbers, or series consisting of many and complicated terms, are thereby introduced. It may, therefore, be fully conceded, that all problems having for their data the probabilities of independent simple events fall within the scope of received methods.

2ndly. Certain of the principles above enumerated, and especially the sixth and seventh, do not presuppose that all the data are the probabilities of simple events. In their peculiar application to questions of causation, they do, however,

以下各章的探讨主要致力于此。除附带涉及外，不打算进入包含补充假说的问题，因为首要的是--即使就此类问题而言(I.17)--首先建立一个建立在坚实而充分的理论基础之上的一般方法。

7. 以下主要是取自拉普拉斯的一个概要，说明已被应用于解决概率问题的各项原理。它们是已经陈述过的基本定义的推论，并可被视为指示了使那些定义可供使用的可能程度。

第一原理。如果 p 是任何事件发生的概率，那么 1 – p 将是其不发生的概率。

第二，两个独立事件同时发生的概率是这两个事件概率的乘积。

第三，两个相依事件同时发生的概率，等于其中一个事件的概率与在该事件发生的条件下另一事件也将发生的概率之乘积。

第四。如果事件 E 发生，则事件 F 也将发生这一概率，等于事件 E 与 F 同时发生的概率除以 E 发生的概率。

第五，两个不能同时发生的事件中任一事件发生的概率，等于它们各自概率之和。

第六，如果一个观察到的事件只能由`先验地同等可能的n个不同原因中的某一个所导致，那么其中任一原因的概率就是一个分数，其分子是在该原因存在的假说下该事件的概率，其分母是相对于所有原因的类似概率之和。

第七。一个未来事件的概率，等于将每一原因的概率乘以若该原因存在则所述未来事件将会发生的概率，再把所得各乘积相加之和。

8. 关于这些原理的范围和相对充分性，可以作出以下观察。

1. 通过这些原理的恰当组合，总是可以表达一个复合事件的概率，该复合事件以任何方式依赖于其各自概率已给定的独立简单事件。实际已解决的大部分问题都属于这一类，而解决它们的困难并非源于概率论所提供指示的不充分，而是源于需要一种分析，使得当由此引入大数的函数或由许多复杂项组成的级数时，那些指示能够被利用。因此，可以充分承认，所有以独立简单事件的概率为其数据的问题都属于公认方法的范围。

其次，上述列举的某些原理，尤其是第六条和第七条，并不预先假定所有数据都是简单事件的概率。然而，在它们对因果性问题的特殊应用中，它们确实如此，

assume, that the causes of which they take account are mutually exclusive, so that no combination of them in the production of an effect is possible. If, as before explained, we transfer the numerical probabilities from the events with which they are connected to the propositions by which those events are expressed, the most general problem to which the aforesaid principles are applicable may be stated in the following order of *data* and *quæsita*.

DATA.

1st. The probabilities of the n conditional propositions:

If the cause A_1 exist, the event E will follow;
" A_2 " E "
.....
" A_n " E "

2nd. The condition that the antecedents of those propositions are mutually conflicting.

REQUIREMENTS.

The probability of the truth of the proposition which declares the occurrence of the event E; also, when that proposition is known to be true, the probabilities of truth of the several propositions which affirm the respective occurrences of the causes $A_1, A_2 \ldots A_n$.

Here it is seen, that the data are the probabilities of a series of compound events, expressed by *conditional* propositions. But the system is obviously a very limited and particular one. For the antecedents of the propositions are subject to the condition of being mutually exclusive, and there is but one consequent, the event E, in the whole system. It does not follow, from our ability to discuss such a system as the above, that we are able to resolve problems whose data are the probabilities of *any* system of conditional propositions; far less that we can resolve problems whose data are the probabilities of *any system of propositions whatever*. And, viewing the subject in its material rather than its formal aspect, it is evident, that the hypothesis of *exclusive* causation is one which is not often realized in the actual world, the phænomena of which seem to be, usually, the products of complex causes, the amount and character of whose co-operation is unknown. Such is, without doubt, the case in nearly all departments of natural or social inquiry in which the doctrine of probabilities holds out any new promise of useful applications.

9. To the above principles we may add another, which has been stated in the following terms by the Savilian Professor of Astronomy in the University of Oxford.[5]

"Principle 8. If there be any number of mutually exclusive hypotheses, $h_1, h_2, h_3, \ldots$ of which the probabilities relative to a particular state of information are $p_1, p_2, p_3, \ldots$ and if new information be given which changes

[5] On certain Questions relating to the Theory of Probabilities; by W. F. Donkin, M. A., F. R. S., &c. Philosophical Magazine, May, 1851.

假定他们所考虑的那些原因是互斥的，以致它们在产生一个结果时不可能有任何组合。如果像前面所解释的那样，我们把数值的概率从与它们相联系的那些事件转移到表达那些事件的那些命题上，那么上述原理所可应用的最一般的问题，可以按照数据与所求的如下次序来陈述。

数据。

第一，条件命题的概率：

如果原因 A^1 存在，事件 E 将随之发生；
？ A^2 ？ E ？
.....
？ 一个 n ？ E ？

第二，那些命题的前件相互冲突的条件。

要求。

宣告事件E发生的那个命题为真的概率；并且，当那个命题已知为真时，肯定各原因A^1,A^2...An各自发生的那些命题为真的概率。

这里可以看到，数据是一系列复合事件的概率，由条件命题表达。但这个系统显然是一个非常有限且特殊的系统。因为各命题的前件都受制于彼此互斥这一条件，而且整个系统中只有一个后件，即事件 E。从我们能够讨论如上这样的系统，并不能推出我们能够解决其数据为任何条件命题系统的概率的问题；更不能推出我们能够解决其数据为任何命题系统的概率的问题。而且，从质料而非形式的方面来看待这一主题，显然，互斥因果关系的假说并不是在现实世界中经常实现的，现实世界的现象似乎通常是复杂原因的产物，而这些原因协同作用的程度与性质是未知的。毫无疑问，在自然探究或社会探究的几乎一切部门中，只要概率学说为有用的应用提供了任何新的希望，情况就是如此。

9.在上述原理之外，我们还可以补充另一条原理，它已由牛津大学萨维尔天文学教授用如下措辞陈述过。[5]

"原理8. 若有任意数目的互斥假说，h^1,h^2,h^3,...它们相对于某一特定信息状态的概率为p^1,p^2,p^3,...，并且若给出新的信息，它改变了

5 论与概率论有关的某些问题；作者 W.F.Donkin，文学硕士，F. R.S.等，《哲学杂志》，1851年5月。

the probabilities of some of them, suppose of h_{m+1} and all that follow, without having otherwise *any reference to the rest*; then the probabilities of these latter have the same ratios to one another, after the new information, that they had before, that is,

$$p'_1 : p'_2 : p'_3 \ldots : p'_m = p_1 : p_2 : p_3 \ldots : p_m,$$

where the accented letters denote the values after the new information has been acquired."

This principle is apparently of a more fundamental character than the most of those before enumerated, and perhaps it might, as has been suggested by Professor Donkin, be regarded as axiomatic. It seems indeed to be founded in the very definition of the measure of probability, as "the ratio of the number of cases favourable to an event to the total number of cases favourable or contrary, and all equally possible." For, adopting this definition, it is evident that in whatever proportion the number of equally possible cases is diminished, while the number of favourable cases remains unaltered, in exactly the same proportion will the probabilities of any events to which these cases have reference be increased. And as the new hypothesis, viz., the diminution of the number of possible cases without affecting the number of them which are favourable to the events in question, increases the probabilities of those events in a constant ratio, the relative measures of those probabilities remain unaltered. If the principle we are considering be then, as it appears to be, inseparably involved in the very definition of probability, it can scarcely, *of itself*, conduct us further than the attentive study of the definition would alone do, in the solution of problems. From these considerations it appears to be doubtful whether, without some aid of a different kind from any that has yet offered itself to our notice, any considerable advance, either in the theory of probabilities as a branch of speculative knowledge, or in the practical solution of its problems can be hoped for. And the establishment, *solely* upon the basis of any such collection of principles as the above, of a method universally applicable to the solution of problems, without regard either to the number or to the nature of the propositions involved in the expression of their data, seems to be impossible. For the attainment of such an object other elements are needed, the consideration of which will occupy the next chapter.

其中某些假说的概率，假定是hm+1及其后所有各项的概率，而并不以其他方式涉及其余各项；那么这些后者的概率在获得新信息之后彼此之间的比值，与它们在此之前所具有的比值相同，即

$$p'_1 : p'_2 : p'_3 \ldots : p'_m = p_1 : p_2 : p_3 \ldots : p_m,$$

其中带重音的字母表示获得新信息之后的值。"

这条原理显然比前面所列举的大多数原理具有更为根本的性质，也许正如唐金(Donkin)教授所提示的那样，它可以被视为公理。它似乎确实建立在概率测度的定义本身之中，即"有利于某一事件的场合数与有利于或不利于该事件、且全都同等可能的场合总数之比"。因为，采纳这一定义，显然，同等可能的场合数无论以何种比例减少，而有利的场合数保持不变，那么这些场合所关涉的任何事件的概率就将以完全相同的比例增加。并且，既然新的假说，即减少可能场合的数目而不影响其中有利于所论事件的那些场合的数目，以恒定的比率增加那些事件的概率，那么这些概率的相对测度就保持不变。那么，如果我们正在考察的这条原理如它看上去那样不可分割地包含在概率的定义本身之中，它本身在解决问题时几乎不能把我们引得比单纯仔细研究该定义所能引得更远。从这些考虑来看，以下一点似乎是可疑的：若无某种与我们迄今所注意到的任何援助都不同的援助，是否还能指望在作为思辨知识一个分支的概率理论中，或在其问题的实际解决中，取得任何可观的进展。而仅仅立足于如上任何这样一组原理的汇集，建立一种普遍适用于解决问题的方法，而既不考虑其数据的表达中所涉及的命题的数目，也不考虑其性质，这似乎是不可能的。为达到这样一个目标，还需要其他元素，对它们的考察将占据下一章。

Chapter XVII

DEMONSTRATION OF A GENERAL METHOD FOR THE SOLUTION OF PROBLEMS IN THE THEORY OF PROBABILITIES.

1. It has been defined (XVI. 2), that "the measure of the probability of an event is the ratio of the number of cases favourable to that event, to the total number of cases favourable or unfavourable, and all equally possible." In the following investigations the term probability will be used in the above sense of "measure of probability."

From the above definition we may deduce the following conclusions.

I. When it is certain that an event will occur, the probability of that event, in the above mathematical sense, is 1. For the cases which are favourable to the event, and the cases which are possible, are in this instance the same.

Hence, also, if p be the probability that an event x will happen, $1-p$ will be the probability that the said event will not happen. To deduce this result directly from the definition, let m be the number of cases favourable to the event x, n the number of cases possible, then $n-m$ is the number of cases unfavourable to the event x. Hence, by definition,

$$\frac{m}{n} = \text{ probability that } x \text{ will happen.}$$
$$\frac{n-m}{n} = \text{ probability that } x \text{ will not happen.}$$

But

$$\frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p.$$

II. The probability of the concurrence of any two events is the product of the probability of either of those events by the probability that if that event occur, the other will occur also.

Let m be the number of cases favourable to the happening of the first event, and n the number of equally possible cases unfavourable to it; then the probability of the first event is, by definition, $\frac{m}{m+n}$. Of the m cases favourable

第十七章

关于概率论中问题求解的一般方法的论证。

1. 前面已经定义过(XVI.2),"一个事件的概率的度量，是有利于该事件的情形数目与有利或不利的、且全都同等可能的情形总数之比。"在以下研究中，概率一词将在上述"概率的度量"的意义上使用。由上述定义我们可以推出以下结论。

I.当一个事件必定发生时，该事件在上述数学意义上的概率为1。因为在这个场合，有利于该事件的情形与可能的情形是相同的。

因此，若p是事件x将发生的概率，则1−p将是所述事件不发生的概率。为直接从定义推出这一结果，设m为有利于事件x的情形数目，n为可能的情形数目，则n−m是不利于事件x的情形数目。因此，按定义，

$\frac{m}{n}$=概率即x将发生。n−m=x将不发生的概率。

$$\frac{n-m}{n}$$

但是

$$\frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p.$$

II.任意两个事件同时发生的概率，是其中任一事件的概率与在该事件发生的条件下另一事件也将发生的概率之积。

Let m be the number of cases favourable to the happening of the first event, and n the number of equally possible cases unfavourable to it; then the probability of the first event is, by definition, $\frac{m}{m+n}$. Of the m cases favourable

to the first event, let l cases be favourable to the conjunction of the first and second events, then, by definition, $\frac{l}{m}$ is the probability that if the first event happen, the second also will happen. Multiplying these fractions together, we have

$$\frac{m}{m+n} \times \frac{l}{m} = \frac{l}{m+n}.$$

But the resulting fraction $\frac{l}{m+n}$ has for its numerator the number of cases favourable to the conjunction of events, and for its denominator, the number $m+n$ of possible cases. Therefore, it represents the probability of the joint occurrence of the two events.

Hence, if p be the probability of any event x, and q the probability that if x occur y will occur, the probability of the conjunction xy will be pq.

III. The probability that if an event x occur, the event y will occur, is a fraction whose numerator is the probability of their joint occurrence, and denominator the probability of the occurrence of the event x.

This is an immediate consequence of Principle 2nd.

IV. The probability of the occurrence of some one of a series of exclusive events is equal to the sum of their separate probabilities.

For let n be the number of possible cases; m_1 the number of those cases favourable to the first event; m_2 the number of cases favourable to the second, &c. Then the separate probabilities of the events are $\frac{m_1}{n}$, $\frac{m_2}{n}$, &c. Again, as the events are exclusive, none of the cases favourable to one of them is favourable to another; and, therefore, the number of cases favourable to some one of the series will be $m_1 + m_2 \ldots$, and the probability of some one of the series happening will be $\frac{m_1 + m_2 \cdots}{n}$. But this is the sum of the previous fractions, $\frac{m_1}{n}$, $\frac{m_2}{n}$, &c. Whence the principle is manifest. 2. DEFINITION.—Two events are said to be independent when the probability of the happening of either of them is unaffected by our expectation of the occurrence or failure of the other.

From this definition, combined with Principle II., we have the following conclusion:

V. The probability of the concurrence of two independent events is equal to the product of the separate probabilities of those events.

For if p be the probability of an event x, q that of an event y regarded as quite independent of x, then is q also the probability that if x occur y will occur. Hence, by Principle II., pq is the probability of the concurrence of x and y

Under the same circumstances, the probability that x will occur and y not occur will be $p(1-q)$. For p is the probability that x will occur, and $1-q$ the probability that y will not occur. In like manner $(1-p)(1-q)$ will be the probability that both the events fail of occurring.

3. There exists yet another principle, different in kind from the above, but necessary to the subsequent investigations of this chapter, before proceeding to the explicit statement of which I desire to make one or two preliminary observations.

to the first event, let l cases be favourable to the conjunction of the first and

第二个事件，那么，按定义，$\frac{l}{m}$是那个概率即如果那个第一个事件发生，第二个也将发生。将这些分数相乘，我们得到

$$\frac{m}{m+n} \times \frac{l}{m} = \frac{l}{m+n}.$$

但所得分数$\frac{l}{m+n}$以有利于事件同时发生的情形数目为其分子，并以可能情形的数目m+n为分母。因此，它表示两个事件同时发生的概率。

因此，若p是任一事件x的概率，而q是若x发生则y将发生的概率，则合取xy的概率将是pq。

III.若事件x发生则事件y将发生的概率，是一个分数，其分子是它们同时发生的概率，分母是事件x发生的概率。

这是第二原理的直接推论。

IV.一系列互斥事件中某一个发生的概率，等于它们各自概率之和。

因为设n为可能情形的数目；m^1为其中有利于第一个事件的情形数目；m^2为有利于第二个事件的情形数目，等等。那么这些事件各自的概率是$\frac{m1}{n}$,$\frac{m2}{n}$等等。又，由于那个事件是互斥的，有利于其中之一的场合没有一个有利于另一个；因此，有利于该系列中某一个的场合数将是 $m^1 + m^2 \ldots$，而该系列中某一个发生的概率将是$\frac{m1+m2..}{n}$。但这就是前面那些分数之和，$\frac{m1}{n}$,$\frac{m2}{n}$等等。由此该原理便显而易见了。2. 定义。--当两个事件中任一事件发生的概率不受我们对另一事件发生或不发生的预期的影响时，这两个事件被称为独立的。

由这一定义，结合原理II.，我们得到以下结论：

V.两个独立事件同时发生的概率，等于这两个事件各自概率之积。

因为若 p 是事件 x 的概率，q 是事件 y 的概率，而 y 被视为与 x 完全独立，那么 q 也是当 x 发生时 y 将发生的概率。因此，根据原理 II.,pq 是 x 与 y 同时发生的概率。在同样的情况下，x 发生而 y 不发生的概率将是 p(1 –q)。因为 p 是 x 将发生的概率，而 1 –q 是 y 将不发生的概率。同样地，(1 –p)(1 –q) 将是两个事件都不发生的概率。

3.还存在另一条原理，其性质与上述原理不同，但对本章随后的研究是必要的；在着手明确陈述它之前，我想先作一两点初步的说明。

I would, in the first place, remark that the distinction between simple and compound events is not one founded in the nature of events themselves, but upon the mode or connexion in which they are presented to the mind. How many separate particulars, for instance, are implied in the terms "To be in health," "To prosper," &c., each of which might still be regarded as expressing a "simple event"? The prescriptive usages of language, which have assigned to particular combinations of events single and definite appellations, and have left unnumbered other combinations to be expressed by corresponding combinations of distinct terms or phrases, is essentially arbitrary. When, then, we designate as simple events those which are expressed by a single verb, or by what grammarians term a simple sentence, we do not thereby imply any real simplicity in the events themselves, but use the term solely with reference to grammatical expression. 4. Now if this distinction of events, as simple or compound, is not founded in their real nature, but rests upon the accidents of language, it cannot affect the question of their mutual dependence or independence. If my knowledge of two simple events is confined to this particular fact, viz., that the probability of the occurrence of one of them is p, and that of the other q; then I regard the events as independent, and thereupon affirm that the probability of their joint occurrence is pq. But the ground of this affirmation is not that the events are simple ones, but that the data afford no information whatever concerning any connexion or dependence between them. When the probabilities of events are given, but all information respecting their dependence withheld, the mind regards them as independent. And this mode of thought is equally correct whether the events, judged according to actual expression, are simple or compound, i.e., whether each of them is expressed by a single verb or by a combination of verbs.

5. Let it, however, be supposed that, together with the probabilities of certain events, we possess some definite information respecting their possible combinations. For example, let it be known that certain combinations are excluded from happening, and therefore that the remaining combinations alone are possible. Then still is the same general principle applicable. The mode in which we avail ourselves of this information in the calculation of the probability of any conceivable issue of events depends not upon the nature of the events whose probabilities and whose limits of possible connexion are given. It matters not whether they are simple or compound. It is indifferent from what source, or by what methods, the knowledge of their probabilities and of their connecting relations has been derived. We must regard the events as independent of any connexion beside that of which we have information, *deeming it of no consequence whether such information has been explicitly conveyed to us in the data, or thence deduced by logical inference.* And this leads us to the statement of the general principle in question, viz.:

VI. The events whose probabilities are given are to be regarded as independent of any connexion but such as is either expressed, or necessarily implied, in the data; and the mode in which our knowledge of that connexion is to be employed is independent of the nature of the source from which such knowledge has been derived.

我首先要指出，简单事件与复合事件之间的区分并非建立在事件本身的性质之中，而是建立在它们呈现于心灵的方式或联系之上。例如，“身体健康”、“兴旺发达”等项中蕴含着多少独立的特称，而其中每一个仍然可以被视为表达了一个“简单事件”？语言的规定性用法——它将单一而确定的名称赋予特定的事件组合，而将无数其他组合留给由不同项或短语的相应组合来表达——本质上是任意的。因此，当我们把那些由单个动词、或由语法学家所称的简单句所表达的事件称为简单事件时，我们并不因此暗示这些事件本身具有任何真正的简单性，而仅仅是参照语法表达来使用这个项。4. 现在，如果这种将事件分为简单或复合的区分并非建立在其真实性质之中，而是依赖于语言的偶然特征，那么它就不能影响它们相互依赖或独立的问题。如果我对两个简单事件的知识仅限于这一特定事实，即其中一个事件发生的概率为p，另一个为q；那么我将这些事件视为独立的，并据此断言它们共同发生的概率为pq。但这种断言的根据并非这些事件是简单事件，而是数据没有提供任何关于它们之间任何联系或依赖的信息。当事件的概率被给出，但所有关于其依赖关系的信息被扣留时，心灵将它们视为独立的。而这种思维方式同样是正确的，无论这些事件按照实际表达来判断是简单的还是复合的，即无论它们每一个是由单个动词表达还是由动词的组合表达。

5. 然而，假定除了某些事件的概率之外，我们还拥有关于它们可能组合的某些确定信息。例如，已知某些组合被排除而不能发生，因此只有其余的组合是可能的。那么，同样的普遍原理仍然适用。我们在计算任何可以设想的事件结局的概率时利用这一信息的方式，并不取决于那些其概率及其可能联系的限制被给出的事件的性质。它们是简单的还是复合的，无关紧要。关于它们的概率以及它们之间联系关系的知识来自何种来源、通过何种方法获得，也无关紧要。我们必须把这些事件视为独立于我们所获信息之外的任何联系，认为这种信息究竟是在数据中明确传达给我们的，还是由此通过逻辑推论得出的，都无关紧要。这就引导我们陈述所讨论的普遍原理，即：

VI.其概率已给出的那些事件，应被视为独立于任何联系，除非这种联系在数据中已被表达或必然隐含；而我们运用关于那种联系的知识的方式，独立于这种知识所由获得的来源的性质。

The practical importance of the above principle consists in the circumstance, that whatever may be the nature of the events whose probabilities are given,—whatever the nature of the event whose probability is sought, we are always able, by an application of the Calculus of Logic, to determine the expression of the latter event as a definite combination of the former events, and definitely to assign the whole of the implied relations connecting the former events with each other. In other words, we can determine what that combination of the given events is whose probability is required, and what combinations of them are alone possible. It follows then from the above principle, that we can reason upon those events as if they were simple events, whose conditions of possible combination had been directly given by experience, and of which the probability of some definite combination is sought. The possibility of a general method in probabilities depends upon this reduction.

6. As the investigations upon which we are about to enter are based upon the employment of the Calculus of Logic, it is necessary to explain certain terms and modes of expression which are derived from this application.

By the event x, I mean that event of which the proposition which affirms the occurrence is symbolically expressed by the equation

$$x = 1.$$

By the event $\phi(x, y, z, \dots)$, I mean that event of which the occurrence is expressed by the equation

$$\phi(x, y, z, \dots) = 1.$$

Such an event may be termed a compound event, in relation to the simple events x, y, z, which its conception involves. Thus, if x represent the event "It rains," y the event "It thunders," the separate occurrences of those events being expressed by the logical equations

$$x = 1, \quad y = 1,$$

then will $x(1-y) + y(1-x)$ represent the event or state of things denoted by the Proposition, "It either rains or thunders, but not both;" the expression of that state of things being

$$x(1-y) + y(1-x) = 1.$$

If for brevity we represent the function $phi(x, y, z, \dots)$, used in the above acceptation by V, it is evident (VI. 13) that the law of duality

$$V(1-V) = 0,$$

will be identically satisfied.

The simple events x, y, z will be said to be "conditioned" when they are not free to occur in every possible combination; in other words, when some compound event depending upon them is precluded from occurring. Thus the events denoted by the propositions, "It rains," "It thunders," are "conditioned"

上述原理的实际重要性在于这样一种情况：无论那些其概率被给出的事件的性质如何--无论那个其概率被寻求的事件的性质如何，我们总是能够通过应用逻辑演算，把后一事件表达为前一些事件的确定组合，并明确地指定连接前一些事件彼此之间的全部隐含关系。换言之，我们能够确定所要求其概率的是给定事件的何种组合，以及它们的哪些组合是唯一可能的。于是，由上述原理可以推出，我们可以对这些事件进行推理，就好像它们是简单事件一样，其可能组合的条件已由经验直接给出，而其中某个确定组合的概率是被寻求的。概率中一般方法的可能性就取决于这一还原。

6.由于我们即将着手的研究以逻辑演算的运用为基础，因此有必要解释某些源自这一应用的术语和表达方式。

所谓事件x，我指的是这样一个事件：肯定其发生的那个命题由方程

$$x = 1.$$

By the event $\phi(x, y, z, \dots)$, I mean that event of which the occurrence is expressed by the equation

$$\phi(x, y, z, \dots) = 1.$$

以记号方式表达出来。这样一个事件，相对于其概念所包含的简单事件x、y、z，可以称为复合事件。因此，若x表示事件"下雨",y表示事件"打雷"，这些事件各自的发生由逻辑方程

$$x = 1, \quad y = 1,$$

来表达，那么x(1 – y) + y(1 – x)将表示命题"或者下雨或者打雷，但不同时发生"所指称的事件或事态；该事态的表达为

$$x(1-y) + y(1-x) = 1.$$

为简洁起见，若我们用 V 表示上述意义下所使用的函数 phi(x, y, z, ...)，则显然(VI.13)对偶律

$$V(1-V) = 0,$$

将恒被满足。

当简单事件 x、y、z 不能自由地以一切可能的组合出现时，亦即当依赖于它们的某个复合事件被排除而无法发生时，这些简单事件便被称为"受条件的"。因此，由命题"下雨"、"打雷"所表示的事件是"受条件的"

if the event denoted by the proposition, "It thunders, but does not rain," is excluded from happening, so that the range of possible is less than the range of conceivable combination. Simple unconditioned events are by definition independent.

Any compound event is similarly said to be conditioned if it is assumed that it can only occur under a certain condition, that is, in combination with some other event constituting, by its presence, that condition.

7. We shall proceed in the natural order of thought, from simple and unconditioned, to compound and conditioned events.

Proposition I.

1st. *If p, q, r are the respective probabilities of any unconditioned simple events x, y, z, the probability of any compound event V will be $[V]$, this function $[V]$ being formed by changing, in the function V, the symbols x, y, z into p, q, r, &c.*

2ndly. *Under the same circumstances, the probability that if the event V occur, any other event V' will also occur, will be $\frac{[VV']}{V}$, wherein $[VV']$ denotes the result obtained by multiplying together the logical functions V and V', and changing in the result x, y, z, &c. into p, q, r, &c.*

Let us confine our attention in the first place to the possible combinations of the two simple events, x and y, of which the respective probabilities are p and q. The primary combinations of those events (V.11), and their corresponding probabilities, are as follows:

EVENTS.		PROBABILITIES.
xy,	Concurrence of x and y,	pq.
$x(1-y)$,	Occurrence of x without y,	$p(1-q)$.
$(1-x)y$,	Occurrence of y without x,	$(1-p)q$.
$(1-x)(1-y)$,	Conjoint failure of x and y,	$(1-p)(1-q)$.

We see that in these cases the probability of the compound event represented by a constituent is the same function of p and q as the logical expression of that event is of x and y; and it is obvious that this remark applies, whatever may be the number of the simple events whose probabilities are given, and whose *joint existence or failure* is involved in the compound event of which we seek the probability.

Consider, in the second place, any *disjunctive* combination of the above constituents. The compound event, expressed in ordinary language as the occurrence of "either the event x without the event y, or the event y without the event x" is symbolically expressed in the form $x(1-y)+y(1-x)$, and its probability, determined by Principles IV. and V., is $p(1-q)+q(1-p)$. The latter of these expressions is the same function of p and q as the former is of x and y. And it is obvious that this is also a particular illustration of a general rule.

如果由命题"打雷，但不下雨"所表示的事件被排除而不会发生，以致可能的范围小于可设想的组合的范围。简单的无条件事件按定义是独立的。

任何复合事件同样地被称为受条件的，如果假定它只能在一定条件下发生，即与某个其他事件相结合而发生，而该其他事件以其出现构成那个条件。

7. 我们将按照思想的自然顺序进行，从简单的、无条件的事件到复合的、受条件的事件。

命题一。

1. 若 ， ，分别是任意无条件简单事件 ， 的概率，则任意复合事件的概率将为[V] ，此函数[V] 由在函数中将记号 ， 变为 ， ， ，&c.而成。

2. 在同样的情况下，若事件发生，则任何其他事件也将发生的概率，将为[VV']即把逻辑函数与相乘，并在结果中将 ， ， ，&c.变为 ， ， ，&c.所得的结果。$\frac{[VV']}{V}$ *denotes*

让我们首先把注意力限于两个简单事件 x 与 y 的可能组合，它们的各自概率为 p 与 q。这些事件的原初组合(V.11)及其相应的概率如下：

EVENTS.		PROBABILITIES.
xy,	x 与 y 同时发生，x 发生而 y 不发生，y 发生而 x 不发生，x 与 y 共同不发生，	pq.
$x(1-y)$,		$p(1-q)$.
$(1-x)y$,		$(1-p)q$.
$(1-x)(1-y)$,		$(1-p)(1-q)$.

我们看到，在这些情形中，由一个成分所表示的复合事件的概率，是 p 与 q 的同一函数，正如该事件的逻辑表达式是 x 与 y 的同一函数一样；而且显然，无论其概率已给出的简单事件的数目是多少，也无论这些事件的共同存在或不发生牵涉到我们所求其概率的那个复合事件之中，这一评述都适用。

其次，考虑上述成分的任何析取组合。用日常语言表达为"或者事件 x 发生而事件 y 不发生，或者事件 y 发生而事件 x 不发生"的复合事件，用符号表示为 x(1 – y) + y(1 – x)，其概率由原理 iv.和 v.确定，为 p(1 – q) + q(1 – p)。后一表达式是 p 和 q 的同一函数，正如前一表达式是 x 和 y 的同一函数。显然，这也是某条普遍规则的一个具体例证。

The events which are expressed by any two or more constituents are mutually exclusive. The only possible combination of them is a *disjunctive* one, expressed in ordinary language by the conjunction *or*, in the language of symbolical logic by the sign +. Now the probability of the occurrence of some one out of a set of mutually exclusive events is the sum of their separate probabilities, and is expressed by connecting the expressions for those separate probabilities by the sign +. Thus the law above exemplified is seen to be *general*. The probability of any unconditioned event V will be found by changing in V the symbols $x, y, z, \ldots$ into $p, q, r, \ldots$

8. Again, by Principle III., the probability that if the event V occur, the event V' will occur with it, is expressed by a fraction whose numerator is the probability of the joint occurrence of V and V', and denominator the probability of the occurrence of V.

Now the expression of that event, or state of things, which is constituted by the joint occurrence of the events V and V', will be formed by multiplying together the expressions V and V' according to the rules of the Calculus of Logic; since whatever constituents are found in both V and V' will appear in the product, and no others. Again, by what has just been shown, the probability of the event represented by that product will be determined by changing therein x, y, z into $p, q, r, \ldots$ Hence the numerator sought will be what $[VV']$ by definition represents. And the denominator will be $[V]$, wherefore

$$\text{Probability that if } V \text{ occur, } V' \text{ will occur with it } = \frac{[VV']}{[V]}.$$

9. For example, if the probabilities of the simple events x, y, z are p, q, r respectively, and it is required to find the probability that if either x or y occur, then either y or z will occur, we have for the logical expressions of the antecedent and consequent—

$$\text{1st. Either } x \text{ or } y \text{ occurs, } x(1-y)+y(1-x).$$
$$\text{2nd. Either } y \text{ or } z \text{ occurs, } y(1-z)+z(1-y).$$

If now we multiply these two expressions together according to the rules of the Calculus of Logic, we shall have for the expression of the concurrence of antecedent and consequent,

$$xz(1-y)+y(1-x)(1-z).$$

Changing in this result x, y, z into p, q, r, and similarly transforming the expression of the antecedent, we find for the probability sought the value

$$\frac{pr(1-q)+q(1-p)(1-r)}{p(1-q)+q(1-p)}.$$

The special function of the calculus, in a case like the above, is to supply the office of the reason in determining what are the conjunctures involved at once in the consequent and the antecedent. But the advantage of this application

由任何两个或多个成分所表达的事件是互斥的。它们唯一可能的组合是一个析取组合，在普通语言中由连词或表达，在符号逻辑的语言中由记号+表达。现在，从一组互斥事件中某个事件发生的概率是它们各自概率之和，并通过用记号+连接那些各自概率的表达式来表达。因此，上述例示的规律被视为是普遍的。任何无条件事件V的概率将通过把V中的记号x, y, z, ...变为p, q, r, ...而求得

8. 又，根据原理 iii.，若事件 V 发生，则事件 V ′将随之发生的概率，用一个分数表示，其分子是 V 与 V ′共同发生的概率，分母是 V 发生的概率。

现在，由事件V和V ′的共同发生所构成的那个事件或事物状态的表达式，将按照逻辑演算的规则，通过把表达式V和V ′相乘而形成；因为凡在V和V ′两者中都被发现的成分，都将出现在乘积中，且没有其他成分。再者，根据刚才所表明的，由那个乘积所表示的事件的概率，将通过把其中的x, y, z变为p, q, r, ...来确定。因此，所求的分子将是[V V ′]按定义所表示的东西。而分母将是[V]，因此

$$\text{Probability that if } V \text{ occur, } V' \text{ will occur with it } = \frac{[VV']}{[V]}.$$

9. 例如，若简单事件 x, y, z 的概率分别为 p, q, r，而要求找出若 x 或 y 发生，则 y 或 z 将发生的概率，我们便得到前件与后件的逻辑表达式如下--

$$\text{1st. Either } x \text{ or } y \text{ occurs, } x(1-y)+y(1-x).$$
$$\text{2nd. Either } y \text{ or } z \text{ occurs, } y(1-z)+z(1-y).$$

现在，若我们按照逻辑演算的规则把这两个表达式相乘，我们便得到前件与后件共同发生的表达式，

$$xz(1-y)+y(1-x)(1-z).$$

在此结果中把 x, y, z 换成 p, q, r,，并同样地变换前件的表达式，我们便求得所求概率的值为

$$\frac{pr(1-q)+q(1-p)(1-r)}{p(1-q)+q(1-p)}.$$

在如上情形中，演算的特殊职能在于代替理性的职责，以确定究竟有哪些情况同时牵涉于后件与前件之中。然而这一应用的优点

is almost entirely prospective, and will be made manifest in a subsequent proposition.

Proposition II.

10. *It is known that the probabilities of certain simple events* $x, y, z, \ldots$ *are* $p, q, r, \ldots$ *respectively when a certain condition* V *is satisfied;* V *being in expression a function of* $x, y, z, \ldots$. *Required the absolute probabilities of the events* $x, y, z, \ldots$, *that is, the probabilities of their respective occurrence independently of the condition* V.

Let, p', q', r', &c., be the probabilities required, i. e. the probabilities of the events x, y, z,.., regarded not only as simple, but as independent events. Then by Prop. I. the probabilities that these events will occur when the condition V, represented by the logical equation $V = 1$, is satisfied, are

$$\frac{[xV]}{[V]}, \quad \frac{[yV]}{[V]}, \quad \frac{[zV]}{[V]}, \text{ \&c.},$$

in which $[xV]$ denotes the result obtained by multiplying V by x, according to the rules of the Calculus of Logic, and changing in the result x, y, z, into p', q', r', &c. But the above conditioned probabilities are by hypothesis equal to $p, q, r, \ldots$ respectively. Hence we have,

$$\frac{[xV]}{[V]} = p, \quad \frac{[yV]}{[V]} = q, \quad \frac{[zV]}{[V]} = r, \text{ \&c.},$$

from which system of equations equal in number to the quantities $p', q', r', \ldots$, the values of those quantities may be determined.

Now xV consists simply of those constituents in V of which x is a factor. Let this sum be represented by V_x, and in like manner let yV be represented by Vy, &c. Our equations then assume the form

$$\frac{[V_x]}{[V]} = p, \quad \frac{[V_y]}{[V]} = q, \text{ \&c.},$$

where $[V_x]$ denotes the results obtained by changing in V_x the symbols x, y, z, &c., into p', q', r', &c.

To render the meaning of the general problem and the principle of its solution more evident, let us take the following example. Suppose that in the drawing of balls from an urn attention had only been paid to those cases in which the balls drawn were either of a particular colour, "white," or of a particular composition, "marble," or were marked by both these characters, no record having been kept of those cases in which a ball that was neither white nor of marble had been drawn. Let it then have been found, that whenever the supposed condition was satisfied, there was a probability p that a white ball would be drawn, arid a probability q that a marble ball would be drawn: and from these data alone let it be required to find the probability that in the next drawing, without reference

几乎完全是前瞻性的，并将在随后的一个命题中显示出来。

命题二。

10. 已知某些简单事件在某一条件得到满足时的概率分别为 ；在表达式中它们是的函数。求事件 ， 的绝对概率，即它们各自独立于条件 *V*.而发生的概率。

设 p′、q′、r′等等为所求的概率，即事件 x、y、z、.....的概率，这些事件不仅被视为简单的，而且被视为独立的事件。于是根据命题 i.，当由逻辑方程 V = 1 所表示的条件 V 得到满足时，这些事件将发生的概率为

$$\frac{[xV]}{[V]}, \quad \frac{[yV]}{[V]}, \quad \frac{[zV]}{[V]}, \text{ \&c.},$$

其中[xV]表示按照逻辑演算的规则把 V 乘以 x，并在结果中把 x、y、z 换成 p′、q′、r′等等而得到的结果。但上述受条件的概率按假说分别等于 p, q, r, ...。因此我们有，

$$\frac{[xV]}{[V]} = p, \quad \frac{[yV]}{[V]} = q, \quad \frac{[zV]}{[V]} = r, \text{ \&c.},$$

由这个方程组的方程数目与量 p′, q′, r′, ...的数目相等，这些量的值便可确定。

现在 xV 仅仅由 V 中以 x 为因子的那些成分组成。令此和用 V x 表示，并同样地令 yV 用 V y 表示，等等。我们的方程于是采取如下形式

$$\frac{[V_x]}{[V]} = p, \quad \frac{[V_y]}{[V]} = q, \text{ \&c.},$$

where $[V_x]$ denotes the results obtained by changing in V_x the symbols x, y, z, &c., into p', q', r', &c.

为了使一般问题的意义及其解法的原则更加明显，让我们举下面的例子。假设在从瓮中抽球时，只注意了那些所抽出的球或是具有某种特定颜色"白"，或是具有某种特定构成"大理石"，或同时带有这两个特征的情形，而没有记录那些所抽出的球既不是白也不是大理石的场合。于是，设已发现，每当所设想的条件得到满足时，抽到一个白球的概率为 p，抽到一个大理石球的概率为q：并且仅从这些数据出发，要求求出在下一次抽取中，不参照

at all to the condition above mentioned, a white ball will be drawn; also the probability that a marble ball will be drawn.

Here if x represent the drawing of a white ball, y that of a marble ball, the condition V will be represented by the logical function

$$xy + x(1-y) + (1-x)y.$$

Hence we have

$$V_x = xy + x(1-y) = x, \quad V_y = xy + (1-x)y = y;$$

whence

$$[V_x] = p; \quad [V_y] = q;$$

and the final equations of the problem are

$$\frac{p'}{p'q' + p'(1-q') + q'(1-p')} = p, \quad \frac{q'}{p'q' + p'(1-q') + q'(1-p')} = q;$$

from which we find

$$p' = \frac{p+q-1}{q}, \quad q' = \frac{p+q-1}{p}.$$

It is seen that p' and q' are respectively proportional to p and q, as by Professor Donkin's principle they ought to be. The solution of this class of problems might indeed, by a direct application of that principle, be obtained.

To meet a possible objection, I here remark, that the above reasoning does not require that the drawings of a white and a marble ball should be independent, in virtue of the physical constitution of the balls. The assumption of their independence is indeed involved in the solution, but it does not rest upon any prior assumption as to the nature of the balls, and their relations, or freedom from relations, of form, colour, structure, &c. It is founded upon our total ignorance of all these things. Probability always has reference to the state of our actual knowledge, and its numerical value varies with varying information.

Proposition III.

11. To determine in any question of probabilities the logical connexion of the quæsitum with the data; that is, to assign the event whose probability is sought, as a logical function of the event whose probabilities are given.

Let S, T, &c., represent any compound events whose probabilities are given, S and T being in expression known functions of the symbols x, y, z, &c., representing simple events. Similarly let W represent any event whose probability is sought, W being also a known function of x, y, z, &c. As $S, T, \ldots W$ must satisfy the fundamental law of duality, we are permitted to replace them by single logical symbols, $s, t, \ldots w$. Assume then

$$s = S, t = T, w = W.$$

在上述条件下，抽到白球的概率；以及抽到大理石球的概率。

这里，如果 x 表示抽到白球，y 表示抽到大理石球，那么条件 V 将由逻辑函数表示

$$xy + x(1-y) + (1-x)y.$$

因此我们有

$$V_x = xy + x(1-y) = x, \quad V_y = xy + (1-x)y = y;$$

由此

$$[V_x] = p; \quad [V_y] = q;$$

而问题的最终方程是

$$\frac{p'}{p'q' + p'(1-q') + q'(1-p')} = p, \quad \frac{q'}{p'q' + p'(1-q') + q'(1-p')} = q;$$

由此我们求得

$$p' = \frac{p+q-1}{q}, \quad q' = \frac{p+q-1}{p}.$$

可以看出，p ′和 q ′分别与 p 和 q 成比例，正如按照 Donkin 教授的原理它们应当如此。这一类问题的解法，实际上可以通过直接应用该原理而得到。

为了回应一个可能的反对意见，我在此说明，上述推理并不要求白球和弹珠球的抽取依据球的物理构造而相互独立。对它们独立性的假定确实包含在解答之中，但它并不依赖于任何关于球的性质及其关系、或其在形状、颜色、结构等方面有无关系的先在假定。它建立在我们对所有这些事物的完全无知之上。概率始终与我们实际知识的状态相关，其数值随信息的变化而变化。

命题三。

11. 在任何概率问题中，确定所求事件与已知数据之间的逻辑联系；也就是说，把所求其概率的事件，表示为已知其概率的事件的逻辑函数。

设 S、T 等表示任何其概率已给定的复合事件，S 和 T 在表达式中是记号 x、y、z 等的已知函数，这些记号表示简单事件。类似地，设 W 表示任何其概率被寻求的事件，W 也是 x、y、z 等的已知函数。由于 S, T, ...W 必须满足对偶性的基本定律，我们被允许用单个逻辑记号 s, t, ...w 来替换它们。于是假定

$$s = S, t = T, w = W.$$

These, by the definition of $S, T, \ldots W$, will be a series of logical equations connecting the symbols $s, t, \ldots w$, with the symbols $x, y, z \ldots$

By the methods of the Calculus of Logic we can eliminate from the above system any of the symbols $x, y, z, \ldots$, representing events whose probabilities are not given, and determine w as a developed function of s, t, &c., and of such of the symbols x, y, z, &c., if any such there be, as correspond to events whose probabilities are given. The result will be of the form

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D,$$

where A, B, C, and D comprise among them all the possible *constituents* which can be formed from the symbols s, t, &c., i. e. from all the symbols representing events whose probabilities are given.

The above will evidently be the complete expression of the relation sought. For it fully determines the event W, represented by the single symbol w, as a function or combination of the events similarly denoted by the symbols s, t, &c., and it assigns by the laws of the Calculus of Logic the condition

$$D - 0,$$

as connecting the events s, t, &c., among themselves. We may, therefore, by Principle VI., regard s, t, &c., as *simple* events, of which the combination w, and the condition with which it is associated D, are definitely determined.

Uniformity in the logical processes of reduction being desirable, I shall here state the order which will generally be pursued.

12. By (VIII. 8), the primitive equations are reducible to the forms

$$\begin{aligned} &s(1-S) + S(1-s) = 0; \\ &t(1-T) + T(1-t) = 0; \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &w(1-W) - W(1-w) = 0; \end{aligned} \tag{1}$$

under which they can be added together without impairing their significance. We can then eliminate the symbols x, y, z, either separately or together. If the latter course is chosen, it is necessary, after adding together the equations of the system, to develop the result with reference to all the symbols to be eliminated, and equate to 0 the product of all the coefficients of the constituents (VII. 9).

As w is the symbol whose expression is sought, we may also, by Prop. III. Chap. IX., express the result of elimination in the form

$$Ew + E'(1-w) = 0.$$

E and E' being successively determined by making in the general system (1), $w = 1$ and $w = 0$, and eliminating the symbols x, y, z, ... Thus the single equations from which E and E' are to be respectively determined become

$$\begin{aligned} &s(1-S) + S(1-s) + t(1-T) + T(1-t) \ldots + 1 - W = 0; \\ &s(1-S) + S(1-s) + t(1-T) + T(1-t) + W = 0. \end{aligned}$$

These, by the definition of $S, T, \ldots W$, will be a series of logical equations connecting the symbols $s, t, \ldots w$, with the symbols $x, y, z \ldots$

通过逻辑演算的方法，我们可以从上述系统中消去记号 x, y, z, ...中的任何一个，这些记号表示其概率未被给出的事件，并把 w 确定为 s、t 等等的展开式函数，以及--如果存在的话--那些对应于其概率已被给出的事件的记号 x、y、z 等等的展开式函数。结果将具有如下形式

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D,$$

其中 A、B、C 和 D 共同包含了所有可能的成分，这些成分可以由记号 s、t 等等，即由所有表示其概率已被给出的事件的记号构成。

上述内容显然就是所求关系的完整表达。因为它完全确定了由单一记号 w 所表示的事件 W，将其表示为以类似方式由记号 s、t 等等所表示的事件的函数或组合，并且它按照逻辑演算的规律指定了条件

$$D - 0,$$

作为在 s、t 等等彼此之间建立联系的条件。因此，我们可以根据原理 vi.，把 s、t 等等视为简单事件，它们的组合 w 以及与之相关联的条件 D 都是确定地给出的。

由于化简的逻辑过程应当保持一致，我在此说明一般将遵循的顺序。

12. 由(viii.8)，原始方程可化为以下形式

$$\begin{aligned} &s(1-S) + S(1-s) = 0; \\ &t(1-T) + T(1-t) = 0; \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &w(1-W) - W(1-w) = 0; \end{aligned} \tag{1}$$

在此条件下它们可以相加而不损害其意义。然后我们可以分别或一起消去记号 x、y、z。如果选择后一种做法，则有必要在将系统的方程相加之后，就所有待消去的记号展开结果，并令各成分系数的乘积等于 0(vii.9)。

由于 w 是所求其表达式的记号，我们还可以根据第 ix 章命题 iii.，将消元的结果表示为如下形式

$$Ew + E'(1-w) = 0.$$

E 和 E ′通过依次在一般系统(1)中令 w = 1 和 w = 0，并消去记号 x、y、z、...而分别确定。于是，分别用以确定 E 和 E ′的单个方程变为

$$\begin{aligned} &s(1-S) + S(1-s) + t(1-T) + T(1-t) \ldots + 1 - W = 0; \\ &s(1-S) + S(1-s) + t(1-T) + T(1-t) + W = 0. \end{aligned}$$

From these it only remains to eliminate x, y, z, &c., and to determine w by subsequent development. In the process of elimination we may, if needful, avail ourselves of the simplifications of Props. I. and II. Chap. IX.

13. Should the data, beside informing us of the probabilities of events, further assign among them any explicit connexion, such connexion must be logically expressed, and the equation or equations thus formed be introduced into the general system.

Proposition IV.

14. *Given the probabilities of any system of events; to determine by a general method the consequent or derived probability of any other event.*

As in the last Proposition, let S, T, &c., be the events whose probabilities are given, W the event whose probability is sought, these being known functions of x, y, z, &c. Let us represent the data as follows:

$$\begin{aligned} &\text{Probability of } S = p; \\ &\text{Probability of } T = q; \end{aligned} \tag{1}$$

and so on, p, q, &c., being known numerical values. If then we represent the compound event S by s, T by t, and W by w, we find by the last proposition,

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D; \tag{2}$$

A, B, C, and D being functions of s, t, &c. Moreover the data (1) are transformed into

$$\text{Prob. } s = p, \quad \text{Prob. } t = q, \text{ &c.} \tag{3}$$

Now the equation (2) is resolvable into the system

$$\left.\begin{aligned} w &= A + qC \\ D &= 0, \end{aligned}\right\} \tag{4}$$

q being an indefinite class symbol (VI. 12). But since by the properties of constituents (V. Prop. III.), we have

$$A + B + C + D = 1,$$

the second equation of the above system may be expressed in the form

$$A + B + C = 1.$$

If we represent the function $A + B + C$ by V, the system (4) becomes

$$w = A + qC; \tag{5}$$

$$V = 1. \tag{6}$$

由此只剩下消去 x、y、z 等等，并通过随后的展开式确定 w。在消元过程中，如有需要，我们可以利用第 ix 章命题 i.和 ii.的化简。

13. 如果数据除了告知我们各事件的概率之外，还进一步在它们之间指定了任何明确的联系，那么这种联系必须在逻辑上加以表达，并将由此形成的一个或多个方程引入一般系统。

命题四。

14. 给定任意事件系统的概率；以一般方法确定
方法，任何其他事件的后续概率或派生概率。

如上一命题中那样，设 S、T 等等为其概率已被给出的事件，W 为所求其概率的事件，这些事件是 x、y、z 等等的已知函数。让我们将数据表示如下：

$$\begin{aligned} &\text{的概率} = \\ &\text{T 的概率} = q; \end{aligned} \tag{1}$$

等等，p、q等为已知数值。若以s表示复合事件S，t表示T，w表示W，则由上一命题可得，

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D; \tag{2}$$

A, B, C, and D being functions of s, t, &c. Moreover the data (1) are transformed into

$$\text{Prob. } s = p, \quad \text{Prob. } t = q, \text{ &c.} \tag{3}$$

现在方程(2)可分解为该系统

$$\left.\begin{aligned} w &= A + qC \\ D &= 0, \end{aligned}\right\} \tag{4}$$

q 是一个不定类记号(VI.12)。但由于根据成分的性质(V.命题 iii.)，我们有

$$A + B + C + D = 1,$$

上述系统的第二个方程可以表示为以下形式

$$A + B + C = 1.$$

如果我们用 V 表示函数 A + B + C，则系统(4)变为

$$w = A + qC; \tag{5}$$

$$V = 1. \tag{6}$$

Let us for a moment consider this result. Since V is the sum of a series of constituents of s, t, &c., it represents the compound event in which the simple events involved are those denoted by s, t, &c. Hence (6) shows that the events denoted by s, t, &c., and whose probabilities are p, q, &c., have such probabilities not as *independent events*, but as events subject to a certain condition V. Equation (5) expresses w as a similarly conditioned combination of the same events.

Now by Principle VI. the mode in which this knowledge of the connexion of events has been obtained does not influence the mode in which, when obtained, it is to be employed. We must reason upon it as if experience had presented to us the events s, t, &c., as simple events, free to enter into every combination, but possessing, when actually subject to the condition V, the probabilities p, q, &c., respectively.

Let then $p', q', \ldots$, be the corresponding probabilities of such events, when the restriction V is removed. Then by Prop. II. of the present chapter, these quantities will be determined by the system of equations,

$$\frac{[V_s]}{[V]} = p, \qquad \frac{[V_t]}{[V]} = q, \text{ &c.;} \tag{7}$$

and by Prop. I. the probability of the event w under the same condition V will be

$$\text{Prob. } w = \frac{[A + cC]}{[V]}; \tag{8}$$

wherein V_s denotes the sum of those constituents in V of which s is a factor, and $[V_s]$ what that sum becomes when $s, t, \ldots$, are changed into $p', q', \ldots$, respectively. The constant c represents the probability of the indefinite event q; it is, therefore, arbitrary, and admits of any value from 0 to 1.

Now it will be observed, that the values of,p', q', &c., are determined from (7) only in order that they may be substituted in (8), so as to render Prob. w a function of known quantities, p, q, &c. It is obvious, therefore, that instead of the letters p', q', &c., we might employ any others as s, t, &c., in the same *quantitative* acceptations. This particular step would simply involve a change of meaning of the symbols s, t, &c.—their ceasing to be *logical*, and becoming *quantitative*. The systems (7) and (8) would then become

$$\frac{V_s}{V} = p, \quad \frac{V_t}{V} = q, \text{ &c.;} \tag{9}$$

$$\text{Prob. } w = \frac{A + cC}{V}. \tag{10}$$

In employing these, it is only necessary to determine from (9) s, t, &c., regarded as quantitative symbols, in terms of p, q, &c., and substitute the resulting values in (10). It is evident, that s, t, &c., inasmuch as they represent *probabilities*, will be positive proper fractions.

The system (9) may be more symmetrically expressed in the form

$$\frac{V_s}{p} = \frac{V_t}{q} \ldots = V. \tag{11}$$

让我们暂时考虑这一结果。由于 V 是 s、t 等的成分项级数之和，它表示其中所涉及的简单事件是由 s、t 等所表示的那些事件的复合事件。因此(6)表明，由 s、t 等表示、其概率为 p、q 等的事件具有这样的概率，并非作为独立事件，而是作为服从某一条件 V 的事件。方程(5)将 w 表达为同样受条件的相同事件的组合。

现在，根据原理 vi.，获得这种关于事件联系的知识的方式，并不影响在获得之后运用它的方式。我们必须对它进行推理，就好像经验把事件 s、t 等等作为简单事件呈现给我们一样，这些事件可以自由地进入每一种组合，但在实际受条件 V 约束时，分别具有概率 p、q 等等。

那么，设 p′, q′, ... ，为当限制 V 被移除时这类事件的相应概率。那么根据本章命题 ii.，这些量将由如下方程组确定，

$$\frac{[V_s]}{[V]} = p, \qquad \frac{[V_t]}{[V]} = q, \text{ &c.;} \tag{7}$$

and by Prop. I. the probability of the event w under the same condition V will be

$$\text{Prob. } w = \frac{[A + cC]}{[V]}; \tag{8}$$

其中 Vs 表示 V 中那些以 s 为因子的成分之和，而 [Vs] 表示当 s、t、.....分别变为 p′、q′、.....时该和变成什么。常数 c 表示不定事件 q 的概率；因此它是任意的，可以取 0 到 1 之间的任何值。现在将会注意到，p′、q′ 等的值仅由 (7) 确定，只是为了将它们代入 (8)，从而使 Prob. w 成为已知量 p、q 等的函数。因此显然，我们可以用任何其他字母如 s、t 等以相同的定量含义来代替字母 p′、q′ 等。这一特称步骤仅仅涉及记号 s、t 等的含义变化——它们不再是逻辑的，而成为数值的。系统 (7) 和 (8) 于是变为

$$\frac{V_s}{V} = p, \quad \frac{V_t}{V} = q, \text{ &c.;} \tag{9}$$

$$\text{Prob. } w = \frac{A + cC}{V}. \tag{10}$$

在运用这些方程时，只需从(9)中把 s、t 等等视为定量记号，用 p、q 等等表示出来，并将所得的值代入(10)。显然，s、t 等等，由于它们表示概率，将是正的真分数。

系统(9)可以更对称地表示为以下形式

$$\frac{V_s}{p} = \frac{V_t}{q} \ldots = V. \tag{11}$$

Or we may express both (9) and (10) together in the symmetrical system

$$\frac{V_s}{p} = \frac{V_t}{q} \ldots = \frac{A + cC}{u} = V; \tag{12}$$

wherein u represents Prob. w.

15. It remains to interpret the constant c assumed to represent the probability of the indefinite event q. Now the *logical* equation

$$w = A + qC,$$

interpreted in the reverse order, implies that if either the event A take place, or the event C in connexion with the event q, the event w will take place, and not otherwise. Hence q represents that condition under which, if the event C take place, the event w will take place. But the probability of q is c. Hence, therefore, $c =$ probability that if the event C take place the event w will take place.

Wherefore by Principle II.,

$$c = \frac{\text{Probability of concurrence of } C \text{ and } w}{\text{Probability of } C}.$$

We may hence determine the nature of that new experience from which the actual value of c may be obtained. For if we substitute in C for s, t, &c., their original expressions as functions of the simple events x, y, z, &c., we shall form the expression of that event whose probability constitutes the denominator of the above value of c; and if we multiply that expression by the original expression of w, we shall form the expression of that event whose probability constitutes the numerator of c, and *the ratio of the frequency of this event to that of the former one, determined by new observations* will give the value of c. Let it be remarked here, that the constant c does not *necessarily* make its appearance in the solution of a problem. It is only when the data are insufficient to render determinate the probability sought, that this arbitrary element presents itself, and in this case it is seen that the final logical equation (2) or (5) informs us how it is to be determined.

If that new experience by which c may be determined cannot be obtained, we can still, by assigning to c its limiting values 0 and 1, determine the limits of the probability of w. These are

$$\begin{array}{ll} \text{Minor limit of Prob. } w & = \frac{A}{V}. \\ \text{Superior limit} & = \frac{A+C}{V}. \end{array}$$

Between these limits, it is certain that the probability sought must lie independently of all new experience which does not absolutely contradict the past.

If the expression of the event C consists of many constituents, the logical value of w being of the form

$$w = A + \frac{0}{0}C_1 + \frac{0}{0}C_2 + \text{ \&c.},$$

或者我们可以把(9)和(10)一起用对称系统来表示

$$\frac{V_s}{p} = \frac{V_t}{q} \ldots = \frac{A + cC}{u} = V; \tag{12}$$

其中u表示Prob.w。

15. 剩下要解释的是常数 c，它被假定表示不定事件 q 的概率。现在，逻辑方程

$$w = A + qC,$$

按相反顺序解释，意味着如果事件 A 发生，或者事件 C 与事件 q 相关联地发生，则事件 w 将发生，否则不发生。因此 q 表示这样一种条件：在该条件下，如果事件 C 发生，则事件 w 将发生。但 q 的概率是 c。因此，c = 如果事件 C 发生则事件 w 将发生的概率。

因此，根据原则 ii.,

$$c = \frac{\text{概率的同时发生的C与w}}{C \quad \text{的概率}}$$

我们由此可以确定那种新经验的性质，从它那里可以获得 c 的实际值。因为如果我们在 C 中用 s、t 等作为简单事件 x、y、z 等的函数来替换它们的原始表达式，我们就会形成那个事件的表达式，其概率构成上述 c 值的分母；而如果我们用 w 的原始表达式乘以那个表达式，我们就会形成那个事件的表达式，其概率构成 c 的分子，并且这个事件的频率与前一个事件的频率之比，由新的观察确定将给出 c 的值。这里需要指出，常数 c 并不必然出现在问题的解中。只有当数据不足以确定所寻求的概率时，这个任意元素才会出现，而在这种情况下可以看出，最终的逻辑方程(2)或(5)告诉我们如何确定它。

如果无法获得那种可以确定 c 的新经验，我们仍然可以通过赋予 c 其极限值 0 和 1，来确定 w 的概率的极限。这些极限是

$$\begin{array}{ll} \text{Minor limit of Prob. } w & = \frac{A}{V}. \\ \text{Superior limit} & = \frac{A+C}{V}. \end{array}$$

在这些极限之间，可以肯定，所求的概率必定落在其中，而不依赖于任何不与过去绝对矛盾的新经验。

如果事件 C 的表达式由许多成分构成，w 的逻辑值具有如下形式

$$w = A + \frac{0}{0}C_1 + \frac{0}{0}C_2 + \text{ \&c.},$$

we can, instead of employing their aggregate as above, present the final solution in the form

$$\text{Prob. } w = \frac{A + c_1C_1 + c_2C_2 + \text{ \&c.}}{V}.$$

Here c_1 = probability that if the event C_1 occur, the event w will occur, and so on for the others. Convenience must decide which form is to be preferred.

16. The above is the complete theoretical solution of the problem proposed. It may be added, that it is applicable equally to the case in which any of the events mentioned in its original statement are conditioned. Thus, if one of the data is the probability p, that if the event x occur the event y will occur; the probability of the occurrence of x not being given, we must assume Prob. $x = c$ (an arbitrary constant), then Prob. $xy = cp$, and these two conditions must be introduced into the data, and employed according to the previous method. Again, if it is sought to determine the probability that if an event x occur an event y will occur, the solution will assume the form

$$\text{Prob. sought } = \frac{\text{Prob. } xy}{\text{Prob. } x},$$

the numerator and denominator of which must be separately determined by the previous general method.

17. We are enabled by the results of these investigations to establish a general rule for the solution of questions in probabilities.

GENERAL RULE.

CASE I.—*When all the events are unconditioned.*

Form the symbolical expressions of the events whose probabilities are given or sought.

Equate such of those expressions as relate to compound events to a new series of symbols, s, t, &c., which symbols regard as representing the events, no longer as compound but simple, to whose expressions they have been equated.

Eliminate from the equations thus formed all the logical symbols, except those which express events, s, t, &c., whose respective probabilities p, q, &c. are given, or the event w whose probability is sought, and determine w as a developed function of s, t, &c. in the form

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D.$$

Let $A + B + C = V$, and let V_s represent the aggregate of those constituents in V which contain s as a factor, V_t of those which contain t as a factor, and thus for all the symbols whose probabilities are given.

Then, passing from Logic to Algebra, form the equations

$$\frac{V_s}{p} = \frac{V_t}{q} = V, \tag{1}$$

$$\text{Prob. } w = \frac{A + cC}{V}, \tag{2}$$

we can, instead of employing their aggregate as above, present the final solution in the form

$$\text{Prob. } w = \frac{A + c_1C_1 + c_2C_2 + \text{ \&c.}}{V}.$$

Here c_1 = probability that if the event C_1 occur, the event w will occur, and so on for the others. Convenience must decide which form is to be preferred.

16. 以上是所提出问题的完整理论解。可以补充说，它同样适用于其原始陈述中提到的任何事件是有条件的情况。因此，如果数据之一是概率 p，即如果事件 x 发生，事件 y 将发生；事件 x 发生的概率未给出，我们必须假设 Prob.x = c(一个任意常数)，那么 Prob.xy = cp，这两个条件必须引入数据中，并按照先前的方法使用。再者，如果寻求确定如果事件 x 发生则事件 y 将发生的概率，解将采取以下形式

$$\text{Prob. sought } = \frac{\text{Prob. } xy}{\text{Prob. } x},$$

其分子和分母必须分别由前述一般方法确定。

17. 借助这些研究的结果，我们能够为概率问题的求解建立一条一般规则。

一般规则。

情形 I.--当所有事件都是无条件的。

构造其概率已被给出或所求的事件的记号表达式。

将那些与复合事件相关的表达式等同于一系列新的记号，s、t等，将这些记号视为表示事件，不再是复合的而是简单的，等同于它们所对应的表达式。

从如此形成的方程中消去所有逻辑记号，除了那些表示事件的记号，s、t等，其各自的概率p、q等是给定的，或者事件w其概率是所求的，并将w确定为s、t等的展开式函数，形式为

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D.$$

设A + B + C = V，并设V s表示V中包含s作为因子的那些成分的总和，V t表示包含t作为因子的那些成分的总和，以此类推对于所有其概率给定的记号。

然后，从逻辑学过渡到代数学，建立方程

$$\frac{V_s}{p} = \frac{V_t}{q} = V, \tag{1}$$

$$\text{Prob. } w = \frac{A + cC}{V}, \tag{2}$$

from (1) determine s, t, &c. as functions of p, q, &c., and substitute their values in (2). The result will express the solution required.

Or form the symmetrical system of equations

$$\frac{V_s}{p} = \frac{V_t}{q} \ldots = \frac{A + cC}{u} = \frac{V}{1}, \tag{3}$$

where u represents the probability sought.

If c appear in the solution, its interpretation will be

$$c = \frac{\text{Prob. } Cw}{\text{Prob. } c},$$

and this interpretation indicates the nature of the experience which is necessary for its discovery.

Case II.—*When some of the events are conditioned.*

If there be given the probability p that if the event X occur, the event Y will occur, and if the probability of the antecedent X be not given, resolve the proposition into the two following, viz.:

$$\begin{aligned} \text{Probability of } X &= c, \\ \text{Probability of } XY &= cp. \end{aligned}$$

If the quæsitum be the probability that if the event W occur, the event Z will occur, determine separately, by the previous case, the terms of the fraction

$$\frac{\text{Prob. } WZ}{\text{Prob. } W},$$

and the fraction itself will express the probability sought.

It is understood in this case that X, Y, W, Z may be any compound events whatsoever. The expressions XY and WZ represent the products of the symbolical expressions of X and Y and of W and Z, formed according to the rules of the Calculus of Logic.

The determination of the single constant c may in certain cases be resolved into, or replaced by, the determination of a series of arbitrary constants $c_1, c_2 \ldots$ according to convenience, as previously explained.

18. It has been stated (I. 12) that there exist two distinct definitions, or modes of conception, upon which the theory of probabilities may be made to depend, one of them being connected more immediately with Number, the other more directly with Logic. We have now considered the consequences which flow from the numerical definition, and have shown how it conducts us to a point in which the necessity of a connexion with Logic obviously suggests itself. We have seen to some extent what is the nature of that connexion; and further, in what manner the peculiar processes of Logic, and the more familiar ones of quantitative Algebra, are involved in the same general method of solution, each of these so accomplishing its own object that the two processes may be regarded as supplementary to each other. It remains to institute the reverse order of investigation, and, setting out from a definition of probability

从(1)确定s、t等作为p、q等的函数，并将它们的值代入(2)。结果将表达所需的解。或者形成对称方程组

$$\frac{V_s}{p} = \frac{V_t}{q} \ldots = \frac{A + cC}{u} = \frac{V}{1}, \tag{3}$$

where u represents the probability sought.

If c appear in the solution, its interpretation will be

$$c = \frac{\text{Prob. } Cw}{\text{Prob. } c},$$

而这种解释指示了其发现所需的经验的性质。

情形二--当部分事件带有条件限制时。

如果给定概率p，即如果事件X发生，则事件Y将发生，并且如果前件X的概率未给定，将命题分解为以下两个，即：

$$\begin{aligned} \text{Probability of } X &= c, \\ \text{Probability of } XY &= cp. \end{aligned}$$

如果所求的是概率，即如果事件W发生，则事件Z将发生，通过前一种情况分别确定分数的各项

$$\frac{\text{Prob. } WZ}{\text{Prob. } W},$$

而该分数本身将表示所求的概率。

在此情况下理解为X、Y、W、Z可以是任何复合事件。表达式XY和WZ表示X和Y以及W和Z的符号表达式的乘积，按照逻辑演算的规则形成。

单个常数c的确定在某些情况下可以根据方便分解为或替换为一系列任意常数c^1, c^2 ...的确定，如前所述。

18. 已经指出(I.12)，存在两个不同的定义或概念模式，概率论可以依赖于它们，其中一个与数更直接相关，另一个与逻辑更直接相关。我们现在已经考虑了从数值定义中产生的后果，并展示了它如何引导我们到达一个点，在那里与逻辑联系的必要性显然自我提示。我们已经在某种程度上看到了那种联系的性质；并且进一步，逻辑的独特过程和定量代数的更熟悉过程如何以相同的一般方法参与解决，每一个都如此完成自己的目标，以至于这两个过程可以被视为相互补充。现在剩下的是建立相反顺序的研究，并从概率的定义出发

in which the logical relation is more immediately involved, to show how the numerical definition would thence arise, and how the same general method, equally dependent upon both elements, would finally, but by a different order of procedure, be established.

That between the symbolical expressions of the logical calculus and those of Algebra there exists a close analogy, is a fact to which attention has frequently been directed in the course of the present treatise. It might even be said that they possess a community of forms, and, to a very considerable degree, a community of laws. With a single exception in the latter respect, their difference is only one of interpretation. Thus the same expression admits of a logical or of a quantitative interpretation, according to the particular meaning which we attach to the symbols it involves. The expression xy represents, under the former condition, a concurrence of the events denoted by x and y; under the latter, the product of the numbers or quantities denoted by x and y. And thus every expression denoting an event, simple or compound, admits, under another system of interpretation, of a meaning purely quantitative. Here then arises the question, whether there exists any principle of transition, in accordance with which the logical and the numerical interpretations of the same symbolical expression shall have an intelligible connexion. And to this question the following considerations afford an answer.

19. Let it be granted that there exists such a feeling as expectation, a feeling of which the object is the occurrence of events, and which admits of differing degrees of intensity. Let it also be granted that this feeling of expectation accompanies our knowledge of the circumstances under which events are produced, and that it varies with the degree and kind of that knowledge. Then, without assuming, or tacitly implying, that the intensity of the feeling of expectation, viewed as a mental emotion, admits of precise numerical measurement, it is perfectly legitimate to inquire into the possibility of a mode of numerical estimation which shall, at least, satisfy these following conditions, viz., that the numerical value which it assigns shall increase when the known circumstances of an event are felt to justify a stronger expectation, shall diminish when they demand a weaker expectation, and shall remain constant when they obviously require an equal degree of expectation.

Now these conditions at least will be satisfied, if we assume the fundamental principle of expectation to be this, viz., that the laws for the expression of expectation, viewed as a numerical element, shall be the same as the laws for the expression of the expected event viewed as a logical element. Thus if $\phi(x, y, z)$ represent any unconditional event compounded in any manner of the events x, y, z, let the same expression $\phi(x, y, z)$, according to the above principle, denote the expectation of that event; x, y, z representing no longer the simple events involved, but the expectations of those events.

For, in the first place, it is evident that, under this hypothesis, the probability of the occurrence of some one of a set of mutually exclusive events will be equal to the sum of the separate probabilities of those events. Thus if the alternation

其中逻辑关系更直接地涉及，以显示数值定义如何由此产生，以及同样的通用方法，同等依赖于两个元素，最终但通过不同的程序顺序，将如何被建立。

逻辑演算的符号表达式与代数的符号表达式之间存在着密切的类比，这是本论著过程中经常引起注意的事实。甚至可以说它们具有共同的形式，并且在相当程度上具有共同的规律。在后一方面只有一个例外，它们的区别仅在于解释。因此，同一个表达式根据我们赋予其中记号的具体意义，可以接受逻辑解释或定量解释。表达式 xy 在前一种情况下表示由 x 和 y 表示的事件的共同发生；在后一种情况下，表示由 x 和 y 表示的数或量的乘积。因此，每一个表示事件(简单的或复合的)的表达式，在另一种解释体系下，都接受纯粹定量的意义。于是这里产生了一个问题：是否存在任何过渡原则，根据它，同一符号表达式的逻辑解释和数值解释将具有可理解的关联。对于这个问题，以下考虑提供了一个答案。

19. 让我们承认存在一种称为期望的情感，这种情感的对象是事件的发生，并且允许有不同强度的程度。让我们也承认这种期望情感伴随着我们对事件产生所处环境的认识，并且它随着那种知识的程度和种类而变化。那么，在不假设或默示期望情感的强度(被视为一种心理情绪)允许精确的数值测量的情况下，探究一种数值估计模式的可能性是完全正当的，这种模式至少应满足以下条件，即：当已知的事件环境被认为证明更强的期望是合理时，它所赋予的数值应增加；当它们要求较弱的期望时，应减少；而当它们明显需要同等程度的期望时，应保持不变。

现在，如果我们假定期望的基本原理是这样的，即：将期望视为一个数值的元素，其表达式的规律应与将所期望的事件视为一个逻辑的元素，其表达式的规律相同，那么至少这些条件将得到满足。因此，如果 φ(x, y, z) 表示由事件 x、y、z 以任何方式复合而成的任一无条件事件，则根据上述原理，让同一表达式 φ(x, y, z) 表示该事件的期望；x、y、z 不再表示所涉及的简单事件，而是表示那些事件的期望。

因为，首先，显而易见的是，在此假说下，一组互斥事件中某个事件发生的概率将等于那些事件各自概率之和。因此，如果所述的交替

in question consist of n mutually exclusive events whose expressions are

$$\phi_1(x,y,z), \phi_2(x,y,z), \dots \phi_n(x,y,z),$$

the expression of that alternation will be

$$\phi_1(x,y,z) + \phi_2(x,y,z) \dots + \phi_n(x,y,z) = 1;$$

the literal symbols x, y, z being logical, and relating to the simple events of which the three alternatives are compounded: and, by hypothesis, the expression of the probability that some one of those alternatives will occur is

$$\phi_1(x,y,z) + \phi_2(x,y,z) \dots + \phi_n(x,y,z),$$

x, y, z here denoting the probabilities of the above simple events. Now this expression increases, *cæteris paribus*, with the increase of the number of the alternatives which are involved, and diminishes with the diminution of their number; which is agreeable to the condition stated.

Furthermore, if we set out from the above hypothetical definition of the measure of probability, we shall be conducted, either by necessary inference or by successive steps of suggestion, which might perhaps be termed *necessary*, to the received numerical definition. We are at once led to recognise unity (1) as the proper numerical measure of certainty. For it is certain that any event x or its contrary $1 - x$ will occur. The expression of this proposition is

$$x + (1 - x) = 1,$$

whence, by hypothesis, $x + (1 - x)$, the measure of the probability of the above proposition, becomes the measure of certainty. But the value of that expression is 1, whatever the particular value of x may be. Unity, or 1, is therefore, on the hypothesis in question, the measure of certainty.

Let there, in the next place, be n mutually exclusive, but equally possible events, which we will represent by $t_1, t_2, \dots t_n$. The proposition which affirms that some one of these must occur will be expressed by the equation

$$t_1 + t_2 \dots + t_n = 1;$$

and, as when we pass in accordance with the reasoning of the last section to numerical probabilities, the same equation remains true in form, and as the probabilities $t_1, t_2 \dots t_n$ are equal, we have

$$nt_1 = 1,$$

whence $t_l = \frac{1}{n}$, and similarly $t_2 = \frac{1}{n}$, $t_n = \frac{1}{n}$. Suppose it then required to determine the probability that some one event of the partial series $t_1, t_2 \dots t_m$ will occur, we have for the expression required

$$\begin{aligned} t_1 + t_2 \dots + t_m &= \frac{1}{n} + \frac{1}{m} \dots \qquad \text{to } m \text{ terms} \\ &= \frac{m}{n}. \end{aligned}$$

由n个互斥事件组成，其表达式为

$$\phi_1(x,y,z), \phi_2(x,y,z), \dots \phi_n(x,y,z),$$

那种交替的表达将是

$$\phi_1(x,y,z) + \phi_2(x,y,z) \dots + \phi_n(x,y,z) = 1;$$

文字记号x、y、z是逻辑的，并且涉及三个备选所由组成的简单事件：并且，根据假说，那些备选中某个将发生的概率的表达式是

$$\phi_1(x,y,z) + \phi_2(x,y,z) \dots + \phi_n(x,y,z),$$

x、y、z在此表示上述简单事件的概率。现在这个表达式增加，在其他条件相同的情况下，随着所涉及的备选数量的增加而增加，随着其数量的减少而减少；这与所述条件相符。

此外，如果我们从上述概率测度的假说定义出发，我们将被引导，无论是通过必然推论还是通过逐步提示，这些提示或许可以称为必然的，到公认的数值定义。我们立即被引导认识到统一(1)作为确定性的适当数值测度。因为任何事件x或其反面1 – x将发生是确定的。这个命题的表达式是

$$x + (1 - x) = 1,$$

因此，根据假说，x +(1 – x)，上述命题的概率测度，成为确定性的测度。但该表达式的值是1，无论x的特定值可能是什么。因此，统一，或1，在所述假说下，是确定性的测度。

接下来，设有n个互斥但等可能的事件，我们将用t 1 , t 2 , ...t n表示。肯定这些中某个必须发生的命题将由方程表达

$$t_1 + t_2 \dots + t_n = 1;$$

并且，当我们按照上一节的推理过渡到数值概率时，同样的方程在形式上仍然成立，并且由于概率t 1 , t 2 ...t n是相等的，我们有

$$nt_1 = 1,$$

由此 t l = 1 n ,并且类似地t2=1 n ,tn=1 n .假设它那么要求去确定部分序列 t 1 , t 2 ...t m 中某一事件将发生的概率，我们有所求的表达式

$$\begin{aligned} t_1 + t_2 \dots + t_m &= \frac{1}{n} + \frac{1}{m} \dots \qquad \text{to } m \text{ terms} \\ &= \frac{m}{n}. \end{aligned}$$

Hence, therefore, if there are m cases favourable to the occurrence of a particular alternation of events out of n possible and equally probable cases, the probability of the occurrence of that alternation will be expressed by the fraction $\frac{m}{n}$.

Now the occurrence of any event which may happen in different equally possible ways is really equivalent to the occurrence of an alternation, i.e., of some one out of a set of alternatives. Hence the probability of the occurrence of any event may be expressed by a fraction whose numerator represents the number of cases favourable to its occurrence, and denominator the total number of equally possible cases. But this is the rigorous numerical definition of the measure of probability. That definition is therefore involved in the more peculiarly *logical* definition, the consequences of which we have endeavoured to trace.

20. From the above investigations it clearly appears, 1st, that whether we set out from the ordinary numerical definition of the measure of probability, or from the definition which assigns to the numerical measure of probability such a law of value as shall establish a formal identity between the logical expressions of events and the algebraic expressions of their values, we shall be led to the same system of practical results. 2ndly, that either of these definitions pursued to its consequences, and considered in connexion with the relations which it inseparably involves, conducts us, by inference or suggestion, to the other definition. To a scientific view of the theory of probabilities it is essential that both principles should be viewed together, in their mutual bearing and dependence.

因此，如果存在m个有利于特定事件交替发生的案例，从n个可能且等概率的案例中，该交替发生的概率将由分数m n ·表达

—

现在，任何可能以不同的同等可能方式发生的事件，其发生实际上等同于一个择一的发生，即等同于一组备选者中某一个的发生。因此，任何事件发生的概率可以用一个分数表示，其分子表示有利于其发生的情况数目，分母表示同等可能的情况总数。但这正是概率测度的严格数值定义。因此，该定义包含在更为独特的逻辑的定义之中，我们一直试图追溯的正是后者的推论。

20. 由上述考察可以清楚看出：第一，无论我们从概率测度的通常数值定义出发，还是从这样一种定义出发--该定义赋予概率的数值测度这样一种取值法则，使得事件的逻辑表达式与其值的代数表达式之间建立起形式上的同一性--我们都将被引向同一套实际结果体系。第二，这两个定义中的任何一个，若被推究到其后果，并结合于它不可分割地卷入的那些关系来考察，都会通过推论或暗示把我们引向另一个定义。对于概率论的科学观点来说，至关重要的是这两个原则应当在其相互关联与依赖中一并加以考察。

Chapter XVIII

ELEMENTARY ILLUSTRATIONS OF THE GENERAL METHOD IN PROBABILITIES.

1. It is designed here to illustrate, by elementary examples, the general method demonstrated in the last chapter. The examples chosen will be chiefly such as, from their simplicity, permit a ready verification of the solutions obtained. But some intimations will appear of a higher class of problems, hereafter to be more fully considered, the analysis of which would be incomplete without the aid of a distinct method determining the necessary conditions among their data, in order that they may represent a possible experience, and assigning the corresponding limits of the final solutions. The fuller consideration of that method, and of its applications, is reserved for the next chapter.

2. Ex. 1.—The probability that it thunders upon a given day is p, the probability that it both thunders and hails is q, but of the connexion of the two phænomena of thunder and hail, nothing further is supposed to be known. Required the probability that it hails on the proposed day.

Let x represent the event—It thunders.

Let y represent the event—It hails.

Then xy will represent the event—It thunders and hails; and the data of the problem are

$$\text{Prob.}, x = p, \quad \text{Prob.}, xy = q.$$

There being here but one compound event xy involved, assume, according to the rule,

$$xy = u. \tag{1}$$

Our data then become

$$\text{Prob.}, x = p, \quad \text{Prob.}, u = q; \tag{2}$$

and it is required to find $\text{Prob.}, y$. Now (1) gives

$$y = \frac{u}{x} = ux + \frac{1}{0}u(1-x) + 0(1-u)x + \frac{0}{0}(1-u)(1-x).$$

第十八章

概率论中一般方法的初等例示

1. 此处旨在通过初等例子来说明上一章所论证的一般方法。所选的例子主要是那些因其简单性而允许对所得解答进行迅速验证的例子。但也会出现一些关于更高一类问题的提示，这些问题将在以后更充分地考虑，若没有一种独特的方法来确定其数据之间的必要条件，以使它们能够表示一种可能的经验，并指定最终解答的相应界限，那么对它们的分析将是不完整的。对该方法及其应用的更充分的考虑，留待下一章。

2. 例1.-某天打雷的概率是p，既打雷又下冰雹的概率是q，但关于打雷和下冰雹这两种现象的联系，不再假设知道其他任何信息。求在提议的那天下冰雹的概率。

令表示事件--打雷。

设表示事件--下冰雹。

Then xy will represent the event—It thunders and hails; and the data of the problem are

$$\text{Prob.}, x = p, \quad \text{Prob.}, xy = q.$$

There being here but one compound event xy involved, assume, according to the rule,

$$xy = u. \tag{1}$$

我们的数据于是变为

$$\text{Prob.}, x = p, \quad \text{Prob.}, u = q; \tag{2}$$

需要求 Prob ., y。现在(1)给出

$$y = \frac{u}{x} = ux + \frac{1}{0}u(1-x) + 0(1-u)x + \frac{0}{0}(1-u)(1-x).$$

Hence (XVII. 17) we find

$$V = ux + (1-u)x + (1-u)(1-x),$$
$$V_x = ux + (1-u)x = x, \quad V_u = ux;$$

and the equations of the General Rule, viz.,

$$\frac{V_x}{p} = \frac{V_u}{q} = V.$$
$$\text{Prob.}, y = \frac{A + cC}{V}$$

become, on substitution, and observing that $A = ux$, $C = (1-u)(1-x)$, and that V reduces to $x + (1-u)(1-x)$,

$$\frac{x}{p} = \frac{ux}{q} = x + (1-u)(1-x), \tag{3}$$
$$\text{Prob.}, y = \frac{ux + c(1-u)(1-x)}{x + (1-u)(1-x)}, \tag{4}$$

from which we readily deduce, by elimination of x and u,

$$\text{Prob.}, y = q + c(l-p). \tag{5}$$

In this result c represents the unknown probability that if the event $(1-u)(1-x)$ happen, the event y will happen. Now $(l-u)(l-x) = (l-xy)(1-x) = 1-x$, on actual multiplication. Hence c is the unknown probability that if it do not thunder, it will hail.

The general solution (5) may therefore be interpreted as follows:—The probability that it hails is equal to the probability that it thunders and hails, q, together with the probability that it does not thunder, $1-p$, multiplied by the probability c, that if it does not thunder it will hail. And common reasoning verifies this result.

If c cannot be numerically determined, we find, on assigning to it the limiting values 0 and 1, the following limits of Prob., y, viz.:

$$\text{Inferior limit} = q.$$
$$\text{Superior limit} = q + 1 - p.$$

3. Ex. 2.—The probability that one or both of two events happen is p, that one or both of them fail is q. What is the probability that only one of these happens?

Let x and y represent the respective events, then the data are—

$$\text{Prob. } xy + x(1-y) + (1-x)y = p,$$
$$\text{Prob. } x(1-y) + (1-x)y + (1-x)(1-y) = q;$$

因此(XVII.17)我们得到

$$V = ux + (1-u)x + (1-u)(1-x),$$
$$V_x = ux + (1-u)x = x, \quad V_u = ux;$$

以及一般规则的方程式，即，

$$\frac{V_x}{p} = \frac{V_u}{q} = V.$$
$$\text{Prob.}, y = \frac{A + cC}{V}$$

become, on substitution, and observing that $A = ux$, $C = (1-u)(1-x)$, and that V reduces to $x + (1-u)(1-x)$,

$$\frac{x}{p} = \frac{ux}{q} = x + (1-u)(1-x), \tag{3}$$
$$\text{Prob.}, y = \frac{ux + c(1-u)(1-x)}{x + (1-u)(1-x)}, \tag{4}$$

由此，通过消去x和u，我们容易推导出，

$$\text{Prob.}, y = q + c(l-p). \tag{5}$$

在这个结果中，c 表示未知概率：如果事件 (1–u)(1–x) 发生，则事件 y 将发生。现在 (1–u)(1–x) = (1–xy)(1–x) = 1–x，经实际乘法运算。因此 c 是未知概率：如果不打雷，就会下冰雹。因此，通解 (5) 可以解释如下：——下冰雹的概率等于打雷且下冰雹的概率 q，加上不打雷的概率 1–p 乘以如果它不打雷就会下冰雹的概率 c。而常识推理验证了这一结果。如果 c 不能数值地确定，我们通过赋予其极限值 0 和 1，得到 Prob. y 的以下极限，即：

下限 = q。
$$\text{Superior limit} = q + 1 - p.$$

3. **例2.** - 两个事件中一个或两个发生的概率是p，其中一个或两个失败的概率是q。这些中只有一个发生的概率是多少?让x和y表示各自的事件，那么数据是-

$$\text{Prob. } xy + x(1-y) + (1-x)y = p,$$
$$\text{Prob. } x(1-y) + (1-x)y + (1-x)(1-y) = q;$$

and we are to find

$$\text{Prob. } x(1-y)+y(1-x).$$

Here all the events concerned being compound, assume

$$\begin{gathered} xy+x(1-y)+(1-x)y=s, \\ x(1-y)+(1-x)y+(1-x)(1-y)=t, \\ x(1-y)+(1-x)y=w. \end{gathered}$$

Then eliminating x and y, and determining w as a developed function of s and t, we find

$$w=st+0s(1-t)+0(1-s)t+\frac{1}{0}(1-s)(1-t).$$

Hence $A=st, C=0, V=st+s(1-t)+(1-s)t=s+(1-s)t, V_s=s, V_t=t$; and the equations of the General Rule (XVII. 17) become

$$\frac{s}{p}=\frac{t}{q}=s+(1-s)t, \tag{1}$$

$$\text{Prob. } w=\frac{st}{s+(1-s)t};$$

whence we find, on eliminating s and t,

$$\text{Prob. } w=p+q-1.$$

Hence $p+q-1$ is the measure of the probability sought. This result may be verified as follows:—Since p is the probability that one or both of the given events occur, $1-p$ will be the probability that they both fail; and since q is the probability that one or both fail, $1-q$ is the probability that they both happen. Hence $1-p+1-q$, or $2-p-q$, is the probability that they either both happen or both fail. But the only remaining alternative which is possible is that one alone of the events happens. Hence the probability of this occurrence is $1-(2-p-q)$, or $p+q-1$, as above. 4. Ex. 3.—The probability that a witness A speaks the truth is p, the probability that another witness B speaks the truth is q, and the probability that they disagree in a statement is r. What is the probability that if they agree, their statement is true?

Let x represent the hypothesis that A speaks truth; y that B speaks truth; then the hypothesis that A and B disagree in their statement will be represented by $x(1-y)+y(1-x)$; the hypothesis that they agree in statement by $xy+(1-x)(1-y)$, and the hypothesis that they agree in the truth by xy. Hence we have the following data:

$$\text{Prob. } x=p, \quad \text{Prob. } y=q, \quad \text{Prob. } x(1-y)+y(1-x)=r,$$

from which we are to determine

$$\frac{\text{Prob. } xy}{\text{Prob. } xy+(1-x)(1-y)}.$$

and we are to find

$$\text{Prob. } x(1-y)+y(1-x).$$

Here all the events concerned being compound, assume

$$\begin{gathered} xy+x(1-y)+(1-x)y=s, \\ x(1-y)+(1-x)y+(1-x)(1-y)=t, \\ x(1-y)+(1-x)y=w. \end{gathered}$$

然后消去x和y，并将w确定为s和t的展开式函数，我们发现

$$w=st+0s(1-t)+0(1-s)t+\frac{1}{0}(1-s)(1-t).$$

Hence $A=st, C=0, V=st+s(1-t)+(1-s)t=s+(1-s)t, V_s=s, V_t=t$; and the equations of the General Rule (XVII. 17) become

$$\frac{s}{p}=\frac{t}{q}=s+(1-s)t, \tag{1}$$

$$\text{Prob. } w=\frac{st}{s+(1-s)t};$$

由此消去s与t，我们得到，

$$\text{Prob. } w=p+q-1.$$

因此，p + q – 1 就是所求概率的度量。这一结果可以如下验证：--由于 p 是给定事件中一个或两个发生的概率，1 – p 将是它们都失败的概率；又由于 q 是一个或两个失败的概率，1 – q 是它们都发生的概率。因此 1 – p + 1 – q，或 2 – p – q，是它们要么都发生要么都失败的概率。但唯一剩下的可能替代情况是事件中仅有一个发生。因此这种情况发生的概率是 1 –(2 – p – q)，或 p + q – 1，如上所述。4. 例3。--证人 A 说真话的概率是 p，另一证人 B 说真话的概率是 q，他们在陈述中不一致的概率是 r。如果他们一致，其陈述为真的概率是多少?

设 x 表示 A 说真话的假说；y 表示 B 说真话；那么 A 和 B 在陈述中不一致的假说将由 x(1 – y) + y(1 – x) 表示；他们在陈述中一致的假说由 xy +(1 – x)(1 – y) 表示，他们在真理上一致的假说由 xy 表示。因此我们有以下数据：

$$\text{Prob. } x=p, \quad \text{Prob. } y=q, \quad \text{Prob. } x(1-y)+y(1-x)=r,$$

我们要由此确定

$$\frac{\text{Prob. } xy}{\text{Prob. } xy+(1-x)(1-y)}.$$

But as Prob. $x(1-y)+y(1-x)=r$, it is evident that Prob. $xy+(1-x)(1-y)$ will be $1-r$; we have therefore to seek

$$\frac{\text{Prob. } xy}{1-r}.$$

Now the compound events concerned being in expression, $x(1-y)+y(1-x)$ and xy, let us assume

$$\left.\begin{array}{rl} x(1-y)+y(1-x) & =s \\ xy & =w \end{array}\right\} \tag{1}$$

Our data then are Prob. $x=p$, Prob. $y=q$, Prob. $s=r$, and we are to find Prob. w.

The system (1) gives, on reduction,

$$\{x(1-y)+y(1-x)\}(1-s)+s\{xy+(1-x)(1-y)\} \\ +xy(1-w)+w(1-xy)=0;$$

whence

$$\begin{aligned} w &= \frac{x(1-y)(1-s)+y(1-x)(1-s)+sxy+s(1-x)(1-y)+xy}{2xy-1} \\ &= \frac{1}{0}xys+xy(1-s)+0x(1-y)s+\frac{1}{0}x(1-y)(1-s) \\ &\quad +0(1-x)ys+\frac{1}{0}(1-x)(1-y)s+\frac{1}{0}(1-x)y(1-s) \\ &\quad +0(1-x)(1-y)(1-s). \end{aligned} \tag{2}$$

In the expression of this development, the coefficient $\frac{1}{0}$ has been made to replace every equivalent form (X. 6). Here we have

$$V=xy(1-s)+x(1-y)s+(1-x)ys+(1-x)(1-y)(1-s);$$

whence, passing from Logic to Algebra,

$$\frac{xy(1-s)+x(1-y)s}{p}=\frac{xy(1-s)+(1-x)ys}{q} \\ =\frac{x(1-y)s+(1-x)ys}{r} \\ =xy(1-s)+x(1-y)s+(1-x)ys+(1-x)(1-y)(1-s).$$

$$\text{Prob. } w=\frac{xy(1-s)}{xy(1-s)+x(1-y)s+(1-x)ys+(1-x)(1-y)(1-s)},$$

from which we readily deduce

$$\text{Prob. } w=\frac{p+q-r}{2};$$

But as Prob. $x(1-y)+y(1-x)=r$, it is evident that Prob. $xy+(1-x)(1-y)$ will be $1-r$; we have therefore to seek

$$\frac{\text{Prob. } xy}{1-r}.$$

现在所涉及的复合事件在表达式中是x(1 – y) + y(1 – x)和xy，让我们假设

$$\left.\begin{array}{rl} x(1-y)+y(1-x) & =s \\ xy & =w \end{array}\right\} \tag{1}$$

那么我们的数据是Prob.x = p,Prob.y = q,Prob.s = r，我们要找Prob.w。

该系统(1)经化简后给出，

$$\{x(1-y)+y(1-x)\}(1-s)+s\{xy+(1-x)(1-y)\} \\ +xy(1-w)+w(1-xy)=0;$$

由此

$$\begin{aligned} w &= \frac{x(1-y)(1-s)+y(1-x)(1-s)+sxy+s(1-x)(1-y)+xy}{2xy-1} \\ &= \frac{1}{0}xys+xy(1-s)+0x(1-y)s+\frac{1}{0}x(1-y)(1-s) \\ &\quad +0(1-x)ys+\frac{1}{0}(1-x)(1-y)s+\frac{1}{0}(1-x)y(1-s) \\ &\quad +0(1-x)(1-y)(1-s). \end{aligned} \tag{2}$$

在这一展开式中，系数[1]

0 有被使得去替换

每一个等价形式(X.6)。这里我们有

$$V=xy(1-s)+x(1-y)s+(1-x)ys+(1-x)(1-y)(1-s);$$

由此，从逻辑学过渡到代数学，

$$\frac{xy(1-s)+x(1-y)s}{p}=\frac{xy(1-s)+(1-x)ys}{q} \\ =\frac{x(1-y)s+(1-x)ys}{r} \\ =xy(1-s)+x(1-y)s+(1-x)ys+(1-x)(1-y)(1-s).$$

$$\text{Prob. } w=\frac{xy(1-s)}{xy(1-s)+x(1-y)s+(1-x)ys+(1-x)(1-y)(1-s)},$$

由此我们容易推得

$$\text{Prob. } w=\frac{p+q-r}{2};$$

whence we have

$$\frac{\text{Prob. } xy}{1-r} = \frac{p+q-r}{2(1-r)} \tag{3}$$

for the value sought.

If in the same way we seek the probability that if A and B agree in their statement, that statement will be false, we must replace the second equation of the system (1) by the following, viz.:

$$(1-x)(1-y) = w;$$

the final logical equation will then be

$$\begin{aligned} w = \frac{1}{0}xys + 0xy(1-s) + 0x(1-y)s + \frac{1}{0}x(1-y)(1-s) \\ +0(1-x)ys + \frac{1}{0}(1-x)y(1-s) + \frac{1}{0}(1-x)(1-y)s \\ +(1-x)(1-y)(1-s); \end{aligned} \tag{4}$$

whence, proceeding as before, we finally deduce

$$\text{Prob. } w = \frac{2-p-q-r}{2}. \tag{5}$$

Wherefore we have

$$\frac{\text{Prob. } (1-x)(1-y)}{1-r} = \frac{2-p-g-r}{2(1-r)} \tag{6}$$

for the value here sought.

These results are mutually consistent. For since it is certain that the joint statement of A and B must be either true or false, the second members of (3) and (5) ought by addition to make 1. Now we have identically,

$$\frac{p+q-r}{2(1-r)} + \frac{2-p-q-r}{2(1-r)} = 1.$$

It is probable, from the simplicity of the results (5) and (6), that they might easily be deduced by the application of known principles; but it is to be remarked that they do not fall directly within the scope of known *methods*. The number of the data exceeds that of the simple events which they involve. M. Cournot, in his very able work, "Exposition de la Theorie des Chances," has proposed, in such cases as the above, to select from the original premises different sets of data, each set equal in number to the simple events which they involve, to assume that those simple events are independent, determine separately from the respective sets of the data their probabilities, and comparing the different values thus found for the same elements, judge how far the assumption of independence is justified. This method can only approach to correctness when the said simple events prove, according to the above criterion, to be nearly or quite independent; and in the questions of testimony and of judgment, in which such an hypothesis

由此我们得到

$$\frac{\text{Prob. } xy}{1-r} = \frac{p+q-r}{2(1-r)} \tag{3}$$

以求所需之值。

如果以同样的方式我们寻求概率，即如果A和B在他们的陈述中一致，那个陈述将是假的，我们必须用以下替换系统(1)的第二个方程，即：

$$(1-x)(1-y) = w;$$

那么最终的逻辑方程将是

$$\begin{aligned} w = \frac{1}{0}xys + 0xy(1-s) + 0x(1-y)s + \frac{1}{0}x(1-y)(1-s) \\ +0(1-x)ys + \frac{1}{0}(1-x)y(1-s) + \frac{1}{0}(1-x)(1-y)s \\ +(1-x)(1-y)(1-s); \end{aligned} \tag{4}$$

由此，按照先前的方法继续推演，我们最终推导出

$$\text{Prob. } w = \frac{2-p-q-r}{2}. \tag{5}$$

因此我们有

$$\frac{\text{Prob. } (1-x)(1-y)}{1-r} = \frac{2-p-g-r}{2(1-r)} \tag{6}$$

for the value here sought.

这些结果是相互一致的。因为既然A和B的联合陈述必须是真或假是确定的，(3)和(5)的第二项通过相加应该等于1。现在我们有恒等式，

$$\frac{p+q-r}{2(1-r)} + \frac{2-p-q-r}{2(1-r)} = 1.$$

从结果(5)和(6)的简单性来看，很可能它们可以容易地通过应用已知原理推导出来；但值得注意的是，它们并不直接属于已知方法的范围。数据的数量超过了它们所涉及的简单事件的数量。库尔诺先生在其实力非凡的著作《概率理论阐述》中提出，在上述情况下，从原始前提中选择不同的数据集，每组数据在数量上等于它们所涉及的简单事件，假设那些简单事件是独立的，分别从各自的数据集确定其概率，并比较由此为同一元素找到的不同值，判断独立性假设在多大程度上是合理的。这种方法只有在上述简单事件根据上述标准被证明是近乎或完全独立时才能接近正确；而在证词和判断的问题中，这种假说

is adopted, it seems doubtful whether it is justified by actual experience of the ways of men.

5. Ex. 4.—From observations made during a period of general sickness, there was a probability p that any house taken at random in a particular district was visited by fever, a probability q that it was visited by cholera, and a probability r that it escaped both diseases, and was not in a defective sanitary condition as regarded cleanliness and ventilation. What is the probability that any house taken at random was in a defective sanitary condition?

With reference to any house, let us appropriate the symbols x, y, z, as follows, viz.:

The symbol x to the visitation of fever.
y " cholera.
z defective sanitary condition.

The events whose probabilities are given are then denoted by x, y, and $(1-x)(1-y)(1-z)$, the event whose probability is sought is z. Assume then,

$$(1-x)(1-y)(1-z) = w;$$

then our data are,

$$\text{Prob. } x = p, \quad \text{Prob. } y = q, \quad \text{Prob. } w = r,$$

and we are to find Prob. z. Now

$$\begin{aligned} z = \frac{(1-x)(1-y)-w}{(1-x)(1-y)} \\ = \frac{1}{0}xyw + \frac{0}{0}xy(1-w) + \frac{1}{0}x(1-y) + \frac{0}{0}x(1-y)(1-w) \\ + \frac{1}{0}(1-x)yw + \frac{0}{0}(1-x)y(1-w) + 0(1-x)(1-y)w \\ + (1-x)(1-y)(1-w). \quad (1) \end{aligned}$$

The value of V deduced from the above is

$$V = xy(1-w) + x(1-y)(1-w) + (1-x)y(1-w) \\ +(1-x)(1-y)w + (1-x)(1-y)(1-w) = 1 - w + w(1-x)(1-y);$$

and similarly reducing V_x, V_y, V_w, we get

$$V_x = x(1-w), \quad V_y = y(1-w), \quad V_w = w(1-x)(1-y);$$

furnishing the algebraic equations

$$\frac{x(1-w)}{p} = \frac{y(1-w)}{q} = \frac{w(1-x)(1-y)}{r} = 1 - w + w(1-x)(1-y). \quad (2)$$

As respects those terms of the development characterized by the coefficients $\frac{0}{0}$, I shall, instead of collecting them into a single term, present them, for the sake of variety (XVII. 18), in the form

$$\frac{0}{0}x(1-w) + \frac{0}{0}(1-x)y(1-w); \quad (3)$$

被采用，它似乎令人怀疑是否被人类行为方式的实际经验所证明。

5. 例4.-从一段普遍疾病期间所作的观察，在特定区域随机选取的任何房屋被热病访问的概率是p，被霍乱访问的概率是q，并且逃脱两种疾病，并且在清洁和通风方面不处于有缺陷的卫生条件的概率是r。随机选取的任何房屋处于有缺陷的卫生条件的概率是多少？

关于任何房屋，让我们采用记号 x, y, z， 如下，即：

符号x表示对热病的探访。
霍乱。卫生条件不良。

那么，其概率已给出的事件由 x、y 和 (1 – x)(1 – y)(1 – z) 表示，其概率被寻求的事件是 z。那么假设，

$$(1-x)(1-y)(1-z) = w;$$

那么我们的数据是，

$$\text{Prob. } x = p, \quad \text{Prob. } y = q, \quad \text{Prob. } w = r,$$

而我们要去求 Prob.z。现在

$$\begin{aligned} z = \frac{(1-x)(1-y)-w}{(1-x)(1-y)} \\ = \frac{1}{0}xyw + \frac{0}{0}xy(1-w) + \frac{1}{0}x(1-y) + \frac{0}{0}x(1-y)(1-w) \\ + \frac{1}{0}(1-x)yw + \frac{0}{0}(1-x)y(1-w) + 0(1-x)(1-y)w \\ + (1-x)(1-y)(1-w). \quad (1) \end{aligned}$$

The value of V deduced from the above is

$$V = xy(1-w) + x(1-y)(1-w) + (1-x)y(1-w) \\ +(1-x)(1-y)w + (1-x)(1-y)(1-w) = 1 - w + w(1-x)(1-y);$$

and similarly reducing V_x, V_y, V_w, we get

$$V_x = x(1-w), \quad V_y = y(1-w), \quad V_w = w(1-x)(1-y);$$

furnishing the algebraic equations

$$\frac{x(1-w)}{p} = \frac{y(1-w)}{q} = \frac{w(1-x)(1-y)}{r} = 1 - w + w(1-x)(1-y). \quad (2)$$

关于展开式中由系数$\frac{0}{0}$表征的那些项，为求变化(xvii.18)，我将不把它们归并为单一的一-项，而是以如下形式呈现它们：

$$\frac{0}{0}x(1-w) + \frac{0}{0}(1-x)y(1-w); \quad (3)$$

the value of Prob. z will then be

$$\text{Prob. } z = \frac{(1-x)(1-y)(1-w) + cx(1-w) + c'(1-x)y(1w)}{1-w+w(1-x)(1-y)}. \qquad (4)$$

From (2) and (4) we deduce

$$\text{Prob. } z = \frac{(1-p-r)(1-q-r)}{1-r} + cp + c'\frac{q(1-p-r)}{1-r},$$

as the expression of the probability required. If in this result we make $c = 0$, and $c' = 0$, we find for an inferior limit of its value $\frac{(1-p-r)(1-q-r)}{1-r}$; and if we make $c = 1$, $c' = 1$, we obtain for its superior limit $1 - r$.

6. It appears from inspection of this solution, that the premises chosen were exceedingly defective. The constants c and c' indicate this, and the corresponding terms (3) of the final logical equation show how the deficiency is to be supplied. Thus, since

$$x(1-w) = x\{1-(1-x)(1-y)(1-z)\} = x,$$
$$(1-x)y(1-w) = (1-x)y\{1-(1-x)(1-y)(1-z)\} = (1-x)y,$$

we learn that c is the probability that if any house was visited by fever its sanitary condition is defective, and that c' is the probability that if any house was visited by cholera without fever, its sanitary condition was defective.

If the terms of the logical development affected by the coefficient $\frac{0}{0}$ had been collected together as in the direct statement of the general rule, the final solution would have assumed the following form:

$$\text{Prob. } z = \frac{(1-p-r)(1-q-r)}{1-r} + c\left(p+q-\frac{pq}{1-r}\right)$$

c here representing the probability that if a house was visited by either or both of the diseases mentioned, its sanitary condition was defective. This result is perfectly consistent with the former one, and indeed the *necessary* equivalence of the different forms of solution presented in such cases may be formally established.

The above solution may be verified in particular cases. Thus, taking the second form, if $c = 1$ we find Prob. $z = 1-r$, a correct result. For if the presence of either fever or cholera *certainly* indicated a defective sanitary condition, the probability that any house would be in a defective sanitary state would be simply equal to the probability that it was *not* found in that category denoted by z, the probability of which would, by the data, be $1-r$, Perhaps the general verification of the above solution would be difficult.

The constants p, q, and r in the above solution are subject to the conditions

$$p + r \overset{=}{<} 1, \quad q + r \overset{=}{<} 1.$$

那么 Prob.z 的值将是

$$\text{Prob. } z = \frac{(1-x)(1-y)(1-w) + cx(1-w) + c'(1-x)y(1w)}{1-w+w(1-x)(1-y)}. \qquad (4)$$

从(2)和(4)我们推得

$$\text{Prob. } z = \frac{(1-p-r)(1-q-r)}{1-r} + cp + c'\frac{q(1-p-r)}{1-r},$$

as the expression of the probability required. If in this result we make $c = 0$, and $c' = 0$, we find for an inferior limit of its value $\frac{(1-p-r)(1-q-r)}{1-r}$; and if we make $c = 1$, $c' = 1$, we obtain for its superior limit $1 - r$.

6. 从对这一解答的检查可以看出，所选的前提极为不完善。常数 c 和 c′表明了这一点，而最终逻辑方程的相应项(3)则表明这种缺陷应如何弥补。因此，由于

$$x(1-w) = x\{1-(1-x)(1-y)(1-z)\} = x,$$
$$(1-x)y(1-w) = (1-x)y\{1-(1-x)(1-y)(1-z)\} = (1-x)y,$$

我们得知 c 是若任何房屋曾遭热病侵袭则其卫生状况不良的概率，而 c′是若任何房屋曾遭不伴热病的霍乱侵袭则其卫生状况不良的概率。

如果逻辑展开式中受系数00影响的各项按照一般规则的直接陈述方式收集在一起，则最终解答将采取以下形式：

$$\text{Prob. } z = \frac{(1-p-r)(1-q-r)}{1-r} + c\left(p+q-\frac{pq}{1-r}\right)$$

此处 c 表示若某房屋曾遭上述疾病之一或两者侵袭则其卫生状况不良的概率。这一结果与前一结果完全一致，而且事实上，在此类情形下所给出的不同解答形式之间的必然等价性可以形式地建立。

上述解答可在特例中验证。于是，取第二种形式，若 c = 1，我们得到 Prob.z = 1 – r，这是一个正确的结果。因为如果任一热病或霍乱必然表明卫生状况有缺陷，那么任何房屋处于有缺陷卫生状态的概率就简单地等于它未被发现于由 z 所表示的那一范畴中的概率，根据数据，该概率为 1 – r。也许上述解答的一般验证会是困难的。

上述解答中的常数 p、q 和 r 受以下条件约束

$$p + r \overset{=}{<} 1, \quad q + r \overset{=}{<} 1.$$

7. Ex. 5.—Given the probabilities of the premises of a hypothetical syllogism to find the probability of the conclusion.

Let the syllogism in its naked form be as follows:

Major premiss: If the proposition Y is true X is true.
Minor premiss: If the proposition Z is true Y is true.
Conclusion: If the proposition Z is true X is true.

Suppose the probability of the major premiss to be p, that of the minor premiss q.

The data then are as follows, representing the proposition X by x, &c., and assuming c and c' as arbitrary constants:

$$\text{Prob. } y = c, \qquad \text{Prob. } xy = cp;$$
$$\text{Prob. } z = c', \qquad \text{Prob. } yz = c'q;$$

from which we are to determine,

$$\frac{\text{Prob. } xz}{\text{Prob. } z} \text{ or } \frac{\text{Prob. } xz}{c'}.$$

Let us assume,

$$xy = u, \quad yz = v, \quad xz = w,$$

then, proceeding according to the usual method to determine w as a developed function of y, z, w, and v, the symbols corresponding to propositions whose probabilities are given, we find

$$\begin{aligned} w = {} & uzvy + 0u(1-z)(1-v)y + 0(1-u)zvy \\ & + \frac{0}{0}(1-u)z(1-v)(1-y) + 0(1-u)(1-z)(1-v)y \\ & + 0(1-u)(1-z)(1-v)(1-y) + \text{ terms whose coefficients are } \frac{1}{0}; \end{aligned}$$

and passing from Logic to Algebra,

$$\begin{aligned} \frac{uzvy + u(1-z)(1-v)y}{cp} &= \frac{uzvy + (1-u)zvy + (1-u)z(1-v)(1-y)}{c'} \\ &= \frac{uzvy + (1-u)zvy}{c'q} \\ = \frac{uzvy + u(1-z)(1-v)y + (1-u)zvy + (1-u)(1-z)(1-v)y}{c} &= V. \end{aligned}$$

$$\text{Prob. } w = \frac{uzvy + a(1-u)z(1-v)(1-y)}{V},$$

wherein

$$\begin{aligned} V = {} & uzvy + u(1-z)(1-v)y + (1-u)zvy + (1-u)z(1-v)(1-y) \\ & + (1-u)(1-z)(1-v)y + (1-u)(1-z)(1-v)(1-y), \end{aligned}$$

7. 例5--已知假言三段论各前提的概率，求结论的概率。

设三段论以其赤裸形式如下：

大前提：若命题 Y 为真，则 X 为真。小前提：若命题 Z 为真，则 Y 为真。结论：若命题 Z 为真，则 X 为真。

设大前提的概率为 p，小前提的概率为 q。

于是数据如下，以 x 表示命题 X，等等，并假定 c 和 c′为任意常数：

$$\text{Prob. } y = c, \qquad \text{Prob. } xy = cp;$$
$$\text{Prob. } z = c', \qquad \text{Prob. } yz = c'q;$$

我们要从中确定，

$$\frac{\text{Prob. } xz}{\text{Prob. } z} \text{ or } \frac{\text{Prob. } xz}{c'}.$$

让我们假设

$$xy = u, \quad yz = v, \quad xz = w,$$

然后，按照通常的方法，将 w 确定为 y、z、w 和 v 的展开函数，这些记号对应于其概率已给出的命题，我们求得

$$\begin{aligned} w = {} & uzvy + 0u(1-z)(1-v)y + 0(1-u)zvy \\ & + \frac{0}{0}(1-u)z(1-v)(1-y) + 0(1-u)(1-z)(1-v)y \\ & + 0(1-u)(1-z)(1-v)(1-y) + \text{ terms whose coefficients are } \frac{1}{0}; \end{aligned}$$

而从逻辑过渡到代数，

$$\begin{aligned} \frac{uzvy + u(1-z)(1-v)y}{cp} &= \frac{uzvy + (1-u)zvy + (1-u)z(1-v)(1-y)}{c'} \\ &= \frac{uzvy + (1-u)zvy}{c'q} \\ = \frac{uzvy + u(1-z)(1-v)y + (1-u)zvy + (1-u)(1-z)(1-v)y}{c} &= V. \end{aligned}$$

$$\text{Prob. } w = \frac{uzvy + a(1-u)z(1-v)(1-y)}{V},$$

其中

$$\begin{aligned} V = {} & uzvy + u(1-z)(1-v)y + (1-u)zvy + (1-u)z(1-v)(1-y) \\ & + (1-u)(1-z)(1-v)y + (1-u)(1-z)(1-v)(1-y), \end{aligned}$$

the solution of this system of equations gives

$$\text{Prob. } w = c'pq + ac'(1-q),$$

whence

$$\frac{\text{Prob. } xy}{c'} = pq + a(1-q),$$

the value required. In this expression the arbitrary constant a is the probability that if the proposition Z is true and Y false, X is true. In other words, it is the probability, that if the minor premiss is false, the conclusion is true.

This investigation might have been greatly simplified by assuming the proposition Z to be true, and then seeking the probability of X. The data would have been simply

$$\text{Prob. } y = q, \quad \text{Prob. } xy = pq;$$

whence we should have found Prob. $x = pq + a(1-q)$. It is evident that under the circumstances this mode of procedure would have been allowable, but I have preferred to deduce the solution by the direct and unconditioned application of the method. The result is one which ordinary reasoning verifies, and which it does not indeed require a calculus to obtain. General methods are apt to appear most cumbrous when applied to cases in which their aid is the least required.

Let it be observed, that the above method is equally applicable to the categorical syllogism, and not to the syllogism only, but to every form of deductive ratiocination. Given the probabilities separately attaching to the premises of *any* train of argument; it is always possible by the above method to determine the consequent probability of the truth of a conclusion legitimately drawn from such premises. It is not needful to remind the reader, that the truth and the correctness of a conclusion are different things.

8. One remarkable circumstance which presents itself in such applications deserves to be specially noticed. It is, that propositions which, when true, are equivalent, are not necessarily equivalent when regarded only as probable. This principle will be illustrated in the following example.

Ex. 6.—Given the probability p of the disjunctive proposition "Either the proposition Y is true, or both the propositions X and Y are false," required the probability of the conditional proposition, "If the proposition X is true, Y is true."

Let x and y be appropriated to the propositions X and Y respectively. Then we have

$$\text{Prob. } y + (1-x)(1-y) = p,$$

from which it is required to find the value of $\frac{\text{Prob. } xy}{\text{Prob. } x}$.

$$\text{Assume} y + (1-x)(1-y) = t. \tag{1}$$

Eliminating y we get

$$(1-x)(1-t) = 0.$$

该方程组的解给出

$$\text{Prob. } w = c'pq + ac'(1-q),$$

由此

$$\frac{\text{Prob. } xy}{c'} = pq + a(1-q),$$

所需的值。在此表达式中，任意常数 a 是若命题 Z 为真而 Y 为假则 X 为真的概率。换言之，它是若小前提为假则结论为真的概率。

本探究若假定命题 Z 为真，然后求 X 的概率，本可大大简化。数据将简单地是

$$\text{Prob. } y = q, \quad \text{Prob. } xy = pq;$$

由此我们本应求得 Prob.x = pq + a(1 – q)。显然，在这种情况下，这种程序方式本会是允许的，但我更愿意通过该方法直接而无条件的应用来推出解答。这一结果是一般推理所验证的，而且确实不需要微积分来获得。一般方法在被应用于最不需要其帮助的情形时，往往显得最为累赘。

请注意，上述方法同样适用于直言三段论，而且不仅适用于三段论，还适用于每一种演绎推理形式。给定分别附着于任何论证序列的前提的概率；通过上述方法，总是可以确定从这些前提合法地得出的结论为真的后承概率。无需提醒读者，结论的真与结论的正确是不同的事情。

8. 在此类应用中出现的有一值得特别注意的显著情形。这就是：当为真时等价的命题，在仅被视为或然时并不必然等价。这一原理将在下例中说明。

例6--已知析取命题"或者命题 Y 为真，或者命题 X 和 Y 两者皆假"的概率为 p，求条件命题"若命题 X 为真，则 Y 为真"的概率。

Let x and y be appropriated to the propositions X and Y respectively. Then we have

$$\text{Prob. } y + (1-x)(1-y) = p,$$

from which it is required to find the value of $\frac{\text{Prob. } xy}{\text{Prob. } x}$.

$$\text{Assume} y + (1-x)(1-y) = t. \tag{1}$$

消去 y 得 Eliminating y we get

$$(1-x)(1-t) = 0.$$

whence

$$x = \frac{0}{0}t + 1 - t;$$

and proceeding in the usual way,

$$\text{Prob. } x = 1 - p + cp. \tag{2}$$

Where c is the probability that if either Y is true, or X and Y false, X is true.

Next to find Prob. xy. Assume

$$xy = w. \tag{3}$$

Eliminating y from (1) and (3) we get

$$z(1 - t) = 0;$$

whence, proceeding as above,

$$\text{Prob. } z = cp,$$

c having the same interpretation as before. Hence

$$\frac{\text{Prob. } xy}{\text{Prob. } x} = \frac{cp}{1 - p + cp},$$

for the probability of the truth of the conditional proposition given.

Now in the science of pure Logic, which, as such, is conversant only with truth and with falsehood, the above disjunctive and conditional propositions are equivalent. They are true and they are false together. It is seen, however, from the above investigation, that when the disjunctive proposition has a probability p, the conditional proposition has a different and partly indefinite probability $\frac{cp}{1-p+cp}$. Nevertheless these expressions are such, that when *either of them becomes 1 or 0, the other assumes the same value.* The results are, therefore, perfectly consistent, and the logical transformation serves to verify the formula deduced from the theory of probabilities.

The reader will easily prove by a similar analysis, that if the probability of the conditional proposition were given as p, that of the disjunctive proposition would be $1 - c + cp$, where c is the arbitrary probability of the truth of the proposition X.

9. Ex. 7.—Required to determine the probability of an event x, having given either the first, or the first and second, or the first, second, and third of the following data, viz.:

1st. The probability that the event x occurs, or that it alone of the three events x, y, z, fails, is p.

2nd. The probability that the event y occurs, or that it alone of the three events x, y, z, fails, is q.

3rd. The probability that the event z occurs, or that it alone of the three events x, y, z, fails, is r.

由此

$$x = \frac{0}{0}t + 1 - t;$$

并按照通常方式进行，

$$\text{Prob. } x = 1 - p + cp. \tag{2}$$

Where c is the probability that if either Y is true, or X and Y false, X is true.

Next to find Prob. xy. Assume

$$xy = w. \tag{3}$$

从(1)和(3)中消去y，我们得到

$$z(1 - t) = 0;$$

由此，按上述方法进行，

$$\text{Prob. } z = cp,$$

c 具有与之前相同的解释。因此

$$\frac{\text{Prob. } xy}{\text{Prob. } x} = \frac{cp}{1 - p + cp},$$

对于给定的条件命题为真的概率。

然而，在纯逻辑科学中，就其本身而言，它只与真和假打交道，上述析取命题和条件命题是等价的。它们同真且同假。然而，从上述探究可以看出，当析取命题具有概率 p 时，条件命题具有一个不同的且部分不确定的概率

~~cp~~ ~~1 − p~~ + cp ·然而这些表达式是这样的：当它们中的任一个变为*1*或*0*时，另一个取相同的值。因此，结果完全一致，而逻辑变换有助于验证从概率论推出的公式。

读者通过类似的分析将容易证明，若条件命题的概率给定为 p，则析取命题的概率将为 1 – c + cp，其中 c 是命题 X 为真的任意概率。

9. 例7--要求确定事件 x 的概率，已知下列数据中的第一项，或第一项和第二项，或第一项、第二项和第三项，即：

1. 事件 x 发生，或在三个事件 x、y、z 中唯独它不发生的概率为 p。2. 事件 y 发生，或在三个事件 x、y、z 中唯独它不发生的概率为 q。3. 事件 z 发生，或在三个事件 x、y、z 中唯独它不发生的概率为 r。

SOLUTION OF THE FIRST CASE.

Here we suppose that only the first of the above data is given. We have then,

$$\text{Prob. } \{x + (1-x)yz\} = p,$$

to find Prob. x,

$$\text{Let} x + (1-x)yz = s,$$

then eliminating yz as a single symbol, we get,

$$x(1-s) = 0.$$

Hence

$$x = \frac{0}{1-s} = \frac{0}{0}s + 0(1-s),$$

whence, proceeding according to the rule, we have

$$\text{Prob. } x = cp, \tag{1}$$

where c is the probability that if x occurs, or alone fails, the former of the two alternatives is the one that will happen. The limits of the solution are evidently 0 and p.

This solution appears to give us no information beyond what unassisted good sense would have conveyed. It is, however, all that the single datum here assumed really warrants us in inferring. We shall in the next solution see how an addition to our data restricts within narrower limits the final solution.

SOLUTION OF THE SECOND CASE.

Here we assume as our data the equations

$$\text{Prob. } \{x + (1-x)yz\} = p,$$
$$\text{Prob. } \{y + (1-y)xz\} = q.$$

Let us write

$$x + (1-x)yz = s,$$
$$y + (1-y)xz = q,$$

from the first of which we have, by (VIII. 7),

$$\{x + (1-x)yz\}(1-s) + s\{1 - x - (1-x)yz\} = 0,$$
$$\text{or} (x + \bar{x}yz)\bar{s} + s\bar{x}(1-yz) = 0;$$

provided that for simplicity we write $\bar{x}$ for $1-x$, $\bar{y}$ for $1-y$, and so on. Now, writing for $1-yz$ its value in constituents, we have

$$(x + \bar{x}yz)\bar{s} + s\bar{x}(y\bar{z} + \bar{y}z + \bar{y}\bar{z}) = 0,$$

第一种情形的解。

这里我们假设只给出上述数据中的第一个。我们有

那么，

$$\text{Prob. } \{x + (1-x)yz\} = p,$$

求 Prob.x,

$$\text{Let} x + (1-x)yz = s,$$

then eliminating yz as a single symbol, we get,

$$x(1-s) = 0.$$

Hence

$$x = \frac{0}{1-s} = \frac{0}{0}s + 0(1-s),$$

whence, proceeding according to the rule, we have

$$\text{Prob. } x = cp, \tag{1}$$

其中 c 是若 x 发生，或单独不发生，则两个备选者中的前者将会发生的概率。解答的界限显然是 0 和 p。

这一解答似乎除了未受助的良好常识本会传达的内容之外，并未给我们任何信息。然而，它只是此处所假定的单一数据真正使我们有理由推断的全部内容。在下一解答中我们将看到，对数据的增加如何将最终解答限制在更窄的界限内。

第二种情形的解。

这里我们以这些方程作为我们的数据

$$\text{Prob. } \{x + (1-x)yz\} = p,$$
$$\text{Prob. } \{y + (1-y)xz\} = q.$$

让我们写

$$x + (1-x)yz = s,$$
$$y + (1-y)xz = q,$$

from the first of which we have, by (VIII. 7),

$$\{x + (1-x)yz\}(1-s) + s\{1 - x - (1-x)yz\} = 0,$$
$$\text{or} (x + \bar{x}yz)\bar{s} + s\bar{x}(1-yz) = 0;$$

provided that for simplicity we write $\bar{x}$ for $1-x$, $\bar{y}$ for $1-y$, and so on. Now, writing for $1-yz$ its value in constituents, we have

$$(x + \bar{x}yz)\bar{s} + s\bar{x}(y\bar{z} + \bar{y}z + \bar{y}\bar{z}) = 0,$$

an equation consisting solely of positive terms. In like manner we have from the second equation,

$$(y + \bar{y}xz)\bar{t} + t\bar{y}(x\bar{z} + \bar{x}z + \bar{x}\bar{z}) = 0;$$

and from the sum of these two equations we are to eliminate y and z.

If in that sum we make $y = 1$, $z = 1$, we get the result $\bar{s} + \bar{t}$.

If in the same sum we make $y = 1$, $z = 0$, we get the result

$$x\bar{s} + s\bar{x} + \bar{t}.$$

If in the same sum we make $y = 0$, $z = 1$, we get

$$x\bar{s} + s\bar{x} + x\bar{t} + t\bar{x}.$$

And if, lastly, in the same sum we make $y = 0$, $z = 0$, we find

$$x\bar{s} + s\bar{x} + tx + t\bar{x}, \text{ or } x\bar{s} + s\bar{x} + t.$$

These four expressions are to be multiplied together. Now the first and third may be multiplied in the following manner:

$$\begin{gathered}(\bar{s} + \bar{t})(x\bar{s} + s\bar{x} + x\bar{t} + t\bar{x}) \\ = x\bar{s} + x\bar{t} + (\bar{s} + \bar{t})(s\bar{x} + t\bar{x}) \text{ by (IX. Prop. II.)} \\ = x\bar{s} + x\bar{t} + \bar{s}\bar{x}t + s\bar{x}\bar{t}. \qquad (2)\end{gathered}$$

Again, the second and fourth give by (IX. Prop. I.)

$$\begin{gathered}(x\bar{s} + s\bar{x} + \bar{t})(x\bar{s} + s\bar{x} + t) \\ = x\bar{s} + s\bar{x}. \qquad (3)\end{gathered}$$

Lastly, (2) and (3) multiplied together give

$$\begin{gathered}(x\bar{s} + s\bar{x})(x\bar{s} + s\bar{x}\bar{t} + x\bar{t} + t\bar{x}\bar{s}) \\ = x\bar{s} + s\bar{x}(s\bar{x}\bar{t} + x\bar{t} + t\bar{x}\bar{s}) \\ = x\bar{s} + s\bar{x}\bar{t}.\end{gathered}$$

Whence the final equation is

$$(1 - s)x + s(1 - t)(1 - x) = 0,$$

which, solved with reference to x, gives

$$\begin{gathered}x = \frac{s(1-t)}{s(1-t) - (1-s)} \\ = \frac{0}{0}st + s(1-t) + 0(1-s)t + 0(1-s)(1-t),\end{gathered}$$

and, proceeding with this according to the rule, we have, finally,

$$\text{Prob. } x = p(1-q) + cpq. \qquad (4)$$

an equation consisting solely of positive terms. In like manner we have from the second equation,

$$(y + \bar{y}xz)\bar{t} + t\bar{y}(x\bar{z} + \bar{x}z + \bar{x}\bar{z}) = 0;$$

并且从这两个方程之和中我们要消去 y 和 z。

若在该和中令 y = 1,z = 1，我们得到结果 s + ¯t。若在同一和中令 y = 1,z = 0，我们得到结果

$$x\bar{s} + s\bar{x} + \bar{t}.$$

若在同一和式中令 y = 0,z = 1，则得到

$$x\bar{s} + s\bar{x} + x\bar{t} + t\bar{x}.$$

最后，如果在同一和式中令 y = 0、z = 0，则得到

$$x\bar{s} + s\bar{x} + tx + t\bar{x}, \text{ or } x\bar{s} + s\bar{x} + t.$$

这四个表达式要相乘。现在第一和第三可以按以下方式相乘：

$$\begin{gathered}(\bar{s} + \bar{t})(x\bar{s} + s\bar{x} + x\bar{t} + t\bar{x}) \\ = x\bar{s} + x\bar{t} + (\bar{s} + \bar{t})(s\bar{x} + t\bar{x}) \text{ by (IX. Prop. II.)} \\ = x\bar{s} + x\bar{t} + \bar{s}\bar{x}t + s\bar{x}\bar{t}. \qquad (2)\end{gathered}$$

又，第二式和第四式由(IX.命题 i.)给出

$$\begin{gathered}(x\bar{s} + s\bar{x} + \bar{t})(x\bar{s} + s\bar{x} + t) \\ = x\bar{s} + s\bar{x}. \qquad (3)\end{gathered}$$

最后，将(2)与(3)相乘得到

$$\begin{gathered}(x\bar{s} + s\bar{x})(x\bar{s} + s\bar{x}\bar{t} + x\bar{t} + t\bar{x}\bar{s}) \\ = x\bar{s} + s\bar{x}(s\bar{x}\bar{t} + x\bar{t} + t\bar{x}\bar{s}) \\ = x\bar{s} + s\bar{x}\bar{t}.\end{gathered}$$

由此得到最终方程

$$(1 - s)x + s(1 - t)(1 - x) = 0,$$

将其关于 x 求解，得出

$$\begin{gathered}x = \frac{s(1-t)}{s(1-t) - (1-s)} \\ = \frac{0}{0}st + s(1-t) + 0(1-s)t + 0(1-s)(1-t),\end{gathered}$$

并且，按照这一规则进行下去，我们最终得到，

$$\text{Prob. } x = p(1-q) + cpq. \qquad (4)$$

where c is the probability that if the event st happen, x will happen. Now if we form the developed expression of st by multiplying the expressions for s and t together, we find—

c = Prob. that if x; and y happen together, or x and z happen together, and y fail, or y and z happen together, and x fail, the event x will happen.

The limits of Prob. x are evidently $p(1-q)$ and p.

This solution is more definite than the former one, inasmuch as it contains a term unaffected by an arbitrary constant.

SOLUTION OF THE THIRD CASE.

Here the data are— $\begin{aligned} &\text{Prob. } \{x+(1-x)yz\} = p,\\ &\text{Prob. } \{y+(1-y)xz\} = q,\\ &\text{Prob. } \{z+(1-z)xy\} = r. \end{aligned}$

Let us, as before, write $\bar{x}$ for $1-x$, &c., and assume

$$\begin{aligned} x+\bar{x}yz &= s,\\ y+\bar{y}xz &= t,\\ z+\bar{z}xy &= u. \end{aligned}$$

On reduction by (VIII. 8) we obtain the equation

$$\begin{aligned} &(x+\bar{x}yz)\bar{s}+s\bar{x}(y\bar{z}+\bar{y}z+\bar{y}\bar{z})\\ &+(y+\bar{y}xz)\bar{t}+t\bar{y}(z\bar{x}+x\bar{z}+\bar{x}\bar{z})\\ &+(z+\bar{z}xy)\bar{u}+u\bar{z}(x\bar{y}+\bar{x}y+\bar{x}\bar{y})=0. \end{aligned} \tag{5}$$

Now instead of directly eliminating y and z from the above equation, let us, in accordance with (IX. Prop, III.), assume the result of that elimination to be

$$Ex+E'(1-x)=0,$$

then E will be found by making in the given equation $x=1$, and eliminating y and z from the resulting equation, and E' will be found by making in the given equation $x=0$, and eliminating y and z from the result. First, then, making $x=1$, we have

$$\bar{s}+(y+\bar{y}z)\bar{t}+t\bar{y}\bar{z}+(z+y\bar{z})\bar{u}+u\bar{y}\bar{z}=0,$$

and making in the first member of this equation successively $y=1, z=1, y=1, z=0$, &c., and multiplying together the results, we have the expression

$$(\bar{s}+\bar{t}+\bar{u})(\bar{s}+\bar{t}+\bar{u})(\bar{s}+\bar{t}+\bar{u})(\bar{s}+t+u),$$

which is equivalent to

$$(\bar{s}+\bar{t}+\bar{u})(\bar{s}+t+u).$$

This is the expression for E. We shall retain it in its present form. It has already been shown by example (VIII. 3), that the actual reduction of such expressions by multiplication, though convenient, is not necessary.

其中 c 是若事件 st 发生则 x 将发生的概率。现在，若我们将 s 和 t 的表达式相乘，形成 st 的展开表达式，我们求得--

c = 概率：若 x 和 y 一起发生，或 x 和 z 一起发生而 y 不发生，或 y 和 z 一起发生而 x 不发生，则事件 x 将发生。

概率 x 的界限显然是 p(1 – q) 和 p。这一解答比前一解答更为确定，因为它包含一个不受任意常数影响的项。

第三种情形的解。

Here the data are— $\begin{aligned} &\text{Prob. } \{x+(1-x)yz\} = p,\\ &\text{Prob. } \{y+(1-y)xz\} = q,\\ &\text{Prob. } \{z+(1-z)xy\} = r. \end{aligned}$

Let us, as before, write $\bar{x}$ for $1-x$, &c., and assume

$$\begin{aligned} x+\bar{x}yz &= s,\\ y+\bar{y}xz &= t,\\ z+\bar{z}xy &= u. \end{aligned}$$

通过(VIII.8)的约化，我们得到方程

$$\begin{aligned} &(x+\bar{x}yz)\bar{s}+s\bar{x}(y\bar{z}+\bar{y}z+\bar{y}\bar{z})\\ &+(y+\bar{y}xz)\bar{t}+t\bar{y}(z\bar{x}+x\bar{z}+\bar{x}\bar{z})\\ &+(z+\bar{z}xy)\bar{u}+u\bar{z}(x\bar{y}+\bar{x}y+\bar{x}\bar{y})=0. \end{aligned} \tag{5}$$

现在，与其直接从上述方程中消去 y 和 z，不如按照(IX.命题 iii.)假定该消元的结果为

$$Ex+E'(1-x)=0,$$

那么 E 将通过在该给定方程中令 x = 1，并从所得方程中消去 y 和 z 而求得，而 E ′将通过在该给定方程中令 x = 0，并从结果中消去 y 和 z 而求得。首先，令 x = 1，我们有

$$\bar{s}+(y+\bar{y}z)\bar{t}+t\bar{y}\bar{z}+(z+y\bar{z})\bar{u}+u\bar{y}\bar{z}=0,$$

并在此方程第一项中依次令y=1，z=1，y=1，z=0等，将结果相乘，得表达式

$$(\bar{s}+\bar{t}+\bar{u})(\bar{s}+\bar{t}+\bar{u})(\bar{s}+\bar{t}+\bar{u})(\bar{s}+t+u),$$

这等价于

$$(\bar{s}+\bar{t}+\bar{u})(\bar{s}+t+u).$$

这是 E 的表达式。我们将保留其现有形式。已经通过例子(VIII.3)表明，这种表达式通过乘法的实际化简虽然方便，但并非必要。

Again in (5), making $x = 0$, we have

$$yz\bar{s} + s(y\bar{z} + \bar{y}z + \bar{y}\bar{z}) + y\bar{t} + t\bar{y} + z\bar{u} + u\bar{z} = 0;$$

from which, by the same process of elimination, we find for E' the expression

$$(\bar{s} + \bar{t} + \bar{u})(s + \bar{t} + u)(s + t + \bar{u})(s + t + u).$$

The final result of the elimination of y and z from (5) is therefore

$$(\bar{s} + \bar{t} + \bar{u})(\bar{s} + t + u)x + (\bar{s} + \bar{t} + \bar{u})(s + \bar{t} + u)(s + t + \bar{u})(s + t + u)(1 - x) = 0.$$

Whence we have

$$x = \frac{(\bar{s} + \bar{t} + \bar{u})(s + \bar{t} + u)(s + t + \bar{u})(s + t + u)}{(\bar{s} + \bar{t} + \bar{u})(s + \bar{t} + u)(s + t + \bar{u})(s + t + u) - (\bar{s} + \bar{t} + \bar{u})(\bar{s} + t + u)};$$

or, developing the second member,

$$\begin{aligned} x = \frac{0}{0}stu + \frac{1}{0}s\bar{t}u + \frac{1}{0}st\bar{u} + s\bar{t}\bar{u} \\ + \frac{1}{0}\bar{s}tu + 0\bar{s}\bar{t}u + 0\bar{s}t\bar{u} + 0\bar{s}\bar{t}\bar{u}. \end{aligned} \tag{6}$$

Hence, passing from Logic to Algebra,

$$\begin{aligned} \frac{stu + s\bar{t}\bar{u}}{p} = \frac{stu + \bar{s}t\bar{u}}{q} = \frac{stu + \bar{s}\bar{t}u}{r} \\ = stu + s\bar{t}\bar{u} + \bar{s}\bar{t}u + \bar{s}t\bar{u} + \bar{s}\bar{t}\bar{u}. \end{aligned} \tag{7}$$

$$\text{Prob. } x = \frac{s\bar{t}\bar{u} + cstu}{stu + s\bar{t}\bar{u} + \bar{s}\bar{t}u + \bar{s}t\bar{u} + \bar{s}\bar{t}\bar{u}}, \tag{8}$$

To simplify this system of equations, change $\frac{s}{\bar{s}}$ into s, $\frac{t}{\bar{t}}$ into t, &c., and after the change let λ stand for $stu + s + t + 1$. We then have

$$\text{Prob. } x = \frac{s + cstu}{\lambda}, \tag{9}$$

with the relations

$$\frac{stu + s}{p} = \frac{stu + t}{q} = \frac{stu + u}{r} = stu + s + t + u + 1 = \lambda. \tag{10}$$

From these equations we get

$$\begin{aligned} stu + s &= \lambda p, \\ stu + s &= \lambda - t - u - 1, \\ \therefore \lambda p &= \lambda - u - t - 1. \\ u + t &= \lambda(1 - p) - 1. \\ \text{Similarly,} u + s &= \lambda(1 - q) - 1, \\ \text{and} s + t &= \lambda(1 - r) - 1. \end{aligned} \tag{11}$$

又在(5)中，令x = 0，我们得到

$$yz\bar{s} + s(y\bar{z} + \bar{y}z + \bar{y}\bar{z}) + y\bar{t} + t\bar{y} + z\bar{u} + u\bar{z} = 0;$$

由此，通过同样的消元过程，我们为 E ′找到表达式

$$(\bar{s} + \bar{t} + \bar{u})(s + \bar{t} + u)(s + t + \bar{u})(s + t + u).$$

因此，从(5)中消去 y 和 z 的最终结果是

$$(\bar{s} + \bar{t} + \bar{u})(\bar{s} + t + u)x + (\bar{s} + \bar{t} + \bar{u})(s + \bar{t} + u)(s + t + \bar{u})(s + t + u)(1 - x) = 0.$$

Whence we have

$$x = \frac{(\bar{s} + \bar{t} + \bar{u})(s + \bar{t} + u)(s + t + \bar{u})(s + t + u)}{(\bar{s} + \bar{t} + \bar{u})(s + \bar{t} + u)(s + t + \bar{u})(s + t + u) - (\bar{s} + \bar{t} + \bar{u})(\bar{s} + t + u)};$$

or, developing the second member,

$$\begin{aligned} x = \frac{0}{0}stu + \frac{1}{0}s\bar{t}u + \frac{1}{0}st\bar{u} + s\bar{t}\bar{u} \\ + \frac{1}{0}\bar{s}tu + 0\bar{s}\bar{t}u + 0\bar{s}t\bar{u} + 0\bar{s}\bar{t}\bar{u}. \end{aligned} \tag{6}$$

因此，从逻辑过渡到代数，

$$\begin{aligned} \frac{stu + s\bar{t}\bar{u}}{p} = \frac{stu + \bar{s}t\bar{u}}{q} = \frac{stu + \bar{s}\bar{t}u}{r} \\ = stu + s\bar{t}\bar{u} + \bar{s}\bar{t}u + \bar{s}t\bar{u} + \bar{s}\bar{t}\bar{u}. \end{aligned} \tag{7}$$

$$\text{Prob. } x = \frac{s\bar{t}\bar{u} + cstu}{stu + s\bar{t}\bar{u} + \bar{s}\bar{t}u + \bar{s}t\bar{u} + \bar{s}\bar{t}\bar{u}}, \tag{8}$$

To simplify this system of equations, change $\frac{s}{\bar{s}}$ into s, $\frac{t}{\bar{t}}$ into t, &c., and after the change let λ stand for $stu + s + t + 1$. We then have

$$\text{Prob. } x = \frac{s + cstu}{\lambda}, \tag{9}$$

与这些关系

$$\frac{stu + s}{p} = \frac{stu + t}{q} = \frac{stu + u}{r} = stu + s + t + u + 1 = \lambda. \tag{10}$$

由这些方程我们得到

$$\begin{aligned} stu + s &= \lambda p, \\ stu + s &= \lambda - t - u - 1, \\ \therefore \lambda p &= \lambda - u - t - 1. \\ u + t &= \lambda(1 - p) - 1. \\ \text{Similarly,} u + s &= \lambda(1 - q) - 1, \\ \text{and} s + t &= \lambda(1 - r) - 1. \end{aligned} \tag{11}$$

From which equations we find

$$s = \frac{\lambda(1+p-q-r)-1}{2}, \quad t = \frac{\lambda(1+q-r-p)-1}{2},$$
$$u = \frac{\lambda(1+r-p-q)-1}{2}. \tag{12}$$

Now, by (10),

$$stu = \lambda p - s.$$

Substitute in this equation the values of s, t, and u above determined, and we have

$$\{(1+p-q-r)\lambda-1\}\{(1+q-p-r)\lambda-1\}\{(1+r-p-q)\lambda-1\}$$
$$= 4\{(p+q+r-1)\lambda+1\}, \tag{13}$$

an equation which determines λ. The values of s, t, and u, are then given by (12), and their substitution in (9) completes the solution of the problem.

10. Now a difficulty, the bringing of which prominently before the reader has been one object of this investigation, here arises. How shall it be determined, which root of the above equation ought to taken for the value of λ. To this difficulty some reference was made in the opening of the present chapter, and it was intimated that its fuller consideration was reserved for the next one; from which the following results are taken.

In order that the data of the problem may be derived from a possible experience, the quantities p, q, and r must be subject to the following conditions:

$$1+p-q-r \overset{=}{>} 0,$$
$$1+q-p-r \overset{=}{>} 0, \tag{14}$$
$$1+r-p-q \overset{=}{>} 0.$$

Moreover, the value of λ to be employed in the general solution must satisfy the following conditions:

$$\lambda \overset{=}{>} \frac{1}{1+p-q-r}, \quad \lambda \overset{=}{>} \frac{1}{1+q-p-r}, \quad \lambda \overset{=}{>} \frac{1}{1+r-p-q}. \tag{15}$$

Now these two sets of conditions suffice for the limitation of the general solution. It may be shown, that the central equation (13) furnishes but one value of λ, which does satisfy these conditions, and that value of λ is the one required.

Let $1+p-q-r$ be the least of the three coefficients of λ given above, then $\frac{1}{1+p-q-r}$ will be the greatest of those values, above which we are to show that there exists but one value of λ. Let us write (13) in the form

$$\{(1+p-q-r)\lambda-1\}\{(1+q-p-r)\lambda-1\}\{(1+r-p-q)\lambda-1\}$$
$$-4\{(p+q+r-1)\lambda+1\} = 0; \tag{16}$$

由哪些方程我们求得

$$s = \frac{\lambda(1+p-q-r)-1}{2}, \quad t = \frac{\lambda(1+q-r-p)-1}{2},$$
$$u = \frac{\lambda(1+r-p-q)-1}{2}. \tag{12}$$

Now, by (10),

$$stu = \lambda p - s.$$

Substitute in this equation the values of s, t, and u above determined, and we have

$$\{(1+p-q-r)\lambda-1\}\{(1+q-p-r)\lambda-1\}\{(1+r-p-q)\lambda-1\}$$
$$= 4\{(p+q+r-1)\lambda+1\}, \tag{13}$$

一个确定 λ 的方程。然后 s、t 和 u 的值由下式给出
(12) ，将它们代入(9)即完成问题的求解。
10. 现在出现了一个困难，将其突出地呈现在读者面前一直是本次研究的目的之一。应当如何确定，上述方程的哪一个根应被取作 λ 的值。在本章开头已对这个困难有所提及，并暗示更充分的考虑留待下一章；以下结果即取自该章。

为了使问题的数据能够来自可能的经验，量 p、q 和 r 必须满足以下条件：

$$1+p-q-r \overset{=}{>} 0,$$
$$1+q-p-r \overset{=}{>} 0, \tag{14}$$
$$1+r-p-q \overset{=}{>} 0.$$

此外，在一般解中所使用的 λ 的值必须满足以下条件：

$$\lambda \overset{=}{>} \frac{1}{1+p-q-r}, \quad \lambda \overset{=}{>} \frac{1}{1+q-p-r}, \quad \lambda \overset{=}{>} \frac{1}{1+r-p-q}. \tag{15}$$

现在这两组条件足以限制一般解。可以证明，中心方程(13)只提供一个满足这些条件的 λ 值，而那个 λ 的值就是所需要的。

设 1 + p – q – r 为上面给出的 λ 的三个系数中最小的一个，则
$\frac{1}{1+p-q-r}$ 将是那些值中的最大者，在其上我们要证明只存在一个 λ 的值。让我们把(13)写成如下形式

$$\{(1+p-q-r)\lambda-1\}\{(1+q-p-r)\lambda-1\}\{(1+r-p-q)\lambda-1\}$$
$$-4\{(p+q+r-1)\lambda+1\} = 0; \tag{16}$$

and represent the first member by V.

Assume $\lambda = \dfrac{1}{1+p-q-r}$, then V becomes

$$-4\left(\frac{p+q+r-1}{1+p-q-r}+1\right) = -4\left(\frac{2p}{1+p-q-r}\right),$$

which is *negative.* Let $\lambda = \infty$, then V is *positive* and infinite.

Again,

$$\frac{d^2V}{d\lambda^2} = (1+p-q-r)(1+q-p-r)\{(1+r-p-q)\lambda - 1\}$$
$$+ \text{ similar positive terms,}$$

which expression is positive between the limits $\lambda = \frac{1}{1+p-q-r}$ and $\lambda = \infty$.

If then we construct a curve whose abscissa shall be measured by λ, and whose ordinates by V, that curve will, between the limits specified, pass from below to above the abscissa λ, its convexity always being downwards. Hence it will but once intersect the abscissa λ within those limits; and the equation (16) will, therefore, have but one root thereto corresponding.

The solution is, therefore, expressed by (9), λ being that root of (13) which satisfies the conditions (15), and s, t, and u being given by (12). The interpretation of c may be deduced in the usual way.

It appears from the above, that the problem is, in all cases, more or less indeterminate.

and represent the first member by V.

Assume $\lambda = \dfrac{1}{1+p-q-r}$, then V becomes

$$-4\left(\frac{p+q+r-1}{1+p-q-r}+1\right) = -4\left(\frac{2p}{1+p-q-r}\right),$$

这是负的。令 λ = ∞，则 V 是正的且无限的。

再次，

$$\frac{d^2V}{d\lambda^2} = (1+p-q-r)(1+q-p-r)\{(1+r-p-q)\lambda - 1\}$$
$$+ \text{ similar positive terms,}$$

该表达式在极限 λ = $^{11+}$ p – q – r 与λ=∞之间为正。

那么，如果我们构造一条曲线，其横坐标由 λ 度量，纵坐标由 V 度量，该曲线将在指定极限之间从横坐标 λ 的下方穿到上方，其凸性始终向下。因此它将在那些极限内只与横坐标 λ 相交一次；因此方程(16)将只有一个与之对应的根。

因此，解由(9)表示，其中 λ 是(13)中满足条件(15)的那个根，而 s, t 和 u 由(12)给出。c 的解释可以按通常方式推出。

从上述可以看出，该问题在所有情况下都或多或少是不确定的。

Chapter XIX

OF STATISTICAL CONDITIONS.

1. By the term statistical conditions, I mean those conditions which must connect the numerical data of a problem in order that those data may be consistent with each other, and therefore such as statistical observations might actually have furnished. The determination of such conditions constitutes an important problem, the solution of which, to an extent sufficient at least for the requirements of this work, I purpose to undertake in the present chapter, regarding it partly as an independent object of speculation, but partly also as a necessary supplement to the theory of probabilities already in some degree exemplified. The nature of the connexion between the two subjects may be stated as follows:

2. There are innumerable instances, and one of the kind presented itself in the last chapter, Ex. 7, in which the solution of a question in the theory of probabilities is finally dependent upon the solution of an algebraic equation of an elevated degree. In such cases the selection of the proper root must be determined by certain conditions, partly relating to the numerical values assigned in the data, partly to the due limitation of the element required. The discovery of such conditions may sometimes be effected by unaided reasoning. For instance, if there is a probability p of the occurrence of an event A, and a probability q of the concurrence of the said event A, and another event B, it is evident that we must have

$$p \overset{=}{>} q.$$

But for the *general* determination of such relations, a distinct method is required, and this we proceed to establish.

As derived from actual experience, the probability of any event is the result of a process of approximation. It is the limit of the ratio of the number of cases in which the event is observed to occur, to the whole number of equally possible cases which observation records,–a limit to which we approach the more nearly as the number of observations is increased. Now let the symbol n, prefixed to the expression of any class, represent the number of individuals contained in

第十九章

论统计条件。

1. 所谓统计条件，我指的是那些必须联结一个问题的数值数据、以使这些数据彼此一致、因而成为统计观察实际上可能提供的那些条件。这种条件的确定构成一个重要问题，我打算在本章中承担其解答，至少达到足以满足本著作要求的程度，我部分地把它视为一个独立的思辨对象，但部分地也把它视为对前面已在一定程度上例示的概率论的必要补充。这两个主题之间联系的性质可以陈述如下：

2. 有无数实例，上一章例7中就出现了这类实例之一，其中概率论中一个问题的解答最终依赖于一个高次代数方程的解答。在此类情形中，正确根的选择必须由某些条件来确定，这些条件部分涉及数据中所指定的数值，部分涉及所需元素的正当限制。这种条件的发现有时可以借助未经辅助的推理来实现。例如，若有一事件 A 发生的概率 p，以及所述事件 A 与另一事件 B 同时发生的概率 q，则显然我们必须有=

$$p > q.$$

但对于此类关系的一般确定，需要一种独特的方法，我们现在着手建立它。

作为从实际经验中得出的，任何事件的概率都是近似过程的结果。它是观察到事件发生的次数与观察记录的所有同等可能情况的总数之比的极限--随着观察次数的增加，我们越来越接近这个极限。现在让符号 n，前缀于任何类的表达式，表示包含在该类中的个体数目。

that class. Thus, x representing men, and y white beings, let us assume

$$\begin{aligned} nx &= \text{number of men.} \\ nxy &= \text{number of white men.} \\ nx(1-y) &= \text{number of men who are not white; and so on.} \end{aligned}$$

In accordance with this notation $n(1)$ will represent the number of individuals contained in the universe of discourse, and $\frac{n(x)}{n(1)}$ will represent the probability that any individual being, selected out of that universe of being denoted by $n(1)$, is a man. If observation has not made us acquainted with the *total* values of $n(x)$ and $n(1)$, then the probability in question is the limit to which $\frac{n(x)}{n(1)}$ approaches as the number of individual observations is increased.

In like manner if, as will generally be supposed in this chapter, x represent an event of a particular kind observed, $n(x)$ will represent the number of occurrences of that event, $n(1)$ the number of observed events (equally probable) of all kinds, and $\frac{n(x)}{n(1)}$, or its limit, the probability of the occurrence of the event x.

Hence it is clear that any conclusions which may be deduced respecting the ratios of the quantities $n(x)$, $n(y)$, $n(1)$, &c. may be converted into conclusions respecting the probabilities of the events represented by x, y, &c. Thus, if we should find such a relation as the following, viz.,

$$n(x) + n(y) < n(1),$$

expressing that the number of times in which the event x occurs and the number of times in which the event y occurs, are together less than the number of possible occurrences $n(1)$, we might thence deduce the relation,

$$\frac{n(x)}{n(1)} + \frac{n(y)}{n(1)} < 1,$$
$$\text{orProb. } x + \text{Prob. } y < 1.$$

And generally any such statistical relations as the above will be converted into relations connecting the probabilities of the events concerned, by changing $n(1)$ *into* 1*, and any other symbol* $n(x)$ *into Prob.* x*.*

3. First, then, we shall investigate a method of determining the numerical relations of classes or events, and more particularly the major and minor limits of numerical value. Secondly, we shall apply the method to the limitation of the solutions of questions in the theory of probabilities.

It is evident that the symbol n is distributive in its operation. Thus we have

$$n\{xy + (1-x)(1-y)\} = nxy + n(1-x)(1-y)$$
$$nx(1-y) = nx - nxy,$$

and so on. The number of things contained in any class resolvable into distinct groups or portions is equal to the sum of the numbers of things found in those

因此，x 表示人，y 表示白色存在物，让我们假设

$$\begin{aligned} nx &= \text{男人数。} \\ nxy &= \text{白人的人数。} \\ nx(1-y) &= \text{不是白人的男子数目；依此类推。} \end{aligned}$$

按照这种记号，n(1) 将表示论域中所包含的个体数目，而$^{n(x)}_{n(1)}$将表示从由 n(1)所表示的那个存在论域中选出的任何存在个体是男人的概率。如果观察没有使我们得知 n(x) 和 n(1) 的总值，那么所讨论的概率就是$^{n(x)}_{n(1)}$所趋向的极限

随着个别观察次数的增加而趋近。

同样地，如果像本章中一般所假设的那样，x 表示所观察到的一类特定事件，那么 n(x) 将表示该事件发生的次数，n(1) 表示所观察到的各类事件(等概率的)的数目，而$^{n(x)}_{n(1)}$,或其极限，即该事件发生的概率

x.

因此很明显，任何关于量 n(x)、n(y)、n(1) 等的比的结论，都可以转换为关于由 x、y 等表示的事件的概率的结论。因此，如果我们发现如下关系，即，

$$n(x) + n(y) < n(1),$$

表达事件 x 发生的次数与事件 y 发生的次数之和小于可能发生的次数 n(1)，我们可以由此推出关系，

$$\frac{n(x)}{n(1)} + \frac{n(y)}{n(1)} < 1,$$
$$\text{orProb. } x + \text{Prob. } y < 1.$$

一般来说，任何上述统计关系都将通过将 n(1) 变为 1 ，以及将任何其他符号 n(x) 变为 *Prob.*来转换为连接相关事件概率的关系。

3. 首先，我们将研究一种确定类或事件的数值关系的方法，特别是数值的主要和次要极限。其次，我们将把该方法应用于概率论中问题解的限制。

显然，符号 n 在其运算中是分配性的。因此我们有

$$n\{xy + (1-x)(1-y)\} = nxy + n(1-x)(1-y)$$
$$nx(1-y) = nx - nxy,$$

等等。任何可分解为不同组或部分的类中所包含的事物的数目，等于在这些组中找到的事物的数目之和。

separate portions. It is evident, further, that any expression formed of the logical symbols x, y, &c. may be developed or expanded in any way consistent with the laws of the symbols, and the symbol n applied to each term of the result, provided that any constant multiplier which may appear, be placed outside the symbol n; without affecting the value of the result. The expression $n(1)$, should it appear, will of course represent the number of individuals contained in the universe. Thus,

$$\begin{gathered} n(1-x)(1-y) = n(1-x-y+xy) \\ = n(1) - n(x) - n(y) + n(xy). \\ \text{Again,} n\{xy + (1-x)(1-y)\} = n(l-x-y+2xy) \\ = n(1) - nx - ny + 2nxy). \end{gathered}$$

In the last member the term $2nxy$ indicates *twice* the number of individuals contained in the class xy.

4. We proceed now to investigate the numerical limits of classes whose logical expression is given. In this inquiry the following principles are of fundamental importance:

1st. If all the members of a given class possess a certain property x, the total number of individuals contained in the class x will be a superior limit of the number of individuals contained in the given class.

2nd. A minor limit of the number of individuals in any class y will be found by subtracting a major numerical limit of the contrary class, $1-y$, from the number of individuals contained in the universe.

To exemplify these principles, let us apply them to the following problem:

PROBLEM.—Given, $n(1)$, $n(x)$, and $n(y)$, required the superior and inferior limits of nxy.

Here our data are the number of individuals contained in the universe of discourse, the number contained in the class x, and the number in the class y, and it is required to determine the limits of the number contained in the class composed of the individuals that are found at once in the class x and in the class y.

By Principle I. this number cannot exceed the number contained in the class x, nor can it exceed the number contained in the class y. Its major limit will then be the least of the two values $n(x)$ and (y).

By Principle II. a minor limit of the class xy will be given by the expression

$$n(l) - \text{ major limit of } \{x(l-y) + y(l-x) + (1-x)(1-y)\}, \qquad (1)$$

since $x(1-y) + y(1-x) - (1-x)(1-y)$ is the complement of the class xy, i.e. what it wants to make up the universe.

Now $x(1-y) + (1-x)(1-y) = 1-y$. We have therefore for (1),

$$\begin{gathered} n(1) - \text{ major limit of } \{1 - y + y1 - x)\} \\ = n(1) - n(l-y) - \text{ major limit of } y(1-x). \end{gathered}$$

分离的部分。此外，显然，任何由逻辑记号 x、y 等构成的表达式，都可以按照与记号规律相容的任何方式展开或扩充，并将记号 n 施用于结果的每一项，只要把可能出现的任何常数乘数放在记号 n 之外；这不会影响结果的值。表达式 n(1) 如果出现，当然将表示论域中所包含的个体数目。于是，

$$\begin{gathered} n(1-x)(1-y) = n(1-x-y+xy) \\ = n(1) - n(x) - n(y) + n(xy). \\ \text{Again,} n\{xy + (1-x)(1-y)\} = n(l-x-y+2xy) \\ = n(1) - nx - ny + 2nxy). \end{gathered}$$

在最后一个成员中，项 2 nxy 表示两倍包含在类 xy 中的个体数目。

4. 我们现在继续研究其逻辑表达式已给出的类的数值极限。在这项研究中，以下原则具有根本重要性：

第一。如果给定类的所有成员都具有某种性质 x，则包含在类 x 中的个体总数将是包含在给定类中的个体数目的一个上极限。

第二。任何类 y 中个体数目的一个下极限，将通过从论域中包含的个体数目中减去相反类 1 – y 的一个上数值极限来找到。

为了例证这些原则，让我们将它们应用于以下问题：问题。--给定 n(1)、n(x) 和 n(y)，求 nxy 的上极限和下极限。

这里我们的数据是论域中所包含的个体数目、类x中所包含的数目以及类y中的数目，要求确定由同时出现在类x和类y中的个体所构成的类中所包含的数目的界限。

根据原理i，这个数目不能超过类x中所包含的数目，也不能超过类y中所包含的数目。因此它的上界将是两个值n(x)和(y)中较小的那个。

根据原理ii，类xy的下界将由以下表达式给出

$$n(l) - \text{ major limit of } \{x(l-y) + y(l-x) + (1-x)(1-y)\}, \qquad (1)$$

since $x(1-y) + y(1-x) - (1-x)(1-y)$ is the complement of the class xy, i.e. what it wants to make up the universe.

Now $x(1-y) + (1-x)(1-y) = 1-y$. We have therefore for (1),

$$\begin{gathered} n(1) - \text{ major limit of } \{1 - y + y1 - x)\} \\ = n(1) - n(l-y) - \text{ major limit of } y(1-x). \end{gathered}$$

The major limit of $y(l-x)$ is the least of the two values $n(y)$ and $n(1-x)$. Let $n(y)$ be the least, then (2) becomes

$$\begin{aligned} & n(1)-n(1-y)-n(y) \\ =\ & n(1)-n(1)+n(y)-n(y)=0. \end{aligned}$$

Secondly, let $n(1-x)$ be less than $n(y)$, then

$$\text{major limit of } ny(1-x)=n(1-x);$$

therefore (2) becomes

$$\begin{aligned} & n(1)-n(1-y)-n(1-x) \\ =\ & n(1)-n(1)+n(y)-n(1)+n(x) \\ =\ & nx+ny-n(1). \end{aligned}$$

The minor limit of nxy is therefore either 0 or $n(x)+n(y)-n(1)$, according as $n(y)$ is less or greater than $n(1-x)$, or, which is an equivalent condition, according as $n(x)$ is greater or less than $n(1-y)$.

Now as 0 is necessarily a minor limit of the numerical value of *any* class, it is sufficient to take account of the second of the above expressions for the minor limit of $n(xy)$. We have, therefore,

$$\begin{aligned} &\text{Major limit of } n(xy) = \text{ least of values } n(x) \text{ and } n(y). \\ &\text{Minor limit of } n(xy) = n(x)+n(y)-n(1).^1 \end{aligned}$$

Proposition I.

5. *To express the major and minor limits of a class represented by any constituent of the symbols x, y, z, &c., having given the values of $n(x)$, $n(y)$, $n(z)$, &c., and $n(1)$.*

Consider first the constituent xyz.

It is evident that the major numerical limit will be the least of the values $n(x)$, $n(y)$, $n(z)$.

The minor numerical limit may be deduced as in the previous problem, but it may also be deduced from the solution of that problem. Thus:

$$\text{Minor limit of } n(xyz)=n(xy)+n(z)-n(1). \tag{1}$$

[1]The above expression for the minor limit of nxy is applied by Professor De Morgan, by whom it appears to have been first given, to the syllogistic form:

Most men in a certain company have coats.
Most men in the same company have waistcoats.
Therefore some in the company have coats and waistcoats.

y(l – x)的上界是两个值n(y)和n(1 – x)中较小的那个。设n(y)为较小者，则(2)变为

$$\begin{aligned} & n(1)-n(1-y)-n(y) \\ =\ & n(1)-n(1)+n(y)-n(y)=0. \end{aligned}$$

其次，设n(1 – x)小于n(y)，则

$$\text{major limit of } ny(1-x)=n(1-x);$$

因此(2)变为

$$\begin{aligned} & n(1)-n(1-y)-n(1-x) \\ =\ & n(1)-n(1)+n(y)-n(1)+n(x) \\ =\ & nx+ny-n(1). \end{aligned}$$

因此nxy的下界要么是0，要么是n(x) + n(y) – n(1)，取决于n(y)是小于还是大于n(1 – x)，或者，这是一个等价的条件，取决于n(x)是大于还是小于n(1 – y)。

现在，由于0必然是任何类的数值的下界，因此只需考虑上述第二个表达式作为n(xy)的下界即可。于是我们有，

$$\begin{aligned} &\text{Major limit of } n(xy) = \text{ least of values } n(x) \text{ and } n(y). \\ &\text{Minor limit of } n(xy) = n(x)+n(y)-n(1).^1 \end{aligned}$$

命题一。

5. 在给定n(x)、n(y)、n(z)等的值以及n(1)的值的情况下，表示由记号、、、&c.的任一成分所代表的类的上界和下界。

首先考虑构成部分 xyz。

显然，数值上界将是n(x)、n(y)、n(z)中的最小值。

数值下界可以如前一问题那样推导，但也可以从该问题的解答中推导。如下：

$$\text{Minor limit of } n(xyz)=n(xy)+n(z)-n(1). \tag{1}$$

[1]The above expression for the minor limit of nxy is applied by Professor De Morgan, by whom it appears to have been first given, to the syllogistic form:

某公司中大多数男人有外套。同一公司中大多数男人有马甲。

因此，公司里有些人有外套和马甲。

Now this means that $n(xyz)$ is at least as great as the expression $n(xy)+n(z)-n(1)$. But $n(xy)$ is at least as great as $n(x)+n(y)-n(1)$. Therefore $n(xyz)$ is at least as great as

$$n(x)+n(y)-n(1)+n(z)-n(1),$$
$$\text{or} \qquad n(x)+n(y)+n(z)-2n(1).$$

Hence we have

$$\text{Minor limit of } n(xyz) = n(x)+n(y)+n(z)-2n(1).$$

By extending this mode of reasoning we shall arrive at the following conclusions:

1st. The major numerical limit of the class represented by any constituent will be found by prefixing n separately to each factor of the constituent, and taking the least of the resulting values.

2nd. The minor limit will be found by adding all the values above mentioned together, and subtracting from the result as many, less one, times the value of $n(1)$.

Thus we should have

$$\text{Major limit of } nxy(1-z) = \text{ least of the values } nx,\ ny, \text{ and } n(1-z).$$
$$\text{Minor limit of } nxy(1-z) = n(x)+n(y)+n(1-z)-2n(1)$$
$$= nx+n(y)-n(z)-n(1).$$

In the use of general symbols it is perhaps better to regard all the values $n(x)$, $n(y)$, $n(1-z)$, as major limits of $n\{xy(1-z)\}$, since, in fact, it cannot exceed any of them. I shall in the following investigations adopt this mode of expression.

Proposition II.

6. *To determine the major numerical limit of a class expressed by a series of constituents of the symbols x, y, z, &c., the values of $n(x)$, $n(y)$, $n(z)$, &c., and $n(1)$, being given.*

Evidently one mode of determining such a limit would be to form the least possible sum of the major limits of the several constituents. Thus a major limit of the expression

$$n\{xy+(1-x)(1-y)\}$$

would be found by adding the least of the two values nx, ny, furnished by the first constituent, to the least of the two values $n(1-x)$, $n(1-y)$, furnished by the second constituent. If we do not know which is in each case the least value, we must form the four possible sums, and reject any of these which are equal to or exceed $n(1)$. Thus in the above example we should have

$$nx \quad +n(l-x) = n(l).$$
$$n(x)+n(1-y) = n(1)+n(x)-n(y).$$
$$n(y)+n(l-y) = n(l)+n(y)-n(x).$$
$$n(y)+n(1-y) = n(1).$$

Now this means that $n(xyz)$ is at least as great as the expression $n(xy)+n(z)-n(1)$. But $n(xy)$ is at least as great as $n(x)+n(y)-n(1)$. Therefore $n(xyz)$ is at least as great as

$$n(x)+n(y)-n(1)+n(z)-n(1),$$
$$\text{or} \qquad n(x)+n(y)+n(z)-2n(1).$$

Hence we have

$$\text{Minor limit of } n(xyz) = n(x)+n(y)+n(z)-2n(1).$$

By extending this mode of reasoning we shall arrive at the following conclusions:

第一，由任一成分所代表的类的数值上界，可以通过将n分别加到该成分的每个因子上，并取所得值中的最小值来求得。

第二，小极限将通过把上述所有值相加，并从结果中减去n(1)的值的次数减一来求得。

因此我们应有

$$\text{Major limit of } nxy(1-z) = \text{ least of the values } nx,\ ny, \text{ and } n(1-z).$$
$$\text{Minor limit of } nxy(1-z) = n(x)+n(y)+n(1-z)-2n(1)$$
$$= nx+n(y)-n(z)-n(1).$$

在使用一般记号时，或许最好将所有值n(x)、n(y)、n(1 – z)视为n { xy(1 – z) }的上界，因为事实上它不能超过其中任何一个。在以下研究中，我将采用这种表达方式。

命题二。

6. 在给定n(x)、n(y)、n(z)等的值以及n(1)的值的情况下，确定由记号、、、&.的若干成分所表达的类的数值上界。

显然，确定这样一个界限的一种方法是构造各成分上界的最小可能和。因此，表达式

$$n\{xy+(1-x)(1-y)\}$$

的上界可以通过将第一个成分所提供的两个值nx、ny中的较小者，与第二个成分所提供的两个值n(1 – x)、n(1 – y)中的较小者相加来求得。如果我们不知道在每种情况下哪个是最小值，我们就必须构造四个可能的和，并舍弃其中等于或超过n(1)的任何和。因此在上面的例子中，我们将有

$$nx \quad +n(l-x) = n(l).$$
$$n(x)+n(1-y) = n(1)+n(x)-n(y).$$
$$n(y)+n(l-y) = n(l)+n(y)-n(x).$$
$$n(y)+n(1-y) = n(1).$$

Rejecting the first and last of the above values, we have

$$n(1) + n(x) - n(y), \text{ and } n(1) + n(y) - n(x),$$

for the expressions required, one of which will (unless $nx = ny$) be less than $n(l)$, and the other greater. The least must of course be taken.

When two or more of the constituents possess a common factor, as x, that factor can only, as is obvious from Principle I., furnish a single term $n(x)$ in the final expression of the major limit. Thus if $n(x)$ appear as a major limit in two or more constituents, we must, in adding those limits together, replace $nx + nx$ by nx, and so on. Take, for example, the expression $n\{xy + x(1-y)z\}$. The major limits of this expression, immediately furnished by addition, would be—

1. nx.	4. $ny + nx$.
2. $nx + n(1-y)$.	5. $ny + n(1-y)$.
3. $nx + n(z)$.	6. $ny + nz$.

Of these the first and sixth only need be retained; the second, third, and fourth being greater than the first; and the fifth being equal to $n(1)$. The limits are therefore

$$n(x) \text{ and } n(y) + n(z),$$

and of these two values the last, supposing it to be less than $n(1)$, must be taken.

These considerations lead us to the following Rule:

Rule.—*Take one factor from each constituent, and prefix to it the symbol n, add the several terms or results thus formed together, rejecting all repetitions of the same term; the sum thus obtained will be a major limit of the expression, and the least of all such sums will be the major limit to be employed.*

Thus the major limits of the expression

$$xyz + x(1-y)(1-z) + (1-x)(1-y)(1-z)$$

would be

$$n(x) + n(1-y), \text{ and } n(x) + n(1-z),$$
$$\text{or} n(x) + n(1) - n(y), \text{ and } n(x) + n(1) - n(z).$$

If we began with $n(y)$, selected from the first term, and took $n(x)$ from the second, we should have to take $n(1-y)$ from the third term, and this would give

$$n(y) + n(x) + n(1-y), \text{ or } n(1) + n(x).$$

But as this result exceeds $n(1)$, which is an obvious major limit to *every* class, it need not be taken into account.

Proposition III.

舍弃上述首末两值，我们得到

$$n(1) + n(x) - n(y), \text{ and } n(1) + n(y) - n(x),$$

作为所需的表达式，其中一个将(除非nx = ny)小于n(l)，另一个则大于。当然必须取最小值。

当两个或更多成分具有一个共同因子，如 x 时，正如从原理 i.明显可见的，该因子在主极限的最终表达式中只能提供一个单项 n(x)。因此，如果 n(x) 作为主极限出现在两个或更多成分中，我们在把这些极限相加时，必须用 nx 代替 nx + nx，依此类推。例如，取表达式 n { xy + x(1 – y) z }。这个表达式的主极限，由加法直接给出，将是--

1. nx.	4. $ny + nx$.
2. $nx + n(1-y)$.	5. $ny + n(1-y)$.
3. $nx + n(z)$.	6. $ny + nz$.

在这些当中，只需保留第一个和第六个；第二个、第三个和第四个大于第一个；第五个等于n(1)。因此界限为

$$n(x) \text{ and } n(y) + n(z),$$

而在这两个值中，必须取最后一个，假设它小于n(1)。

这些考虑引导我们得出以下规则：

规则--从每个成分中取一个因子，并在其前面加上记号，将由此形成的各项或结果相加，舍弃同一项的所有重复；如此得到的和将是该表达式的一个上界，而所有这样的和中的最小值将是所要采用的上界。

因此，该表达式的主要极限

$$xyz + x(1-y)(1-z) + (1-x)(1-y)(1-z)$$

会是

$$n(x) + n(1-y), \text{ and } n(x) + n(1-z),$$
$$\text{or} n(x) + n(1) - n(y), \text{ and } n(x) + n(1) - n(z).$$

如果我们从第一项中选取n(y)开始，并从第二项中取n(x)，我们就必须从第三项中取n(1 – y)，这将给出

$$n(y) + n(x) + n(1-y), \text{ or } n(1) + n(x).$$

But as this result exceeds $n(1)$, which is an obvious major limit to *every* class, it need not be taken into account.

命题三。

7. To find the minor numerical limit of any class expressed by constituents of the symbols x, y, z, having given $n(x), n(y), n(z) \ldots n(1)$.

This object may be effected by the application of the preceding Proposition, combined with Principle II., but it is better effected by the following method:

Let any two constituents, which differ from one another only by a single factor, be added, so as to form a single class term as $x(1-y)+xy$ form x, and this species of aggregation having been carried on as far as possible, i.e., there having been selected out of the given series of constituents as many sums of this kind as can be formed, each such sum comprising as many constituents as can be collected into a single term, without regarding whether any of the said constituents enter into the composition of other terms, let these ultimate aggregates, together with those constituents which do not admit of being thus added together, be written down as distinct terms. Then the several minor limits of those terms, deduced by Prop. I., will be the minor limits of the expression given, and one only of those minor limits will at the same time be positive.

Thus from the expression $xy+(1-x)y+(1-x)(1-y)$ we can form the aggregates y and $1-x$, by respectively adding the first and second terms together, and the second and third. Hence $n(y)$ and $n(1-x)$ will be the minor limits of the expression given. Again, if the expression given were

$$xyz+x(1-y)z+(1-x)yz+(l-x)(1-y)z \\ +xy(1-z)+(1-x)(1-y)(1-z),$$

we should obtain by addition of the first four terms the single term z, by addition of the first and fifth term the single term xy, and by addition of the fourth and sixth terms the single term $(1-x)(1-y)$; and there is no other way in which constituents can be collected into single terms, nor are there are any constituents left which have not been thus taken account of. The three resulting terms give, as the minor limits of the given expression, the values

$$n(z),\ n(x)+n(y)-n(1), \\ \text{and} n(1-x)+n(1-y)-n(1),\ \text{or}\ n(1)-n(x)-n(y).$$

8. The proof of the above rule consists in the proper application of the following principles: — 1st. The minor limit of any collection of constituents which admit of being added into a single term, will obviously be the minor limit of that single term. This explains the first part of the rule. 2nd. The minor limit of the sum of any two terms which either are distinct constituents, or consist of distinct constituents, but do not admit of being added together, will be the sum of their respective minor limits, if those minor limits are both positive; but if one be positive, and the other negative, it will be equal to the positive minor limit alone. For if the negative one were added, the value of the limit would be diminished, i. e. it would be less for the sum of two terms than for a single term. Now whenever two constituents differ in more than one factor, so as not to admit of being added together, the minor limits of the two cannot be both positive. Thus let the terms be xyz and $(1-x)(1-y)z$, which differ in

7. 求由记号所表示的类的最小数值极限，已知 n(x) , n(y) , n(z) ...n(1) .

这一目标可以通过应用前一个命题并结合原理ii来实现，但用以下方法实现更好：

设任意两个仅相差一个因子的成分相加，以形成单个类项如 x(1 – y) + xy 形成 x，并且这种聚合已尽可能进行，即从给定的成分序列中选出尽可能多的此类和，每个这样的和包含尽可能多的可归入单个项的成分子，而不考虑所述成分中是否有任何成分进入其他项的构成，将这些最终的聚合体连同那些不能如此相加的成分一起，作为不同的项写下。然后，由命题 I.推得的那些项的各个下限，将是给定表达式的下限，并且这些下限中只有一个同时是正的。

因此，从表达式 xy +(1 – x) y +(1 – x)(1 – y) 中，我们可以通过分别将第一项和第二项相加，以及将第二项和第三项相加，构成聚合 y 和 1 – x。因此 n(y) 和 n(1 – x) 将是给定表达式的最小极限。再者，如果给定的表达式是

$$xyz+x(1-y)z+(1-x)yz+(l-x)(1-y)z \\ +xy(1-z)+(1-x)(1-y)(1-z),$$

我们应通过前四项相加得到单项 z，通过第一项和第五项相加得到单项 xy，通过第四项和第六项相加得到单项(1 – x)(1 – y)；并且没有其他方式可将成分归入单项，也没有任何成分未被如此考虑。所得三项给出给定表达式的下限值

$$n(z),\ n(x)+n(y)-n(1), \\ \text{and} n(1-x)+n(1-y)-n(1),\ \text{or}\ n(1)-n(x)-n(y).$$

8. 上述规则的证明在于正确应用以下原理：--第一。任何可相加为单项的成分集合的下限，显然将是该单项的下限。这解释了规则的第一部分。第二。任何两项之和的下限--这两项要么是不同的成分，要么由不同的成分构成，但不能相加在一起--将是它们各自下限之和，如果这两个下限都是正的；但如果一个是正的，另一个是负的，则它将等于正的下限单独。因为如果加上负的，极限的值将减小，即两项之和的值将小于单项的值。现在，只要两个成分在不止一个因子上不同，以致不能相加在一起，两者的下限就不可能都是正的。因此，设项为 xyz 和(1 – x)(1 – y) z，它们在

two factors, the minor limit of the first is $n(x+y+z-2)$, that of the second $n(1-x+1-y+z-2)$, or,

$$\text{1st. } n\{x+y-1-(1-z)\}. \quad \text{2nd. } n\{1-x-y-(1-z)\}.$$

If $n(x+y-1)$ is positive, $n(1-x-y)$ is negative, and the second must be negative. If $n(x+y-1)$ is negative, the first is negative; and similarly for cases in which a larger number of factors are involved. It may in this manner be shown that, according to the mode in which the aggregate terms are formed in the application of the rule, no two minor limits of distinct terms can be added together, for either those terms will involve some common constituent, in which case it is clear that we cannot add their minor limits together,—or the minor limits of the two will not be both positive, in which case the addition would be useless.

PROPOSITION IV.

9. *Given the respective numbers of individuals comprised in any classes, s, t, &c. logically defined, to deduce a system, of numerical limits of any other class w, also logically defined.*

As this is the most general problem which it is meant to discuss in the present chapter, the previous inquiries being merely introductory to it, and the succeeding ones occupied with its application, it is desirable to state clearly its nature and design.

When the classes s, $t \ldots w$ are said to be logically defined, it is meant that they are classes so defined as to enable us to write down their symbolical expressions, whether the classes in question be simple or compound. By the general method of this treatise, the symbol w can then be determined directly as a developed function of the symbols s, t, &c. in the form

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D, \tag{1}$$

wherein A, B, C, and D are formed of the constituents of s, t, &c. How from such an expression the numerical limits of w may in the most general manner be determined, will be considered hereafter. At present we merely purpose to show how far this object can be accomplished on the principles developed in the previous propositions; such an inquiry being sufficient for the purposes of this work. For simplicity, I shall found my argument upon the particular development,

$$w = st + 0s(1-t) + \frac{1}{0}(1-s)t + \frac{0}{0}(1-s)(1-t), \tag{2}$$

in which all the varieties of coefficients present themselves.

Of the constituent $(1-s)(1-t)$, which has for its coefficient $\frac{0}{0}$, it is implied that some, none, or all of the class denoted by that constituent are found in w. It is evident that $n(w)$ will have its highest numerical value when all the members of the class denoted by $(1-s)(1-f)$ are found in w. Moreover, as none of the individuals contained in the classes denoted by $s(1-t)$ and $(1-s)t$

two factors, the minor limit of the first is $n(x+y+z-2)$, that of the second $n(1-x+1-y+z-2)$, or,

$$\text{1st. } n\{x+y-1-(1-z)\}. \quad \text{2nd. } n\{1-x-y-(1-z)\}.$$

如果 n(x + y – 1) 是正的，n(1 – x – y) 是负的，并且第二个必须是负的。如果 n(x + y – 1) 是负的，第一个是负的；对于涉及更多因子的情况也类似。以这种方式可以表明，根据规则应用中聚合项的形成方式，不同项的两个下限不能相加在一起，因为要么那些项将涉及某个共同成分，在这种情况下显然我们不能将它们的下限相加，--要么两者的下限不会都是正的，在这种情况下相加将是无用的。

命题四。

9. 已知逻辑定义的任意类 、 、*&c.*中所包含的个体各自数目，推导出任意其他类的数值极限系统，该类也是逻辑定义的。

由于这是本章打算讨论的最一般的问题，前面的探究只是对它的引导，而后续的探究则涉及其应用，因此有必要清楚地说明其性质和设计。

当类 s、t ...w 被称为逻辑定义时，意思是它们是这样定义的类，使我们能够写出它们的符号表达式，无论所讨论的类是简单还是复合。通过本论著的一般方法，记号 w 随后可以直接确定为记号 s、t、&c.的展开函数，形式为

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D, \tag{1}$$

其中 A、B、C 和 D 由 s、t 等的成分构成。如何从这样的表达式以最一般的方式确定 w 的数值极限，将在以后考虑。目前我们仅旨在表明，根据先前命题中发展的原理，这一目标能在多大程度上实现；这样的探究对于本著作的目的而言是足够的。为简单起见，我将把我的论证建立在特定的展开式上，

$$w = st + 0s(1-t) + \frac{\iota}{0}(1-s)t + \frac{\upsilon}{0}(1-s)(1-t), \tag{2}$$

其中所有种类的系数都呈现出来。

对于成分(1 – s)(1 – t)，其系数为$\frac{0}{0}$,它是隐含 –

即该成分所表示的类中的一些、没有或全部成员被发现于 w 中。显然，当(1 – s)(1 – f) 所表示的类的所有成员都被发现于 w 中时，n(w) 将具有其最高数值。此外，由于 s(1 – t) 和(1 – s) t 所表示的类中包含的个体没有一个

are found in w, the superior numerical limits of w will be identical with those of the class $st + (1-s)(1-t)$. They are, therefore,

$$ns + n(1-t) \text{ and } nt + n(1-s).$$

In like manner a system of superior numerical limits of the development $A + 0B + \frac{0}{0}C + \frac{1}{0}D$, *may be found from those of* $A + C$ *by Prop. 2.*

Again, any minor numerical limit of w will, by Principle II., be given by the expression

$$n(1) - \text{ major limit of } n(1-w),$$

but the development of w being given by (1), that of $1-w$ will obviously be

$$1 - w = 0A + B + \frac{0}{0}C + \frac{1}{0}D.$$

This may be directly proved by the method of Prop. 2, Chap. X. Hence

$$\begin{aligned}\text{Minor limit of } n(w) &= n(1) - \text{ major limit } (B+C)\\ &= \text{ minor limit of } (A+D),\end{aligned}$$

by Principle II., since the classes $A+D$ and $B+C$ are supplementary. Thus the minor limit of the second member of (2) would be $n(t)$, and, generalizing this mode of reasoning, we have the following result:

A system of minor limits of the development

$$A + 0B + \frac{0}{0}C + \frac{1}{0}D$$

will be given by the minor limits of $A + D$.

This result may also be directly inferred. For of minor numerical limits we are bound to seek the greatest. Now we obtain in general a higher minor limit by connecting the class D with A in the expression of w, a combination which, as shown in various examples of the Logic we are permitted to make, than we otherwise should obtain.

Finally, as the concluding term of the development of w indicates the equation $D = 0$, it is evident that $n(D) = 0$. Hence we have

$$\text{Minor limit of } n(D) \overset{=}{<} 0,$$

and this equation, treated by Prop. 3, gives the requisite conditions among the numerical elements $n(s)$, $n(t)$, &c., in order that the problem may be real, and may embody in its data the results of a possible experience,

Thus from the term $\frac{1}{0}(1-s)t$ in the second member of (2) we should deduce

$$\begin{aligned}n(1-s) + n(t) - n(1) &\overset{=}{<} 0,\\ \therefore n(t) &\overset{=}{<} n(s).\end{aligned}$$

These conclusions may be embodied in the following rule:

被发现于 w 中，w 的较高数值极限将与类 st +(1 – s)(1 – t) 的极限相同。因此，它们是

$$ns + n(1-t) \text{ and } nt + n(1-s).$$

In like manner a system of superior numerical limits of the development $A + 0B + \frac{0}{0}C + \frac{1}{0}D$, *may be found from those of* $A + C$ *by Prop. 2.*

Again, any minor numerical limit of w will, by Principle II., be given by the expression

$$n(1) - \text{ major limit of } n(1-w),$$

but the development of w being given by (1), that of $1-w$ will obviously be

$$1 - w = 0A + B + \frac{0}{0}C + \frac{1}{0}D.$$

这可以直接由第10章命题2的方法证明。因此

$$\begin{aligned}\text{Minor limit of } n(w) &= n(1) - \text{ major limit } (B+C)\\ &= \text{ minor limit of } (A+D),\end{aligned}$$

根据原理ii，因为类 A + D 和 B + C 是互补的。因此(2)的第二成员的最小极限将是 n(t)，并且，推广这种推理方式，我们得到以下结果：

一个发展的次要极限系统

$$A + 0B + \frac{0}{0}C + \frac{1}{0}D$$

将由 A + D 的最小极限给出。

这个结果也可以直接推断出来。因为对于最小数值极限，我们必须寻求最大的。现在，我们一般通过将类 D 与 A 在 w 的表达式中连接起来，获得更高的最小极限，正如在我们被允许进行的逻辑学的各种例子中所展示的，这种组合比我们否则会得到的更高。

Finally, as the concluding term of the development of w indicates the equation $D = 0$, it is evident that $n(D) = 0$. Hence we have

$$\text{Minor limit of } n(D) \overset{=}{<} 0,$$

而这个方程，通过命题3处理，给出了数值元素 n(s)、n(t)、&c.之间的必要条件，以便问题可以是真实的，并且可以在其数据中体现可能经验的结果，

因此，从项10(1–s)t在(2) 的第一成员中我们应该推断出

$$\begin{aligned}n(1-s) + n(t) - n(1) &\overset{=}{<} 0,\\ \therefore n(t) &\overset{=}{<} n(s).\end{aligned}$$

这些结论可归纳为以下规则：

10. RULE.—*Determine the expression of the class w as a developed logical function of the symbols s, t, &c. in the form*

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D.$$

Then will

$$\begin{aligned} \text{Maj. lim. } w &= \text{ Maj. lim. } A + C. \\ \text{Min. lim. } w &= \text{ Min. lim. } A + D. \end{aligned}$$

The necessary numerical conditions among the data being given by the inequality

$$\text{Min. lim. } D \overline{=}< n(1).$$

To apply the above method to the limitation of the solutions of questions in probabilities, it is only necessary to replace in each of the formula $n(x)$ by Prob. x, $n(y)$ by Prob. y, &c., and, finally, $n(1)$ by 1. The application being, however, of great importance, it may be desirable to exhibit in the form of a rule the chief results of transformation.

11. Given the probabilities of any events s, t, &c., whereof another event w is a developed logical function, in the form

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D,$$

required the systems of superior and inferior limits of Prob. w, and the conditions among the data.

SOLUTION.—The superior limits of Prob. $(A + C)$, and the inferior limits of Prob. $(A + D)$ will form two such systems as are sought. The conditions among the constants in the data will be given by the inequality,

$$\text{Inf. lim. Prob. } D \overline{=}< 0.$$

In the application of these principles we have always

$$\text{Inf. lim. Prob. } x_1 x_2 \ldots x_n = \text{ Prob. } x_1 + \text{ Prob. } x_2 \ldots + \text{ Prob. } x_n - (n-1).$$

Moreover, the inferior limits can only be determined from *single* terms, either given or formed by aggregation. Superior limits are included in the form $\sum$ Prob. x, Prob. x applying only to symbols which are different, and are taken from different terms in the expression whose superior limit is sought. Thus the superior limits of Prob. $xyz + x(1-y)(1-z)$ are

$$\text{Prob. } x, \quad \text{Prob. } y + \text{ Prob. } (1-z), \text{ and Prob. } z + \text{ Prob. } (1-y).$$

Let it be observed, that if in the last case we had taken Prob. z from the first term, and Prob. $(1-z)$ from the second,—a connexion not forbidden,—we should have had as their sum 1, which as a result would be useless because *à*

10. 规则。- 确定类的表达式，作为记号 、 、 &c.的展开逻辑函数，形式为

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D.$$

那么将

$$\begin{aligned} \text{Maj. lim. } w &= \text{ Maj. lim. } A + C. \\ \text{Min. lim. } w &= \text{ Min. lim. } A + D. \end{aligned}$$

数据之间的必要数值条件由不等式给出

$$\text{Min. lim. } D \overline{=}< n(1).$$

为了将上述方法应用于概率问题解的限制，只需在每个公式中将 n(x) 替换为 Prob.x，将 n(y) 替换为 Prob.y,&c.，最后将 n(1) 替换为 1。然而，由于应用非常重要，以规则的形式展示变换的主要结果可能是可取的。

11. 已知任意事件 s、t、&c.的概率，其中另一个事件 w 是一个展开的逻辑函数，形式为

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D,$$

要求 Prob.w 的上下极限系统，以及数据之间的条件。

解。- Prob.(A + C) 的上限和 Prob.(A + D) 的下限将形成所寻求的两个这样的系统。数据中常数之间的条件将由不等式给出，

$$\text{Inf. lim. Prob. } D \overline{=}< 0.$$

在应用这些原则时，我们始终

Inf.lim.Prob.x ^{1} x ^{2} ...x n = Prob.x ^{1} + Prob.x ^{2} ...+ Prob.x n –(n – 1) .此外，下限只能从单个项确定，无论是给定的还是通过聚合形成的。上限包含在形式中

P Prob.x, Prob.x 仅适用于不同的记号，并且是
取自其上限被寻求的表达式中的不同项。因此 Prob.xyz + x(1 – y)(1 – z) 的上限是

$$\text{Prob. } x, \quad \text{Prob. } y + \text{ Prob. } (1-z), \text{ and Prob. } z + \text{ Prob. } (1-y).$$

请注意，如果在最后一种情况下我们从第一项取 Prob.z，从第二项取 Prob.(1 – z)--这种连接不被禁止--我们将得到它们的和为1，作为结果这将是无用的，因为`*a*

priori necessary. It is obvious that we may reject any limits which do not fall between 0 and 1.

Let us apply this method to Ex. 7, Case III. of the last chapter.

The final logical solution is

$$x = \frac{0}{0}stu + \frac{1}{0}s\bar{t}u + \frac{1}{0}st\bar{u} + s\bar{t}\bar{u} + \frac{1}{0}\bar{s}tu + 0\bar{s}\bar{t}u + 0\bar{s}t\bar{u} + 0\bar{s}\bar{t}\bar{u},$$

the data being

$$\text{Prob. } s = p, \quad \text{Prob. } t = q, \quad \text{Prob. } u = r.$$

We shall seek both the numerical limits of x, and the conditions connecting p, q, and r. The superior limits of x are, according to the rule, given by those of $stu + s\bar{t}\bar{u}$. They are, therefore,

$$p, q + 1 - r, \quad r + 1 - q.$$

The inferior limits of x are given by those of

$$s\bar{t}u + st\bar{u} + s\bar{t}\bar{u} + \bar{s}tu.$$

We may collect the first and third of these constituents in the single term $s\bar{t}$, and the second and third in the single term $s\bar{u}$. The inferior limits of x must then be deduced separately from the terms $s(1-t)$, $s(1-u)$, $(1-s)tu$, which give

$$p + 1 - q - 1, \quad p + 1 - r - 1, \quad 1 - p + q + r - 2,$$
$$\text{or} \qquad p - q, \quad p - r, \quad \text{and } q + r - p - 1.$$

Finally, the conditions among the constants p, q, and r, are given by the terms

$$s\bar{t}u, \quad st\bar{u}, \quad \bar{s}tu,$$

from which, by the rule, we deduce

$$p + 1 - q + r - 2 \overline{=}< 0, \quad p + q + 1 - r - 2 \overline{=}< 0, \quad 1 - p + q + r - 2 \overline{=}< 0.$$
$$\text{or } 1 + q - p - r \overline{=}> 0, \quad 1 + r - p - q \overline{=}> 0, \quad 1 + p - q - r \overline{=}> 0.$$

These are the limiting conditions employed in the analysis of the final solution. The conditions by which in that solution λ is limited, were determined, however, simply from the conditions that the quantities s, t, and u should be positive. Narrower limits of that quantity might, in all probability, have been deduced from the above investigation.

12. The following application is taken from an important problem, the solution of which will be given in the next chapter. There are given,

$$\text{Prob. } x = c_1, \quad \text{Prob. } y = c_2, \quad \text{Prob. } s = c_1p_1, \quad \text{Prob. } t = c_2p_2,$$

先验必然的。显然，我们可以拒绝任何不在0和1之间的极限。

让我们将此方法应用于上一章例7，情况iii。最终的逻辑解是

$$x = \frac{0}{0}stu + \frac{1}{0}s\bar{t}u + \frac{1}{0}st\bar{u} + s\bar{t}\bar{u} + \frac{1}{0}\bar{s}tu + 0\bar{s}\bar{t}u + 0\bar{s}t\bar{u} + 0\bar{s}\bar{t}\bar{u},$$

所给出的数据

$$\text{Prob. } s = p, \quad \text{Prob. } t = q, \quad \text{Prob. } u = r.$$

我们将寻求 x 的数值极限，以及连接 p、q 和 r 的条件。根据规则，x 的上限由 stu + s¯t¯u 的上限给出。因此，它们是

$$p, q + 1 - r, \quad r + 1 - q.$$

x 的下限由以下各项的下限给出

$$s\bar{t}u + st\bar{u} + s\bar{t}\bar{u} + \bar{s}tu.$$

我们可以将这些成分中的第一个和第三个收集到单项 s¯t 中，将第二个和第三个收集到单项 s¯u 中。x 的下限然后必须分别从项 s(1 – t)、s(1 – u)、(1 – s) tu 推导出，这些给出

$$p + 1 - q - 1, \quad p + 1 - r - 1, \quad 1 - p + q + r - 2,$$
$$\text{or} \qquad p - q, \quad p - r, \quad \text{and } q + r - p - 1.$$

Finally, the conditions among the constants p, q, and r, are given by the terms

$$s\bar{t}u, \quad st\bar{u}, \quad \bar{s}tu,$$

由此，根据规则，我们推得

$$p + 1 - q + r - 2 \overline{=}< 0, \quad p + q + 1 - r - 2 \overline{=}< 0, \quad 1 - p + q + r - 2 \overline{=}< 0.$$
$$\text{or } 1 + q - p - r \overline{=}> 0, \quad 1 + r - p - q \overline{=}> 0, \quad 1 + p - q - r \overline{=}> 0.$$

这些是最终解分析中使用的限制条件。然而，在该解中限制 λ 的条件仅仅是从量 s、t 和 u 应为正的条件确定的。从上述探究中很可能可以推导出该量的更窄极限。

12. 以下应用取自一个重要问题，其解将在下一章给出。已知，

$$\text{Prob. } x = c_1, \quad \text{Prob. } y = c_2, \quad \text{Prob. } s = c_1p_1, \quad \text{Prob. } t = c_2p_2,$$

together with the logical equation

$$z = stxy + s\bar{t}x\bar{y} + \bar{s}t\bar{x}y + 0\bar{s}\bar{t}$$
$$+\frac{1}{0}\left\{\begin{array}{r} stx\bar{y} + st\bar{x}y + st\bar{x}\bar{y} + s\bar{t}xy + s\bar{t}\bar{x}y \\ +s\bar{t}\bar{x}\bar{y} + \bar{s}txy + \bar{s}tx\bar{y} + \bar{s}t\bar{x}\bar{y}; \end{array}\right.$$

and it is required to determine the conditions among the constants c_1, c_2, p_1, p_2, and the major and minor limits of z.

First let us seek the conditions among the constants. Confining our attention to the terms whose coefficients are $\frac{1}{0}$, we readily form, by the aggregation of constituents, the following terms, viz.:

$$s(1-x), \qquad t(1-y), \qquad sy(1-t), \qquad tx(1-s);$$

nor can we form any other terms which are not included under these. Hence the conditions among the constants are,

$$n(s) + n(1-x) - n(1) \overset{=}{<} 0,$$
$$n(t) + n(1-y) - n(1) \overset{=}{<} 0,$$
$$n(s) + n(y) + n(1-t) - 2n(1) \overset{=}{<} 0,$$
$$n(t) + n(x) + n(1-s) - 2n(1) \overset{=}{<} 0.$$

Now replace $n(x)$ by c_1, $n(y)$ by c_2, $n(s)$ by c_1p_1, $n(t)$ by c_2p_2, and $n(1)$ by 1, and we have, after slight reductions,

$$c_1p_1 \overset{=}{<} c_1, \qquad c_2p_2 \overset{=}{<} c_2,$$
$$c_1p_1 \overset{=}{<} 1 - c_2(1-p_2), \qquad c_2p_2 \overset{=}{<} 1 - c_1(1-p_1).$$

Such are, then, the requisite conditions among the constants.

Again, the major limits of z are identical with those of the expression

$$stxy + s(1-t)x(1-y) + (1-s)t(1-x)y;$$

which, if we bear in mind the conditions

$$n(s) \overset{=}{<} n(x), \qquad n(t) \overset{=}{<} n(y),$$

above determined, will be found to be

$$n(s) + n(t), \text{ or, } c_1p_1 + c_2p_2,$$
$$n(s) + n(1-x), \text{ or, } 1 - c_1(1-p_1)n(t) + n(1-y), \qquad \text{or, } 1 - c_2(1-p_2).$$

Lastly, to ascertain the minor limits of z, we readily form from the constituents, whose coefficients are 1 or $\frac{1}{0}$, the single terms s and t, nor can any other terms not included under these be formed by selection or aggregation. Hence, for the minor limits of z we have the values c_1p_1 and c_2p_2.

连同逻辑方程

$$z = stxy + s\bar{t}x\bar{y} + \bar{s}t\bar{x}y + 0\bar{s}\bar{t}$$
$$+\frac{1}{0}\left\{\begin{array}{r} stx\bar{y} + st\bar{x}y + st\bar{x}\bar{y} + s\bar{t}xy + s\bar{t}\bar{x}y \\ +s\bar{t}\bar{x}\bar{y} + \bar{s}txy + \bar{s}tx\bar{y} + \bar{s}t\bar{x}\bar{y}; \end{array}\right.$$

并且要求确定常数 c 1、c 2、p 1、p 2之间的条件，以及 z 的主要极限和次要极限。

首先让我们寻求常数之间的条件。将我们的注意力限制在系数为[10]我们容易形成，通过成分的聚合，以下各项，即：

$$s(1-x), \qquad t(1-y), \qquad sy(1-t), \qquad tx(1-s);$$

我们也不能形成任何不包含在这些之下的其他项。因此常数之间的条件是，

$$n(s) + n(1-x) - n(1) \overset{=}{<} 0,$$
$$n(t) + n(1-y) - n(1) \overset{=}{<} 0,$$
$$n(s) + n(y) + n(1-t) - 2n(1) \overset{=}{<} 0,$$
$$n(t) + n(x) + n(1-s) - 2n(1) \overset{=}{<} 0.$$

Now replace $n(x)$ by c_1, $n(y)$ by c_2, $n(s)$ by c_1p_1, $n(t)$ by c_2p_2, and $n(1)$ by 1, and we have, after slight reductions,

$$c_1p_1 \overset{=}{<} c_1, \qquad c_2p_2 \overset{=}{<} c_2,$$
$$c_1p_1 \overset{=}{<} 1 - c_2(1-p_2), \qquad c_2p_2 \overset{=}{<} 1 - c_1(1-p_1).$$

那么，这些就是常数之间所必需的条件。

同样，z 的主要极限与该表达式的极限相同

$$stxy + s(1-t)x(1-y) + (1-s)t(1-x)y;$$

只要我们把那些条件记在心里

$$n(s) \overset{=}{<} n(x), \qquad n(t) \overset{=}{<} n(y),$$

上述已确定的内容，将会被发现是

$$n(s) + n(t), \text{ or, } c_1p_1 + c_2p_2,$$

n(s) + n(1 – x) ， 或，1 – c 1(1 – p 1) n(t) + n(1 – y) ， 或，1 – c 2(1 – p 2) .最后，为了确定 z 的较小极限，我们容易从成分中形成，其系数为 1 或[10]该单个项s和t,也不能任何其他未包含在这些之下的项通过选择或聚合形成。因此，对于 z 的较小极限，我们有值 c 1 p 1和 c 2 p 2。

13. It is to be observed, that the method developed above does not always assign the narrowest limits which it is possible to determine. But it in all cases, I believe, sufficiently limits the solutions of questions in the theory of probabilities.

The problem of the determination of the *narrowest* limits of numerical extension of a class is, however, always reducible to a purely algebraical form.[2] Thus, resuming the equations

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D,$$

let the highest inferior numerical limit of w be represented by the formula $an(s) + bn(t) \ldots + dn(1)$, wherein $a, b, c, \ldots d$ are numerical constants to be determined, and s, t, &c., the logical symbols of which A, B, C, D are constituents. Then

$$an(s) + bn(t) \ldots + dn(1) = \text{ minor limit of } A \text{ subject to the condition } D = 0.$$

Hence if we develop the function

$$as + bt \ldots + d,$$

reject from the result all constituents which are found in D, the coefficients of those constituents which remain, and are found also in A, ought not individually to exceed unity in value, and the coefficients of those constituents which remain, and which are not found in A, should individually not exceed 0 in value. Hence we shall have a series of inequalities of the form $f < 1$, and another series of the form $g \overline{=} 0$, f and g being linear functions of a, b, c, &c. Then those values of $a, b \ldots d$, which, while satisfying the above conditions, give to the function

$$an(s) + bn(t) \ldots + dn(1),$$

its highest value must be determined, and the highest value in question will be the highest minor limit of w. To the above we may add the relations similarly formed for the determination of the relations among the given constants $n(s), n(t) \ldots n(1)$.

14. The following somewhat complicated example will show how the limitation of a solution is effected, when the problem involves an arbitrary element, constituting it the representative of a system of problems agreeing in their data, but unlimited in their quæsita.

PROBLEM.—Of n events $x_1\ x_2 \ldots x_n$, the following particulars are known:

1st. The probability that either the event x_1 will occur, or all the events fail, is p_1.

[2]The author regrets the loss of a manuscript, written about four years ago, in which this method, he believes, was developed at considerable length. His recollection of the contents is almost entirely confined to the *impression* that the principle of the method was the same as above described, and that its sufficiency was proved. The prior methods of this chapter are, it is almost needless to say, easier, though certainly less general.

13. 应当注意，上述展开的方法并不总是给出可能确定的最窄极限。但据我相信，它在所有情况下都充分地限制了概率论中问题的解。

然而，确定一个类的数值外延的最窄界限的问题，总是可以化归为纯粹代数的形式。[2] 因此，重新取方程

$$w = A + 0B + \frac{0}{0}C + \frac{1}{0}D,$$

设 w 的最高下数值极限由公式 an(s) + bn(t) ...+ dn(1) 表示，其中 a, b, c, ...d 是待确定的数值常数，而 s、t 等是逻辑记号，A、B、C、D 是成分。那么

$$an(s) + bn(t) \ldots + dn(1) = \text{ minor limit of } A \text{ subject to 到条件 D = 0。}$$

因此，如果我们展开该函数

$$as + bt \ldots + d,$$

从结果中剔除在 D 中发现的所有成分，那些保留下来的、并且也在 A 中发现的成分的系数，其值各自不应超过 1，而那些保留下来的、并且不在 A 中发现的成分的系数，其值各自不应超过 0。因此

我们将得到一系列形如 f < 1 的不等式,以及另一系列

形式 g < 0,f 和 g 是 a、b、c 等的线性函数。那么,那些 a, b ...d 的值,在满足上述条件的同时,使函数

$$an(s) + bn(t) \ldots + dn(1),$$

取得其最高值，必须被确定，而所论的最高值将是 w 的最高次极限。在上述内容之上，我们还可以补充为确定给定常数 n(s) , n(t) ...n(1) 之间的关系而类似地形成的关系。

14. 以下这个稍显复杂的例子将表明，当问题涉及一个任意元素、使它成为一组数据相同但所求问题不受限制的问题的代表时，解的限制是如何实现的。

问题。--关于 n 个事件 x 1 x 2 ...x n，已知以下具体情况：第一，事件 x 1将发生，或者所有事件都不发生的概率是 p 1。

2 作者遗憾地遗失了一份约四年前写成的手稿，他相信其中曾相当详尽地展开了这一方法。他对内容的记忆几乎完全局限于这样一种印象：该方法的原理与上述相同，并且其充分性得到了证明。几乎无需说明，本章先前的方法更容易，尽管确实不那么一般。

2nd. The probability that either the event x_2 will occur, or all the events fail, is p_2. And so on for the others.

It is required to find the probability of any single event, or combination of events, represented by the general functional form $\phi(x_1 \dots x_n)$, or ϕ.

Adopting a previous notation, the data of the problem are

$$\text{Prob. } (x_1 + \bar{x}_1 \dots \bar{x}_n) = p_1 \dots \text{Prob. } (x_n + \bar{x}_1 \dots \bar{x}_n) = p_n.$$

And Prob. $\phi(x_1 \dots x_n)$ is required.

Assume generally

$$x_r + \bar{x}_1 \dots \bar{x}_n = s_r, \tag{1}$$
$$\phi = w. \tag{2}$$

We hence obtain the collective logical equation of the problem

$$\sum \{ (x_r + \bar{x}_1 \dots \bar{x}_n) \bar{s}_r + s_r (\bar{x}_r - \bar{x}_1 \dots \bar{x}_n) \} + \phi \bar{w} + w \bar{\phi} = 0. \tag{3}$$

From this equation we must eliminate the symbols $x_1, \dots x_n$, and determine w as a developed logical function of $s_1 \dots s_n$.

Let us represent the result of the aforesaid elimination in the form

$$Ew + E'(1 - w) = 0;$$

then will E be the result of the elimination of the same symbols from the equation

$$\sum \{ (x_r + \bar{x}_1 \dots \bar{x}_n) \bar{s}_r + s_r (\bar{x}_r - \bar{x}_1 \dots \bar{x}_n) \} + 1 - \phi = 0. \tag{4}$$

Now E will be the product of the coefficients of all the constituents (considered with reference to the symbols $x_1, x_2 \dots x_n$) which are found in the development of the first member of the above equation. Moreover, ϕ, and therefore $1 - \phi$, will consist of a series of such constituents, having unity for their respective coefficients. In determining the forms of the coefficients in the development of the first member of (4), it will be convenient to arrange them in the following manner:

1st. The coefficients of constituents found in $1 - \phi$.

2nd. The coefficient of $\bar{x}_1, \bar{x}_2 \dots \bar{x}_n$, if found in ϕ.

3rd. The coefficients of constituents found in ϕ, excluding the constituent $\bar{x}_1, \bar{x}_2 \dots \bar{x}_n$.

The above is manifestly an exhaustive classification.

First then; the coefficient of any constituent found in $1 - \phi$, will, in the development of the first member of (4), be of the form

$$1 + \text{ positive terms derived from } \sum.$$

Hence, every such coefficient may be replaced by unity, Prop. I. Chap. IX.

第二，事件 x 2将发生，或者所有事件都不发生的概率是 p 2。其余事件依此类推。

要求求出由一般函数形式 φ(x 1 ...x n) 或 φ 所表示的任何单个事件或事件组合的概率。

沿用前面的记号，本题的数据为

$$\text{Prob. } (x_1 + \bar{x}_1 \dots \bar{x}_n) = p_1 \dots \text{Prob. } (x_n + \bar{x}_1 \dots \bar{x}_n) = p_n.$$

And Prob. $\phi(x_1 \dots x_n)$ is required.

Assume generally

$$x_r + \bar{x}_1 \dots \bar{x}_n = s_r, \tag{1}$$
$$\phi = w. \tag{2}$$

我们由此得到该问题的集体逻辑方程

$$\sum \{ (x_r + \bar{x}_1 \dots \bar{x}_n) \bar{s}_r + s_r (\bar{x}_r - \bar{x}_1 \dots \bar{x}_n) \} + \phi \bar{w} + w \bar{\phi} = 0. \tag{3}$$

从这个方程中我们必须消去记号 x 1 , ...x n，并把 w 确定为 s 1 ...s n 的展开逻辑函数。

让我们把上述消元的结果表示成如下形式

$$Ew + E'(1 - w) = 0;$$

那么 E 将是从该方程中消去相同记号的结果

$$\sum \{ (x_r + \bar{x}_1 \dots \bar{x}_n) \bar{s}_r + s_r (\bar{x}_r - \bar{x}_1 \dots \bar{x}_n) \} + 1 - \phi = 0. \tag{4}$$

现在 E 将是所有成分的系数的乘积(参照记号 x 1 , x 2 ...x n 来考虑)，这些成分出现在上述方程第一个成员的展开式中。此外，φ，因此 1 – φ，将由一系列这样的成分组成，它们各自的系数为 1。在确定(4)的第一个成员的展开式中系数的形式时，将它们按以下方式排列将是方便的：

1st. The coefficients of constituents found in $1 - \phi$.

2nd. The coefficient of $\bar{x}_1, \bar{x}_2 \dots \bar{x}_n$, if found in ϕ.

3rd. The coefficients of constituents found in ϕ, excluding the constituent $\bar{x}_1, \bar{x}_2 \dots \bar{x}_n$.

The above is manifestly an exhaustive classification.

First then; the coefficient of any constituent found in $1 - \phi$, will, in the development of the first member of (4), be of the form

1 + 由X导出的正项

因此，每一个这样的系数都可以用1来代替，见第九章命题一。

Secondly; the coefficient of $\bar{x}_1 \ldots \bar{x}_n$, if found in ϕ, in the development of the first member of (4) will be

$$\sum \bar{s}_r, \text{ or } \bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$$

Thirdly; the coefficient of any other constituent, $x_1 \ldots x_i, \bar{x}_{i+1} \ldots \bar{x}_n$, found in ϕ, in the development of the first member of (4) will be $\bar{s}_1 \ldots \bar{s}_i + s_{i+1} \ldots + s_n$.

Now it is seen, that E is the product of all the coefficients above determined; but as the coefficients of those constituents which are not found in ϕ reduce to unity, E may be regarded as the product of the coefficients of those constituents which are found in ϕ. From the mode in which those coefficients are formed, we derive the following rule for the determination of E, viz., in each constituent found in ϕ, except the constituent $\bar{x}_1\, \bar{x}_2 \ldots \bar{x}_n$, for x_1 write $\bar{s}_1$, for $\bar{x}_l$ write s_1, and so on, and add the results; but for the constituent $\bar{x}_1, \bar{x}_2 \ldots \bar{x}_n$, if it occur in ϕ, write $\bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$, the product of all these sums is E.

To find E' we must in (3) make $w = 0$, and eliminate $x_1, x_2 \ldots x_n$, from the reduced equation. That equation will be

$$\sum \{(x_r + \bar{x}_1 \ldots + \bar{x}_n)\bar{s}_r + s_r(\bar{x}_r - \bar{x}_1 \ldots \bar{x}_n)\} + \phi = 0. \qquad (5)$$

Hence E' will be formed from the constituents in $1-\phi$, i. e. from the constituents *not* found in ϕ in the same way as E is formed from the constituents found in ϕ.

Consider next the equation

$$Ew + E'(1-w) = 0.$$

This gives

$$w = \frac{E'}{E' - E}. \qquad (6)$$

Now E and E' are functions of the symbols $s_1, s_2 \ldots s_n$. The expansion of the value of w will, therefore, consist of all the constituents which can be formed out of those symbols, with their proper coefficients annexed to them, as determined by the rule of development.

Moreover, E and E' are each formed by the multiplication of factors, and neither of them can vanish unless some one of the factors of which it is composed vanishes. Again, any factor, as $\bar{s}_1 \ldots + \bar{s}_n$ can only vanish when all the terms by the addition of which it is formed vanish together, since in development we attribute to these terms the values 0 and 1, only. It is further evident, that no two factors differing from each other can vanish together. Thus the factors $\bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$, and $s_1 + \bar{s}_2 \ldots + \bar{s}_n$, cannot simultaneously vanish, for the former cannot vanish unless $\bar{s}_1 = 0$, or $s_1 = 1$; but the latter cannot vanish unless $s_1 = 0$.

First, let us determine the coefficient of the constituent $\bar{s}_1 \bar{s}_2 \ldots \bar{s}_n$ in the development of the value of w.

The simultaneous assumption $\bar{s}_1 = 1$, $\bar{s}_2 = 1 \ldots \bar{s}_n = 1$, would cause the factor $s_1 + s_2 \ldots + s_n$ to vanish if this should occur in E or E'; and no other

其次； x1.....⁻xn 的系数，若在 φ 中出现，在 (4) 第一项的展开式中将是

$$\sum \bar{s}_r, \text{ or } \bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$$

Thirdly; the coefficient of any other constituent, $x_1 \ldots x_i, \bar{x}_{i+1} \ldots \bar{x}_n$, found in ϕ, in the development of the first member of (4) will be $\bar{s}_1 \ldots \bar{s}_i + s_{i+1} \ldots + s_n$.

现在可以看出，E 是上面确定的所有系数的乘积；但由于那些未出现在 φ 中的成分的系数归结为 1,E 可以被视为那些出现在 φ 中的成分的系数的乘积。从这些系数形成的方式，我们推导出确定 E 的以下规则，即，在 φ 中出现的每个成分中，除了成分 x_1⁻x_2 ...⁻x n，对于 x_1写 s_1，对于 x l 写 s_1，依此类推，并相加结果；但对于成分 x_1 ,⁻x_2 ...⁻x n，如果它出现在 φ 中，写 s_1 +⁻s_2 ...+⁻s n，所有这些和的乘积是 E。

为求 E ′，我们必须在 (3) 中令 w = 0，并从化简后的方程中消去 x_1 , x_2 ...x n。该方程将是

$$\sum \{(x_r + \bar{x}_1 \ldots + \bar{x}_n)\bar{s}_r + s_r(\bar{x}_r - \bar{x}_1 \ldots \bar{x}_n)\} + \phi = 0. \qquad (5)$$

Hence E' will be formed from the constituents in $1-\phi$, i. e. from the constituents *not* found in ϕ in the same way as E is formed from the constituents found in ϕ.

Consider next the equation

$$Ew + E'(1-w) = 0.$$

这给出

$$w = \frac{E'}{E' - E}. \qquad (6)$$

现在 E 和 E ′是记号 s_1 , s_2 ...s n 的函数。因此，w 的值的展开式将包含由这些记号所能形成的全部成分，并按照展开规则所确定的、附于其上的适当系数。

此外，E 和 E ′各自通过因子的乘法形成，并且它们中任何一个都不能消失，除非组成它的某个因子消失。再者，任何因子，如 s_1 ...+⁻s n 只能当通过加法形成它的所有项一起消失时才能消失，因为在展开式中我们只赋予这些项值 0 和 1。进一步明显的是，没有两个彼此不同的因子能一起消失。因此，因子 s_1 +⁻s_2 ...+⁻s n，和 s_1 +⁻s_2 ...+⁻s n，不能同时消失，因为前者不能消失，除非 s_1 = 0，或 s_1 = 1；但后者不能消失，除非 s_1 = 0。

首先，让我们确定成分 s_1⁻s_2 ...⁻s n 在 w 的值的展开式中的系数。

同时假定 s_1 = 1、⁻s_2 = 1 ...⁻s n = 1，如果这出现在 E 或 E ′中，将使因子 s_1 + s_2 ...+ s n 消失；并且没有其他

factor under the same assumption would vanish; but $s_1 + s_2 \ldots + s_n$ does not occur as a factor of either E or E'; neither of these quantities, therefore, can vanish; and, therefore, the expression $\frac{E'}{E'-E}$, is neither 1, 0, nor $\frac{0}{0}$.

Wherefore the coefficient of $\bar{s}_1\ \bar{s}_2 \ldots \bar{s}_n$ *in the expanded value of* w, may be represented by $\frac{1}{0}$.

Secondly, let us determine the coefficient of the constituent $s_1\ s_2 \ldots s_n$. The assumptions $s_1 = 1, s_2 = 1, \ldots s_n = 1$, would cause the factor $\bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$ to vanish. Now this factor is found in E and not in E' whenever ϕ contains both the constituents $x_1\ x_2 \ldots x_n$ and $\bar{x}_1\ \bar{x}_2 \ldots \bar{x}_n$. Here then $\frac{E'}{E'-E}$ becomes $\frac{E'}{E'}$ or 1. The factor $\bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$ is found in E' and not in E, if ϕ contains neither of the constituents $x_1\ x_2 \ldots x_n$ and $\bar{x}_1\ \bar{x}_2 \ldots \bar{x}_n$. Here then $\frac{E'}{E'-E}$ becomes $\frac{0}{-E}$ or 0. Lastly, the factor $\bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$ is contained in both E and E', if one of the constituents $x_1\ x_2 \ldots x_n$ and $\bar{x}_1\ \bar{x}_2 \ldots \bar{x}_n$ is found in ϕ, and one is not. Here then $\frac{E'}{E'-E}$ becomes $\frac{0}{0}$.

The coefficient of the constituent $s_1\ s_2 \ldots s_n$, *will therefore be* 1, 0, *or* $\frac{0}{0}$, *according as* ϕ *contains both the constituents* $x_1\ x_2 \ldots x_n$ *and* $\bar{x}_1\ \bar{x}_2 \ldots \bar{x}_n$, *or neither of them, or one of them and not the other.*

Lastly, to determine the coefficient of any other constituent as $s_1 \ldots s_i\ \bar{s}_{i+1} \ldots \bar{s}_n$.

The assumptions $s_1 = 1, \ldots s_i = 1,\ s_{i+1} = 0,\ s_n = 0$, would cause the factor $\bar{s}_1 \ldots + \bar{s}_i + s_{i+1} \ldots + s_n$ to vanish. Now this factor is found in E, if the constituent $x_1 \ldots x_i\ \bar{x}_{i+1} \ldots \bar{x}_n$ is found in ϕ and in E', if the said constituent is not found in ϕ. In the former case we have $\frac{E'}{E'-E} = \frac{E'}{E'} = 1$; in the latter case we have $\frac{E'}{E'-E} = \frac{0}{0-E} = 0$.

Hence the coefficient of any other constituent $s_1 \ldots s_i,\ \bar{s}_{i+1} \ldots \bar{s}_n$ *is 1 or 0 according as the similar constituent* $x_1 \ldots x_i\ \bar{x}_{i+1} \ldots \bar{x}_n$ *is or is not found in* ϕ.

We may, therefore, practically determine the value of w in the following manner. Rejecting from the given expression of ϕ the constituents $x_1\ x_2 \ldots x_c$ and $\bar{x}_1\ \bar{x}_2 \ldots \bar{x}_n$, should both or either of them be contained in it, let the symbols $x_1,\ x_2, \ldots x_n$, in the result be changed into $s_1,\ s_2, \ldots s_n$ respectively. Let the coefficients of the constituents $s_1\ s_2 \ldots s_n$ and $\bar{s}_1\ \bar{s}_2 \ldots \bar{s}_n$ be determined according to the special rules for those cases given above, and let every other constituent have for its coefficient 0. The result will be the value of w as a function of $s_1, s_2, \ldots s_n$.

As a particular case, let $\phi = x_1$. It is required from the given data to determine the probability of the event x_1.

The symbol x_1 expanded in terms of the entire series of symbols $x_1, x_2, \ldots x_n$, will generate all the constituents of those symbols which have x_1 as a factor. Among those constituents will be found the constituent $x_1\ x_2 \ldots x_n$, but not the constituent $\bar{x}_1\ \bar{x}_2 \ldots \bar{x}_n$.

Hence in the expanded value of x_1 as a function of the symbols $s_1, s_2, \ldots s_n$, the constituent $s_1\ s_2 \ldots s_n$ will have the coefficient $\frac{0}{0}$, and the constituent $\bar{s}_1\ \bar{s}_2 \ldots \bar{s}_n$ the coefficient $\frac{1}{0}$.

If from x_1 we reject the constituent $x_1\ x_2 \ldots x_n$, the result will be $x_1 - x_1 x_2 \ldots x_n$, and changing therein x_1 into s_1 &c., we have $s_1 - s_1 s_2 \ldots s_n$ for the

在同一假设下因子会消失；但 s 1 + s 2 ...+ s n 不作为 E 或 E ′的因子出现；因此，这些量中任何一个都不能消失；因此，表达式 E ′ E ′ – E ′是既不1,0,也不00.

Wherefore the coefficient of $\bar{s}_1\ \bar{s}_2 \ldots \bar{s}_n$ *in the expanded value of* w, may be represented by $\frac{1}{0}$.

Secondly, let us determine the coefficient of the constituent $s_1\ s_2 \ldots s_n$. The assumptions $s_1 = 1, s_2 = 1, \ldots s_n = 1$, would cause the factor $\bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$ to vanish. Now this factor is found in E and not in E' whenever ϕ contains both the constituents $x_1\ x_2 \ldots x_n$ and $\bar{x}_1\ \bar{x}_2 \ldots \bar{x}_n$. Here then $\frac{E'}{E'-E}$ becomes $\frac{E'}{E'}$ or 1. The factor $\bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$ is found in E' and not in E, if ϕ contains neither of the constituents $x_1\ x_2 \ldots x_n$ and $\bar{x}_1\ \bar{x}_2 \ldots \bar{x}_n$. Here then $\frac{E'}{E'-E}$ becomes $\frac{0}{-E}$ or 0. Lastly, the factor $\bar{s}_1 + \bar{s}_2 \ldots + \bar{s}_n$ is contained in both E and E', if one of the constituents $x_1\ x_2 \ldots x_n$ and $\bar{x}_1\ \bar{x}_2 \ldots \bar{x}_n$ is found in ϕ, and one is not. Here then $\frac{E'}{E'-E}$ becomes $\frac{0}{0}$.

成分 s 1 s 2 ...s n 的系数，因此将是 1，0 ，或00，取决于它同时包含成分 x 1 x 2 ...x n 和 x 1 ¯ x 2 ... ¯ x n ，还是两者都不包含，还是包含其中之一而不包含另一个。

Lastly, to determine the coefficient of any other constituent as $s_1 \ldots s_i\ \bar{s}_{i+1} \ldots \bar{s}_n$.

The assumptions $s_1 = 1, \ldots s_i = 1,\ s_{i+1} = 0,\ s_n = 0$, would cause the factor $\bar{s}_1 \ldots + \bar{s}_i + s_{i+1} \ldots + s_n$ to vanish. Now this factor is found in E, if the constituent $x_1 \ldots x_i\ \bar{x}_{i+1} \ldots \bar{x}_n$ is found in ϕ and in E', if the said constituent is not found in ϕ. In the former case we have $\frac{E'}{E'-E} = \frac{E'}{E'} = 1$; in the latter case we have $\frac{E'}{E'-E} = \frac{0}{0-E} = 0$.

Hence the coefficient of any other constituent $s_1 \ldots s_i,\ \bar{s}_{i+1} \ldots \bar{s}_n$ *is 1 or 0 according as the similar constituent* $x_1 \ldots x_i\ \bar{x}_{i+1} \ldots \bar{x}_n$ *is or is not found in* ϕ.

因此，我们可以实际地按以下方式确定w的值。从φ的给定表达式中剔除成分x 1 x 2 ...x c和 x 1 ¯ x 2 ... ¯ x n，如果其中包含它们两者或其中之一，则在结果中将记号x 1 , x 2 , ...x n分别改为s 1 , s 2 , ...s n。成分s 1 s 2 ...s n和 s 1 ¯ s 2 ... ¯ s n的系数，按上文针对这些情形给出的特殊规则确定，并令其他每个成分的系数为0。结果将是w作为s 1 , s 2 , ...s n的函数的值。

作为一个特例，设 φ = x 1。要求从给定数据确定事件 x 1的概率。

记号x 1按记号x 1 , x 2 , ...x n的整个序列展开，将生成以x 1为因子的那些记号的所有成分。在这些成分中将找到成分x 1 x 2 ...x n，但找不到成分 x 1 ¯ x 2 ... ¯ x n。

Hence in the expanded value of x_1 as a function of the symbols $s_1, s_2, \ldots s_n$, the constituent $s_1\ s_2 \ldots s_n$ will have the coefficient $\frac{0}{0}$, and the constituent $\bar{s}_1\ \bar{s}_2 \ldots \bar{s}_n$ the coefficient $\frac{1}{0}$.

If from x_1 we reject the constituent $x_1\ x_2 \ldots x_n$, the result will be $x_1 - x_1 x_2 \ldots x_n$, and changing therein x_1 into s_1 &c., we have $s_1 - s_1 s_2 \ldots s_n$ for the

corresponding portion of the expression of x_1 as a function of $s_1, s_2, \dots s_n$.

Hence the final expression for x_1 is

$$x_1 = s_1 - s_1 s_2 \dots s_n + \frac{0}{0} s_1 s_2 \dots s_n + \frac{1}{0} \bar{s}_1 \bar{s}_2 \dots \bar{s}_n + \text{constituents whose coefficients are } 0. \tag{7}$$

The sum of all the constituents in the above expansion whose coefficients are either 1, 0, or $\frac{0}{0}$, will be $1 - \bar{s}_1 \bar{s}_2 \dots \bar{s}_n$.

We shall, therefore, have the following *algebraic* system for the determination of Prob. x_1, viz.:

$$\text{Prob. } x_1 = \frac{s_1 - s_1 s_2 \dots s_n + c s_1 s_2 \dots s_n}{1 - \bar{s}_1 \bar{s}_2 \dots \bar{s}_n}, \tag{8}$$

with the relations

$$\frac{s_1}{p_1} = \frac{s_2}{p_2} \dots = \frac{s_n}{p_n} = 1 - \bar{s}_1 \bar{s}_2 \dots \bar{s}_n = \lambda. \tag{9}$$

It will be seen, that the relations for the determination of $s_1\ s_2 \dots s_n$ are quite independent of the form of the function ϕ, and the values of these quantities, determined once, will serve for all possible problems in which the data are the same, however the *quæsita* of those problems may vary. The nature of that event, or combination of events, whose probability is sought, will affect only the form of the function in which the determined values of $s_1\ s_2 \dots s_n$ are to be substituted. We have from (9)

$$s_1 = p_1 \lambda, \quad s_2 = p_2 \lambda, \dots s_n = p_n \lambda.$$

Whence

$$1 - (1 - p_1\lambda)(1 - p_2\lambda) \dots (1 - p_n\lambda) = \lambda.$$

Or,

$$1 - \lambda = (1 - p_1\lambda)(1 - p_2\lambda) \dots (1 - p_n\lambda); \tag{10}$$

from which equation the value of λ is to be determined.

Supposing this value determined, the value of Prob. x_1 will be

$$\frac{p_1\lambda - (1-c)p_1 p_2 \dots p_n \lambda^n}{1 - (1 - p_1\lambda)(1 - p_2\lambda) \dots (1 - p_n\lambda)},$$

or, on reduction by (10),

$$\text{Prob. } x_1 = p_1 - (1-c)p_1 p_2 \dots p_n \lambda^{n-1}. \tag{11}$$

Let us next seek the conditions which must be fulfilled among the constants $p_1, p_2, \dots p_n$, and the limits of the value of Prob. x_1.

corresponding portion of the expression of x_1 as a function of $s_1, s_2, \dots s_n$.

Hence the final expression for x_1 is

$$x_1 = s_1 - s_1 s_2 \dots s_n + \frac{0}{0} s_1 s_2 \dots s_n + \frac{1}{0} \bar{s}_1 \bar{s}_2 \dots \bar{s}_n + \text{constituents whose coefficients are } 0. \tag{7}$$

上述展开式中系数为的所有成分之和。因此，我们将得到以下用于确定的代数方程组为 1、0 或00,将是1− s 1 -s 2 ...-s n。Prob.x 1 的，即：

$$\text{Prob. } x_1 = \frac{s_1 - s_1 s_2 \dots s_n + c s_1 s_2 \dots s_n}{1 - \bar{s}_1 \bar{s}_2 \dots \bar{s}_n}, \tag{8}$$

with the relations

$$\frac{s_1}{p_1} = \frac{s_2}{p_2} \dots = \frac{s_n}{p_n} = 1 - \bar{s}_1 \bar{s}_2 \dots \bar{s}_n = \lambda. \tag{9}$$

可以看出，用于确定s 1 s 2 ...s n的关系式完全独立于函数φ的形式，而这些量的值一经确定，便适用于数据相同的所有可能问题，无论那些问题的*quæsita*如何变化。其概率被寻求的那个事件或事件组合的性质，只影响要代入s 1 s 2 ...s n的已确定值的函数的形式。由(9)我们有

$$s_1 = p_1 \lambda, \quad s_2 = p_2 \lambda, \dots s_n = p_n \lambda.$$

Whence

$$1 - (1 - p_1\lambda)(1 - p_2\lambda) \dots (1 - p_n\lambda) = \lambda.$$

Or,

$$1 - \lambda = (1 - p_1\lambda)(1 - p_2\lambda) \dots (1 - p_n\lambda); \tag{10}$$

from which equation the value of λ is to be determined.

Supposing this value determined, the value of Prob. x_1 will be

$$\frac{p_1\lambda - (1-c)p_1 p_2 \dots p_n \lambda^n}{1 - (1 - p_1\lambda)(1 - p_2\lambda) \dots (1 - p_n\lambda)},$$

或，通过(10)化简，

$$\text{Prob. } x_1 = p_1 - (1-c)p_1 p_2 \dots p_n \lambda^{n-1}. \tag{11}$$

Let us next seek the conditions which must be fulfilled among the constants $p_1, p_2, \dots p_n$, and the limits of the value of Prob. x_1.

As there is but one term with the coefficient $\frac{1}{0}$, there is but one condition among the constants, viz.,

$$\text{Minor limit, } (1-s_1)(1-s_2)\dots(1-s_n) \overline{=}< 0.$$

$$\text{Or, } \quad n(1-s_1)+n(1-s_2)\dots+n(1-s_n)-(n-1)n(1) \overline{=}< 0.$$

$$\text{Or, } \quad n(1)-n(s_1)-n(s_2)\dots-n(s_n) \overline{=}< 0.$$

$$\text{Whence } \quad p_1+p_2\dots+p_n \overline{=}> 1,$$

the condition required.

The major limit of Prob. x_1 is the major limit of the sum of those constituents whose coefficients are 1 or $\frac{0}{0}$. But that sum is s_1.

Hence,

$$\text{Major limit, Prob. } x_1 = \text{ major limit } s_1 = p_1.$$

The minor limit of Prob. x_1 will be identical with the minor limit of the expression

$$s_1 - s_1 s_2 \dots s_n + (1-s_1)(1-s_2)\dots(1-s_n).$$

A little attention will show that the different aggregates, terms which can be formed out of the above, each including the greatest possible number of constituents, will be the following, viz.:

$$s_1(1-s_2), \quad s_1(1-s_3), \dots s_1(1-s_n), \quad (1-s_2)(1-s_3)\dots(1-s_n).$$

From these we deduce the following expressions for the minor limit, viz.:

$$p_1-p_2, \quad p_1-p_3\dots p_1-p_n, \quad 1-p_2-p_3\dots-p_n.$$

The value of Prob. x_1 will, therefore, not fall short of any of these values, nor exceed the value of p_1.

Instead, however, of employing these conditions, we may directly avail ourselves of the principle stated in the demonstration of the general method in probabilities. The condition that $s_1, s_2, \dots s_n$ must each be less than unity, requires that λ should be less than each of the quantities $\frac{1}{p_1}, \frac{1}{p_2}, \dots \frac{1}{p_n}$. And the condition that $s_1, s_2, \dots s_n$, must each be greater than 0, requires that λ should also be greater than 0. Now $p_1\ p_2 \dots p_n$ being proper fractions satisfying the condition

$$p_1+p_2\dots+p_n > 1,$$

it may be shown that but one positive value of λ can be deduced from the central equation (10) which shall be less than each of the quantities $\frac{1}{p_1}, \frac{1}{p_2}, \dots \frac{1}{p_n}$. That value of λ is, therefore, the one required.

To prove this, let us consider the equation

$$(1-p_1\lambda)(1-p_2\lambda)\dots(1-p_n\lambda)-1+\lambda = 0.$$

When $\lambda = 0$ the first member vanishes, and the equation is satisfied. Let us examine the variations of the first member between the limits $\lambda = 0$ and

由于只有一个系数为[10,]的项,所以只有一个常数之间的条件，即，

$$\text{Minor limit, } (1-s_1)(1-s_2)\dots(1-s_n) \overline{=}< 0.$$

$$\text{Or, } \quad n(1-s_1)+n(1-s_2)\dots+n(1-s_n)-(n-1)n(1) \overline{=}< 0.$$

$$\text{Or, } \quad n(1)-n(s_1)-n(s_2)\dots-n(s_n) \overline{=}< 0.$$

$$\text{Whence } \quad p_1+p_2\dots+p_n \overline{=}> 1,$$

所需条件。

Prob.x [1]的主要极限是那些系数为 1 或[00.]但那个和是[s1.]的成分之和的主要极限。

因此，

主要极限，Prob.x [1] = 主要极限 s [1] = p [1] .

Prob.x [1]的次要极限将与表达式的次要极限相同

$$s_1 - s_1 s_2 \dots s_n + (1-s_1)(1-s_2)\dots(1-s_n).$$

稍加注意就会表明，由上述内容所能形成的、各自包含尽可能多的成分的不同聚合项，将是以下这些，即：

$$s_1(1-s_2), \quad s_1(1-s_3), \dots s_1(1-s_n), \quad (1-s_2)(1-s_3)\dots(1-s_n).$$

由这些我们推出次极限的如下表达式，即：

$$p_1-p_2, \quad p_1-p_3\dots p_1-p_n, \quad 1-p_2-p_3\dots-p_n.$$

因此，Prob.x [1]的值既不会低于这些值中的任何一个，也不会超过 p [1]的值。

然而，我们可以不采用这些条件，而是直接利用概率中一般方法的论证所陈述的原理。条件 s [1] , s [2] , ...s n 各自必须小于1，这要求 λ 应小于各量[1] p [1,1] p [2,..1] p n .[并且该]条件 s [1] , s [2] , ...s n 各自必须大于0，这要求 λ 也应大于0。现在 p [1] p [2] ...p n 作为满足该条件的真分数

$$p_1+p_2\dots+p_n > 1,$$

可以证明，从中心方程(10)只能推出一个小于量[1] p [1,1] p [2,..1] p n .[该] λ 的值，因此，就是所求的那个值。

为了证明这一点，让我们考虑方程

$$(1-p_1\lambda)(1-p_2\lambda)\dots(1-p_n\lambda)-1+\lambda = 0.$$

当 λ = 0 时，第一项消失，方程得到满足。让我们考察第一项在极限 λ = 0 与

$\lambda = \frac{1}{p_1}$, supposing p_1 the greatest of the values $p_1\ p_2 \ldots p_n$. Representing the first member of the equation by V, we have

$$\frac{dV}{d\lambda} = -p_1(1-p_2\lambda)\ldots(1-p_n\lambda)\ldots - p_n(1-p_1\lambda)\ldots(1-p_{n-1}\lambda) + 1,$$

which, when $\lambda = 0$, assumes the form $-p_1 - p_2 \ldots - p_n + 1$, and is negative in value.

Again, we have

$$\frac{d^2V}{d\lambda^2} = p_1p_2(1-p_3\lambda)(1-p_n\lambda) + \ \&c.,$$

consisting of a series of terms which, under the given restrictions with reference to the value of λ, are *positive*.

Lastly, when $\lambda = \frac{1}{p_1}$, we have

$$V = -1 + \frac{1}{p_1},$$

which is positive.

From all this it appears, that if we construct a curve, the ordinates of which shall represent the value of V corresponding to the abscissa λ, that curve will pass through the origin, and will for small values of λ lie beneath the abscissa. Its convexity will, between the limits $\lambda = 0$ and $\lambda = \frac{1}{p_1}$ be downwards, and at the extreme limit $\frac{1}{p_1}$ the curve will be above the abscissa, its ordinate being positive. It follows from this description, that it will intersect the abscissa once, and only once, within the limits specified, viz., between the values $\lambda = 0$, and $\lambda = \frac{1}{p_1}$.

The solution of the problem is, therefore, expressed by (11), the value of λ being that root of the equation (10), which lies within the limits 0 and $\frac{1}{p_1}, \frac{1}{p_2}, \ldots \frac{1}{p_n}$.

The constant c is obviously the probability, that if the events $x_1, x_2, \ldots x_n$, all happen, or all fail, they will all happen.

This determination of the value of λ suffices for all problems in which the data are the same as in the one just considered. It is, as from previous discussions we are prepared to expect, a determination independent of the form of the function ϕ. Let us, as another example, suppose

$$\phi = \text{ or } w = x_1(1-x_2)\ldots(1-x_n)\ldots + x_n(1-x_1)\ldots(1-x_{n-1}).$$

This is equivalent to requiring the probability, that of the events $x_1, x_2, \ldots x_n$ one, and only one, will happen. The value of w will obviously be

$$w = s_1(1-s_2)\ldots(1-s_n)\ldots + s_n(1-s_1)\ldots(1-s_{n-1}) + \frac{1}{0}(1-s_1)\ldots(1-s_n),$$

λ = 1 p-1,假设p1该最大的值之中p1p2..pn.以该方程的第一项表示为 V，我们有

$$\frac{dV}{d\lambda} = -p_1(1-p_2\lambda)\ldots(1-p_n\lambda)\ldots - p_n(1-p_1\lambda)\ldots(1-p_{n-1}\lambda) + 1,$$

which, when $\lambda = 0$, assumes the form $-p_1 - p_2 \ldots - p_n + 1$, and is negative in value.

Again, we have

$$\frac{d^2V}{d\lambda^2} = p_1p_2(1-p_3\lambda)(1-p_n\lambda) + \ \&c.,$$

由一系列项组成，这些项在关于 λ 值的给定限制下是正的。

最后，当 λ = 1 p 1,我们有

$$V = -1 + \frac{1}{p_1},$$

这是正面的。

由上述一切可见，若我们构造一条曲线，使其纵坐标表示与横坐标 λ 相对应的 V 之值，则该曲线将经过原点，且对于 λ 的较小值将位于横坐标之下。在 λ = 0 与 λ = 1 p 1bedownwards,and在极限1 p 1thecurvewillbeabovetheabscissa,itsordinatebeing为正之间，其凸性将向下。由此描述可知，它将在指定范围内与横坐标相交一次，且仅一次，即在 λ = 0 与 λ = 1 p 1之间。

因此，问题的解由(11)表示，其中 λ 的值是方程(10)的位于 0 与1 p 1,1 p 2,..1 p n 之间的那个根。

常数 c 显然是概率：若事件 x1、x2、.....xn 全部发生或全部失败，它们将全部发生。

对 λ 值的这一确定，足以解决所有数据与刚才所考虑的问题相同的问题。正如根据先前的讨论我们所预料的那样，这是一个与函数 φ 的形式无关的确定。作为另一个例子，让我们假设

$$\phi = \text{ or } w = x_1(1-x_2)\ldots(1-x_n)\ldots + x_n(1-x_1)\ldots(1-x_{n-1}).$$

这等价于要求这样的概率：在事件 x 1 , x 2 , ...x n 中，有一个且仅有一个将发生。显然，w 的值将是

$$w = s_1(1-s_2)\ldots(1-s_n)\ldots + s_n(1-s_1)\ldots(1-s_{n-1}) + \frac{1}{0}(1-s_1)\ldots(1-s_n),$$

from which we should have

$$\begin{aligned}&\text{Prob. } \{x_1(1-x_2)\dots(1-x_n)\dots+x_n(1-x_1)\dots(1-x_{n-1})\}\\&=\frac{s_1(1-s_2)\dots(1-s_n)\dots+s_n(1-s_1)\dots(1-s_{n-1})}{1-(1-s_1)\dots(1-s_n)}\\&=\frac{p_1\lambda(1-p_2\lambda)\dots(1-p_n\lambda)\dots+p_n\lambda(1-p_1\lambda)\dots(1-p_{n-1}\lambda)}{\lambda}\\&=\frac{p_1(1-\lambda)}{1-p_1\lambda}+\frac{p_2(1-\lambda)}{1-p_2\lambda}\dots+\frac{p_n(1-\lambda)}{1-p_n\lambda}\end{aligned}$$

This solution serves well to illustrate the remarks made in the introductory chapter (I. 16) The essential difficulties of the problem are founded in the nature of its data and not in that of its quæsita. The central equation by which λ is determined, and the peculiar discussions connected therewith, are equally pertinent to every form which that problem can be made to assume, by varying the interpretation of the arbitrary elements in its original statement.

由此我们应得

$$\begin{aligned}&\text{Prob. } \{x_1(1-x_2)\dots(1-x_n)\dots+x_n(1-x_1)\dots(1-x_{n-1})\}\\&=\frac{s_1(1-s_2)\dots(1-s_n)\dots+s_n(1-s_1)\dots(1-s_{n-1})}{1-(1-s_1)\dots(1-s_n)}\\&=\frac{p_1\lambda(1-p_2\lambda)\dots(1-p_n\lambda)\dots+p_n\lambda(1-p_1\lambda)\dots(1-p_{n-1}\lambda)}{\lambda}\\&=\frac{p_1(1-\lambda)}{1-p_1\lambda}+\frac{p_2(1-\lambda)}{1-p_2\lambda}\dots+\frac{p_n(1-\lambda)}{1-p_n\lambda}\end{aligned}$$

这个解很好地说明了导论章(I.16)中所作的评论。问题的本质困难在于其数据的性质，而不在于其求问项的性质。用以确定 λ 的中心方程以及与之相关的特殊讨论，对于通过改变原陈述中任意元素的解释而使该问题所能采取的任何形式，都同样适用。

Chapter XX

PROBLEMS RELATING TO THE CONNEXION OF CAUSES AND EFFECTS.

1. So to apprehend in all particular instances the relation of cause and effect, as to connect the two extremes in thought according to the order in which they are connected in nature (for the *modus operandi* is, and must ever be, unknown to us), is the final object of science. This treatise has shown, that there is special reference to such an object in the constitution of the intellectual faculties. There is a sphere of thought which comprehends things only as coexistent parts of a universe; but there is also a sphere of thought (Chap. XI.) in which they are apprehended as links of an unbroken, and, to human appearance, an endless chain—as having their place in an order connecting them both with that which has gone before, and with that which shall follow after. In the contemplation of such a series, it is impossible not to feel the pre-eminence which is due, above all other relations, to the relation of cause and effect.

Here I propose to consider, in their abstract form, some problems in which the above relation is involved. There exists among such problems, as might be anticipated from the nature of the relation with which they are concerned, a wide diversity. From the probabilities of causes assigned *à priori*, or given by experience, and their respective probabilities of association with an effect contemplated, it may be required to determine the probability of that effect; and this either, 1st, absolutely, or 2ndly, under given conditions. To such an object some of the earlier of the following problems relate. On the other hand, it may be required to determine the probability of a particular cause, or of some particular connexion among a system of causes, from observed effects, and the known tendencies of the said causes, singly or in connexion, to the production of such effects. This class of questions will be considered in a subsequent portion of the chapter, and other forms of the general inquiry will also be noticed. I would remark, that although these examples are designed chiefly as illustrations of a *method*, no regard has been paid to the question of ease or convenience in the application of that method. On the contrary, they have been devised, with whatever success, as types of the class of problems which might be expected to arise from the study of the relation of cause and effect in the more complex of its actual and visible manifestations.

第二十章

关于原因与结果联系的问题。

1. 因此，要在所有特称实例中领会原因与结果的关系，以便在思想中按照它们在自然中相联系的次序把这两个极端联系起来(因为*modus operandi*是我们所不知道的，而且必定永远不知道)，这就是科学的最终目标。本论著已经表明，在理智的构造中，对这种目标有特殊的关联。有一个思想领域，它只把事物理解为宇宙中共存的部分；但也有一个思想领域(第xi章)，在其中事物被领会为一条不间断的、并且在人类看来是无尽的链条的环节--它们在一种秩序中各有其位置，这秩序把它们既与那先于它们的东西联系起来，也与那将随它们之后而来的东西联系起来。在沉思这样一个系列时，不可能不感到原因与结果的关系高于一切其他关系而应得的优越地位。

在此我打算以其抽象形式来考虑一些涉及上述关系的问题。从这些问题的性质可以预料，这类问题之间存在着很大的差异。从`*a priori*所指定的、或由经验给出的原因的概率，以及它们各自与所考虑的结果相结合的概率，可能要求确定该结果的概率；而这或者是1st，绝对地，或者是2ndly，在给定条件下。以下一些问题中的某些较早的问题就与这样的目标有关。另一方面，可能要求从观察到的结果，以及从上述原因单独或结合产生此类结果的已知倾向，来确定某一特定原因的概率，或一个原因体系中某种特定联系的概率。这一类问题将在本章的后一部分中考虑，一般探究的其他形式也将予以注意。我要指出，尽管这些例子主要意在说明一种方法，但并未顾及该方法应用中的难易或便利问题。相反，它们被设计出来--无论成功与否--作为一类问题的典型，这类问题可以预料会从对原因与结果关系在其实际而可见的较复杂表现中的研究中产生。

2. PROBLEM I.—The probabilities of two causes A_1 and A_2 are c_1 and c_2 respectively. The probability that if the cause A_1 present itself, an event E will accompany it (whether as a consequence of the cause A_1 or not) is p_1, and the probability that if the cause A_2 present itself, that event E will accompany it, whether as a consequence of it or not, is p_2. Moreover, the event E cannot appear in the absence of both the causes A_1 and A_2. [1] Required the probability of the event E.

The solution of what this problem becomes in the case in which the causes A_1, A_2 are mutually exclusive, is well known to be

$$\text{Prob. } E = c_1p_1 + c_2p_2;$$

and it expresses a particular case of a fundamental and very important principle in the received theory of probabilities. Here it is proposed to solve the problem free from the restriction above stated.

Let us represent

The cause A_1 by x.
The cause A_2 by y.
The effect E by z.

Then we have the following numerical data:

$$\begin{aligned} &\text{Prob. } x = c_1, && \text{Prob. } y = c_2, \\ &\text{Prob. } xz = c_1p_1, && \text{Prob. } yz = c_2p_2. \end{aligned} \tag{1}$$

Again, it is provided that if the causes A_1, A_2 are both absent, the effect E does not occur; whence we have the logical equation

$$(1-x)(1-y) = v(1-z).$$

Or, eliminating v,

$$z(1-x)(1-y) = 0. \tag{2}$$

Now assume,

$$xz = s, \quad yz = t. \tag{3}$$

[1]The mode in which such data as the above might be furnished by experience is easily conceivable. Opposite the window of the room in which I write is a field, liable to be overflowed from two causes, distinct, but capable of being combined, viz., floods from the upper sources of the River Lee, and tides from the ocean. Suppose that observations made on N separate occasions have yielded the following results: On A occasions the river was swollen by freshets, and on P of those occasions it was inundated, whether from this cause or not. On B occasions the river was swollen by the tide, and on Q of those occasions it was inundated, whether from this cause or not. Supposing, then, that the field cannot be inundated in the absence of *both* the causes above mentioned, let it be required to determine the total probability of its inundation.

Here the elements a, b, p, q of the general problem represent the ratios

$$\frac{A}{N}, \frac{P}{A}, \frac{B}{N}, \frac{Q}{B},$$

or rather the values to which those ratios approach, as the value of N is indefinitely increased.

2. 问题 I.- 两个原因 A [1]和 A [2]的概率分别为 c [1]和 c [2]。若原因 A [1]出现，则事件 E 将伴随它(无论是否作为原因 A [1]的结果)的概率为 p [1]，而若原因 A [2]出现，则该事件 E 将伴随它(无论是否作为它的结果)的概率为 p [2]。此外，事件 E 不可能在两个原因 A [1]和 A [2]均不出现的情况下出现。[1]求事件 E 的概率。

在原因 A [1]、A [2]相互排斥的情形下，这个问题所变成的问题的解是众所周知的，即

$$\text{Prob. } E = c_1p_1 + c_2p_2;$$

它表达了公认概率论中一个基本且非常重要的原理的一个特例。这里提出要在不受上述限制的情况下解决这个问题。

让我们表示

The cause A_1 by x.
The cause A_2 by y.
The effect E by z.

于是我们得到如下数值数据：

$$\begin{aligned} &\text{Prob. } x = c_1, && \text{Prob. } y = c_2, \\ &\text{Prob. } xz = c_1p_1, && \text{Prob. } yz = c_2p_2. \end{aligned} \tag{1}$$

再者，如果原因 A [1]、A [2]都不存在，则结果 E 不发生；由此我们得到逻辑方程

$$(1-x)(1-y) = v(1-z).$$

或者，消去 v，

$$z(1-x)(1-y) = 0. \tag{2}$$

现在假设，

$$xz = s, \quad yz = t. \tag{3}$$

1 上述这类数据可能由经验提供的方式是容易设想的。我写作的房间窗外是一片田地，它可能因两种原因而泛滥，这两种原因彼此不同，但能够结合，即来自利河(River Lee)上游水源的洪水和来自海洋的潮汐。假设在N个不同场合所作的观察已产生以下结果：在A个场合，河流因春汛而上涨，而在这些场合中的P个场合，它被淹没了，无论是否由于这个原因。在B个场合，河流因潮汐而上涨，而在这些场合中的Q个场合，它被淹没了，无论是否由于这个原因。那么，假设在上述两种原因都不存在时，这片田地不可能被淹没，要求确定其被淹没的总概率。

这里，一般问题的元素 a、b、p、q 表示比值

$$\frac{A}{N}, \frac{P}{A}, \frac{B}{N}, \frac{Q}{B},$$

或者更确切地说，表示当 N 的值无限增大时那些比值所趋近的值。

Then, reducing these equations (VIII. 7), and connecting the result with (2),

$$xz(1-s)+s(1-xz)+yz(1-t)+t(1-yz)+z(1-x)(1-y)=0. \qquad (4)$$

From this equation, z must be determined as a developed logical function of x, y, s, and t, and its probability thence deduced by means of the data,

$$\text{Prob. } x=c_1, \quad \text{Prob. } y=c_2, \quad \text{Prob. } s=c_1p_1, \quad \text{Prob. } t=c_2p_2. \qquad (5)$$

Now developing (4) with respect to z, and putting $\bar{x}$ for $1-x$, $\bar{y}$ for $1-y$, and[2] so on, we have

$$\begin{aligned}
&(x\bar{s}+s\bar{x}+y\bar{t}+t\bar{y}+\bar{x}\bar{y})z+(s+t)\bar{z}=0,\\
&\therefore z+\frac{s+t}{s+t-x\bar{s}-s\bar{x}-y\bar{t}-t\bar{y}-\bar{x}\bar{y}}\\
&=stxy+\frac{1}{0}stx\bar{y}+\frac{1}{0}st\bar{x}y+\frac{1}{0}st\bar{x}\bar{y}\\
&+\frac{1}{0}s\bar{t}xy+s\bar{t}x\bar{y}+\frac{1}{0}s\bar{t}\bar{x}y+\frac{1}{0}s\bar{t}\bar{x}\bar{y}\\
&+\frac{1}{0}\bar{s}txy+\frac{1}{0}\bar{s}tx\bar{y}+\bar{s}t\bar{x}y+\frac{1}{0}\bar{s}t\bar{x}\bar{y}\\
&+0\bar{s}\bar{t}xy+0\bar{s}\bar{t}x\bar{y}+0\bar{s}\bar{t}\bar{x}y+0\bar{s}\bar{t}\bar{x}\bar{y}. \qquad (6)
\end{aligned}$$

From this result we find (XVII. 17),

$$\begin{aligned}
V&=stxy+s\bar{t}x\bar{y}+\bar{s}t\bar{x}y+\bar{s}\bar{t}xy+\bar{s}\bar{t}\bar{x}\bar{y}\\
&=stxy+s\bar{t}x\bar{y}+\bar{s}t\bar{x}y+\bar{s}\bar{t}.
\end{aligned}$$

Whence, passing from Logic to Algebra, we have the following system of equations, u standing for the probability sought:

$$\begin{aligned}
\frac{stxy+s\bar{t}x\bar{y}+\bar{s}\bar{t}x}{c_1}&=\frac{stxy+\bar{s}t\bar{x}y+\bar{s}\bar{t}y}{c_2}\\
&=\frac{stxy+s\bar{t}x\bar{y}}{c_1p_1}=\frac{stxy+\bar{s}t\bar{x}y}{c_2p_2} \qquad (7)\\
=\frac{stxy+s\bar{t}x\bar{y}+\bar{s}t\bar{x}y}{u}&=\frac{stxy+s\bar{t}x\bar{y}+\bar{s}t\bar{x}y+\bar{s}\bar{t}}{1}=V,
\end{aligned}$$

from which we must eliminate s, t, x, y, and V.

Now if we have any series of equal fractions, as

$$\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}\ldots=\lambda,$$

we know that

$$\frac{la+mb+nc}{la'+mb'+nc'}=\lambda.$$

[2]The original text was "$\bar{y}$ for $1=y$", corrected here by Distributed Proofreaders.

然后，化简这些方程(VIII.7)，并将结果与(2)联系起来，xz(1 – s) + s(1 – xz) + yz(1 – t) + t(1 – yz) + z(1 – x)(1 – y) = 0 .(4) 从这个方程中，z必须被确定为x、y、s和t的展开逻辑函数，其概率由此借助数据推得，

$$\text{Prob. } x=c_1, \quad \text{Prob. } y=c_2, \quad \text{Prob. } s=c_1p_1, \quad \text{Prob. } t=c_2p_2. \qquad (5)$$

Now developing (4) with respect to z, and putting $\bar{x}$ for $1-x$, $\bar{y}$ for $1-y$, and[2] so on, we have

$$\begin{aligned}
&(x\bar{s}+s\bar{x}+y\bar{t}+t\bar{y}+\bar{x}\bar{y})z+(s+t)\bar{z}=0,\\
&\therefore z+\frac{s+t}{s+t-x\bar{s}-s\bar{x}-y\bar{t}-t\bar{y}-\bar{x}\bar{y}}\\
&=stxy+\frac{1}{0}stx\bar{y}+\frac{1}{0}st\bar{x}y+\frac{1}{0}st\bar{x}\bar{y}\\
&+\frac{1}{0}s\bar{t}xy+s\bar{t}x\bar{y}+\frac{1}{0}s\bar{t}\bar{x}y+\frac{1}{0}s\bar{t}\bar{x}\bar{y}\\
&+\frac{1}{0}\bar{s}txy+\frac{1}{0}\bar{s}tx\bar{y}+\bar{s}t\bar{x}y+\frac{1}{0}\bar{s}t\bar{x}\bar{y}\\
&+0\bar{s}\bar{t}xy+0\bar{s}\bar{t}x\bar{y}+0\bar{s}\bar{t}\bar{x}y+0\bar{s}\bar{t}\bar{x}\bar{y}. \qquad (6)
\end{aligned}$$

由这一结果我们得到(XVII.17),

$$\begin{aligned}
V&=stxy+s\bar{t}x\bar{y}+\bar{s}t\bar{x}y+\bar{s}\bar{t}xy+\bar{s}\bar{t}\bar{x}\bar{y}\\
&=stxy+s\bar{t}x\bar{y}+\bar{s}t\bar{x}y+\bar{s}\bar{t}.
\end{aligned}$$

于是，从逻辑过渡到代数，我们得到以下方程组，u 代表所求的概率：

$$\begin{aligned}
\frac{stxy+s\bar{t}x\bar{y}+\bar{s}\bar{t}x}{c_1}&=\frac{stxy+\bar{s}t\bar{x}y+\bar{s}\bar{t}y}{c_2}\\
&=\frac{stxy+s\bar{t}x\bar{y}}{c_1p_1}=\frac{stxy+\bar{s}t\bar{x}y}{c_2p_2} \qquad (7)\\
=\frac{stxy+s\bar{t}x\bar{y}+\bar{s}t\bar{x}y}{u}&=\frac{stxy+s\bar{t}x\bar{y}+\bar{s}t\bar{x}y+\bar{s}\bar{t}}{1}=V,
\end{aligned}$$

由此我们必须消去 s、t、x、y 和 V。

现在，如果我们有一系列相等的分数，如

$$\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}\ldots=\lambda,$$

我们知道

$$\frac{la+mb+nc}{la'+mb'+nc'}=\lambda.$$

2 原文为“ y for 1 = y”，此处由 Distributed Proofreaders 更正。

And thus from the above system of equations we may deduce

$$\frac{\bar{s}t\bar{x}y}{u-c_1p_1}=\frac{s\bar{t}x\bar{y}}{u-c_2p_2}=\frac{\bar{s}\bar{t}}{1-u}=V;$$

whence we have, on equating the product of the three first members to the cube of the last,

$$\frac{s\bar{s}^2t\bar{t}^2x\bar{x}y\bar{y}}{(u-c_1p_1)(u-c_2p_2)(1-u)}=V^3. \tag{8}$$

Again, from the system (7) we have

$$\frac{\bar{s}\bar{t}\bar{x}}{1-u-c_1+c_1p_1}=\frac{\bar{s}\bar{t}\bar{y}}{1-u-c_2+c_2p_2}=\frac{stxy}{c_1p_1+c_2p_2-u}=V,$$

whence proceeding as before

$$\frac{s\bar{s}^2t\bar{t}^2x\bar{x}y\bar{y}}{(1-c_1+c_1p_1-u)(1-c_2+c_2p_2-u)(c_1p_1+c_2p_2-u)}=V^3. \tag{9}$$

Equating the values of V^3 in (8) and (9), we have

$$\begin{aligned}&(u-c_1p_1)(u-c_2p_2)(1-u)\\&=\{1-c_1(1-p_1)-u\}\{1-c_2(1-p_2)-u\}(c_1p_1+c_2p_2-u),\end{aligned}$$

which may be more conveniently written in the form

$$\frac{(u-c_1p_1)(u-c_2p_2)}{c_1p_1+c_2p_2-u}=\frac{\{1-c_1(1-p_1)-u\}\{1-c_2(1-p_2)-u\}}{1-u}. \tag{10}$$

From this equation the value of u may be found. It remains only to determine which of the roots must be taken for this purpose.

3. It has been shown (XIX. 12) that the quantity u, in order that it may represent the probability required in the above case, must exceed each of the quantities c_1p_1, c_2p_2, and fall short of each of the quantities $1-c_1(1-p_1)$, $1-c_2(1-p_2)$ and $c_1p_1+c_2p_2$; the condition among the constants, moreover, being that the three last quantities must individually exceed each of the two former ones. Now I shall show that these conditions being satisfied, the final equation (10) has but one root which falls within the limits assigned. That root will therefore be the required value of u.

Let us represent the lower limits c_1p_1, c_2p_2, by a, b respectively, and the upper limits $1-c_1(1-p_1)$, $1-c_2(1-p_2)$ and $c_1p_1+c_2p_2$ by a', b', c' respectively. Then the general equation may be expressed in the form

$$\begin{aligned}&(u-a)(u-b)(1-u)-(a'-u)(b'-u)(c'-u)=0, \qquad (11)\\&\text{or}(1-a'-b')u^2-\{ab-a'b'+(1-a'-b')c'\}u+ab-a'b'c'=0.\end{aligned}$$

Representing the first member of the above equation by V, we have

$$\frac{d^2V}{du^2}=2(1-a'-b'). \tag{12}$$

因此，由上述方程组我们可以推导出

$$\frac{\bar{s}t\bar{x}y}{u-c_1p_1}=\frac{s\bar{t}x\bar{y}}{u-c_2p_2}=\frac{\bar{s}\bar{t}}{1-u}=V;$$

whence we have, on equating the product of the three first members to the cube of the last,

$$\frac{s\bar{s}^2t\bar{t}^2x\bar{x}y\bar{y}}{(u-c_1p_1)(u-c_2p_2)(1-u)}=V^3. \tag{8}$$

同样，由系统(7)我们得到

$$\frac{\bar{s}\bar{t}\bar{x}}{1-u-c_1+c_1p_1}=\frac{\bar{s}\bar{t}\bar{y}}{1-u-c_2+c_2p_2}=\frac{stxy}{c_1p_1+c_2p_2-u}=V,$$

whence proceeding as before

$$\frac{s\bar{s}^2t\bar{t}^2x\bar{x}y\bar{y}}{(1-c_1+c_1p_1-u)(1-c_2+c_2p_2-u)(c_1p_1+c_2p_2-u)}=V^3. \tag{9}$$

令(8)与(9)中V 3的值相等，我们得到

$$\begin{aligned}&(u-c_1p_1)(u-c_2p_2)(1-u)\\&=\{1-c_1(1-p_1)-u\}\{1-c_2(1-p_2)-u\}(c_1p_1+c_2p_2-u),\end{aligned}$$

which may be more conveniently written in the form

$$\frac{(u-c_1p_1)(u-c_2p_2)}{c_1p_1+c_2p_2-u}=\frac{\{1-c_1(1-p_1)-u\}\{1-c_2(1-p_2)-u\}}{1-u}. \tag{10}$$

From this equation the value of u may be found. It remains only to determine which of the roots must be taken for this purpose.

3. 已经证明(XIX.12)，量u为了表示上述情形中所要求的概率，必须超过量c 1 p 1、c 2 p 2中的每一个，并小于量1 – c 1(1 – p 1)、1 – c 2(1 – p 2)和c 1 p 1 + c 2 p 2中的每一个；此外，常数之间的条件是，后三个量必须各自超过前两个量中的每一个。现在我将证明，这些条件得到满足时，最终方程(10)只有一个根落在所指定的界限之内。因此该根就是u的所需值。

让我们分别用a、b表示下界c 1 p 1、c 2 p 2，并分别用a ′、b ′、c ′表示上界1 – c 1(1 – p 1)、1 – c 2(1 – p 2)和c 1 p 1 + c 2 p 2。于是一般方程可以表示为如下形式

$$\begin{aligned}&(u-a)(u-b)(1-u)-(a'-u)(b'-u)(c'-u)=0, \qquad (11)\\&\text{or}(1-a'-b')u^2-\{ab-a'b'+(1-a'-b')c'\}u+ab-a'b'c'=0.\end{aligned}$$

将上述方程的第一项用V表示，我们有

$$\frac{d^2V}{du^2}=2(1-a'-b'). \tag{12}$$

Now let us suppose a the highest of the lower limits of u, a' the lowest of its higher limits, and trace the progress of the values of V between the limits $u = a$ and $u = a'$.

When $u = a$, we see from the form of the first member of (11) that V is negative, and when $u = a'$ we see that V is positive. Between those limits V varies continuously without becoming infinite, and $\frac{d^2V}{du^2}$ is always of the same sign.

Hence if u represent the abscissa V the ordinate of a plane curve, it is evident that the curve will pass from a point below the axis of u corresponding to $u = a$, to a point above the axis of u corresponding to $u = a'$, the curve remaining continuous, and having its concavity or convexity always turned in the same direction. A little attention will show that, under these circumstances, it must cut the axis of u once, and only once.

Hence between the limits $u = a$, $u = a'$, there exists one value of u, and only one, which satisfies the equation (11). It will further appear, if in thought the curve be traced, that the other value of u will be less than a when the quantity $1 - a' - b'$ is positive and greater than any one of the quantities a', b', c' when $1 - a' - b'$ is negative. It hence follows that in the solution of (11) the positive sign of the radical must be taken. We thus find

$$u = \frac{ab - a'b' + (1 - a' - b')c' + \sqrt{Q}}{2(1 - a' - b')}, \tag{13}$$

where $Q = \{ab - a'b' + (1 - a' - b')c'\}^2 - 4(1 - a' - b')(ab - a'b'c')$.

4. The results of this investigation may to some extent be verified. Thus, it is evident that the probability of the event E must in general exceed the probability of the concurrence of the event E and the cause A_1 or A_2. Hence we must have, as the solution indicates,

$$u > c_1p_1, \quad u > c_2p_2.$$

Again, it is clear that the probability of the effect E must in general be less than it would be if the causes A_1, A_2 were mutually exclusive. Hence

$$u \overline{=}< c_1p_1 + c_2p_2.$$

Lastly, since the probability of the failure of the effect E concurring with the presence of the cause A_1 must, in general, be less than the absolute probability of the failure of E, we have

$$c_1(1 - p_1) \overline{=}< 1 - u,$$
$$\therefore u \overline{=}< 1 - c_1(1 - p_1).$$

Similarly,

$$u \overline{=}< 1 - c_2(1 - p_2).$$

And thus the conditions by which the general solution was limited are confirmed.

现在让我们假设 a 是 u 的下限中的最高者，a′是其上限中的最低者，并追踪 V 的值在 u = a 与 u = a′之间的变化过程。

当 u = a 时，我们从(11)的第一项的形式看出 V 为负，而当 u = a′时，我们看到 V 为正。在这些界限之间，V 连续变化而不变为无穷，并且$\frac{d^2V}{du^2}$ 2isalwaysofthesame符号。

因此，如果u表示横坐标V，纵坐标表示一条平面曲线，那么显然该曲线将从对应于u = a的u轴下方的一点，过渡到对应于u = a′的u轴上方的一点，曲线保持连续，且其凹性或凸性始终朝向同一方向。稍加注意即可看出，在这些情况下，它必定与u轴相交一次，且仅相交一次。

因此，在界限u = a、u = a′之间，存在u的一个值，且仅有一个值，满足方程(11)。进一步可以看出，如果在思想中描出该曲线，那么当量1 – a′ – b′为正且大于量a′、b′、c′中的任何一个时，u的另一个值将小于a，当

1 – a′ – b′为负。由此可知，在(11)的解中，必须取根号的正号。我们于是得到

$$u = \frac{ab - a'b' + (1 - a' - b')c' + \sqrt{Q}}{2(1 - a' - b')}, \tag{13}$$

where $Q = \{ab - a'b' + (1 - a' - b')c'\}^2 - 4(1 - a' - b')(ab - a'b'c')$.

4. 这一研究的结果在某种程度上可以得到验证。因此，显然事件 E 的概率一般必定超过事件 E 与原因 A 1或 A 2同时出现的概率。因此我们必须有，正如解所表明的，

$$u > c_1p_1, \quad u > c_2p_2.$$

再者，显然结果 E 的概率一般必定小于当原因 A 1 , A 2互斥时它所会具有的概率。因此

$$u \overline{=}< c_1p_1 + c_2p_2.$$

最后，由于结果 E 与原因 A 1同时出现之失败的概率一般必定小于 E 失败之绝对概率，我们有

$$c_1(1 - p_1) \overline{=}< 1 - u,$$
$$\therefore u \overline{=}< 1 - c_1(1 - p_1).$$

Similarly,

$$u \overline{=}< 1 - c_2(1 - p_2).$$

And thus the conditions by which the general solution was limited are confirmed.

Again, let $p_1 = 1$, $p_2 = 1$. This is to suppose that when either of the causes A_1, A_2 is present, the event E will occur. We have then $a = c_1$, $b = c_2$, $a' = 1$, $b' = 1$, $c' = c_1 + c_2$, and substituting in (13) we get

$$u = \frac{c_1c_2 - c_1 - c_2 - 1 + \sqrt{\{(c_1c_2 - c_1 - c_2 - 1)^2 + 4(c_1c_2 - c_1 - c_2)\}}}{-2}$$
$$= c_1 + c_2 - c_1c_2 \text{ on reduction}$$
$$= 1 - (1 - c_1)(1 - c_2).$$

Now this is the known expression for the probability that one cause at least will be present, which, under the circumstances, is evidently the probability of the event E.

Finally, let it be supposed that c_1 and c_2 are very small, so that their product may be neglected; then the expression for u reduces to $c_1p_1 + c_2p_2$. Now the smaller the probability of each cause, the smaller, in a much higher degree, is the probability of a conjunction of causes. Ultimately, therefore, such reduction continuing, the probability of the event E becomes the same as if the causes were mutually exclusive.

I have dwelt at greater length upon this solution, because it serves in some respect as a model for those which follow, some of which, being of a more complex character, might, without such preparation, appear difficult.

5. PROBLEM II.—In place of the supposition adopted in the previous problem, that the event E cannot happen when both the causes A_1, A_2 are absent, let it be assumed that the causes A_1, A_2 cannot both be absent, and let the other circumstances remain as before. Required, then, the probability of the event E.

Here, in place of the equation (2) of the previous solution, we have the equation

$$(1 - x)(1 - y) = 0.$$

The developed logical expression of z is found to be

$$z = stxy + \frac{1}{0}stx\bar{y} + \frac{1}{0}st\bar{x}y + \frac{1}{0}st\bar{x}\bar{y}$$
$$+ \frac{1}{0}s\bar{t}xy + s\bar{t}x\bar{y} + \frac{1}{0}s\bar{t}\bar{x}y + \frac{1}{0}s\bar{t}\bar{x}\bar{y}$$
$$+ \frac{1}{0}\bar{s}txy + \frac{1}{0}\bar{s}tx\bar{y} + \bar{s}t\bar{x}y + \frac{1}{0}\bar{s}t\bar{x}\bar{y}$$
$$+ 0\bar{s}\bar{t}xy + 0\bar{s}\bar{t}x\bar{y} + 0\bar{s}\bar{t}\bar{x}y + \frac{1}{0}\bar{s}\bar{t}\bar{x}\bar{y};$$

and the final solution is

$$\text{Prob. } E = u;$$

the quantity u being determined by the solution of the equation

$$\frac{(u - a)(u - b)}{a + b - u} = \frac{(a' - u)(b' - u)}{u - a' - b' + 1}, \qquad (1)$$

再者，令 p 1 = 1,p 2 = 1。这是假设当原因 A 1、A 2中任一者出现时，事件 E 将会发生。我们于是有 a = c 1,b = c 2,a ′ = 1,b ′ = 1,c ′ = c 1 + c 2，代入(13)我们得到

$$u = \frac{c_1c_2 - c_1 - c_2 - 1 + \sqrt{\{(c_1c_2 - c_1 - c_2 - 1)^2 + 4(c_1c_2 - c_1 - c_2)\}}}{-2}$$
$$= c_1 + c_2 - c_1c_2 \text{ on reduction}$$
$$= 1 - (1 - c_1)(1 - c_2).$$

现在这就是至少一个原因将会出现的概率的已知表达式，在此情形下，这显然就是事件 E 的概率。

最后，假设 c 1和 c 2非常小，以致它们的乘积可以忽略；那么 u 的表达式简化为 c 1 p 1 + c 2 p 2。现在，每个原因的概率越小，原因合取的概率就越小，且程度要高得多。因此，最终随着这种简化继续下去，事件 E 的概率变得与原因互斥时相同。

我之所以对这一个解详加论述，是因为它在某些方面可作为随后诸解的范本，其中有些解性质更为复杂，若无这样的准备，可能会显得困难。

5. 问题 II。--代替前一问题中所采用的假设，即当原因 A 1、A 2两者皆不出现时事件 E 不能发生，今假设原因 A 1、A 2不能两者皆不出现，并令其他情形如前不变。于是，求事件 E 的概率。

Here, in place of the equation (2) of the previous solution, we have the equation

$$(1 - x)(1 - y) = 0.$$

The developed logical expression of z is found to be

$$z = stxy + \frac{1}{0}stx\bar{y} + \frac{1}{0}st\bar{x}y + \frac{1}{0}st\bar{x}\bar{y}$$
$$+ \frac{1}{0}s\bar{t}xy + s\bar{t}x\bar{y} + \frac{1}{0}s\bar{t}\bar{x}y + \frac{1}{0}s\bar{t}\bar{x}\bar{y}$$
$$+ \frac{1}{0}\bar{s}txy + \frac{1}{0}\bar{s}tx\bar{y} + \bar{s}t\bar{x}y + \frac{1}{0}\bar{s}t\bar{x}\bar{y}$$
$$+ 0\bar{s}\bar{t}xy + 0\bar{s}\bar{t}x\bar{y} + 0\bar{s}\bar{t}\bar{x}y + \frac{1}{0}\bar{s}\bar{t}\bar{x}\bar{y};$$

而最终的解决方案是

$$\text{Prob. } E = u;$$

量 u 由方程的解确定

$$\frac{(u - a)(u - b)}{a + b - u} = \frac{(a' - u)(b' - u)}{u - a' - b' + 1}, \qquad (1)$$

wherein $a = c_1p_1$, $b = c_2p_2$, $a' = 1 - c_1(1 - p_1)$, $b' = 1 - c_2(1 - p_2)$.

The conditions of limitation are the following:—That value of u must be chosen which exceeds each of the three quantities

$$a, b, \text{ and } a' + b' - 1,$$

and which at the same time falls short of each of the three quantities

$$a', b', \text{ and } a + b.$$

Exactly as in the solution of the previous problem, it may be shown that the quadratic equation (1) will have one root, and only one root, satisfying these conditions. The conditions themselves were deduced by the same rule as before, excepting that the minor limit $a' + b' - 1$ was found by seeking the major limit of $1 - z$.

It may be added that the constants in the data, beside satisfying the conditions implied above, viz., that the quantities a', b', and $a + b$, must individually exceed a, b, and $a' + b' - 1$, must also satisfy the condition $c_1 + c_2 \overline{=}> 1$. This also appears from the application of the rule.

6. PROBLEM III.—The probabilities of two events A and B are a and b respectively, the probability that if the event A take place an event E will accompany it is p, and the probability that if the event B take place, the same event E will accompany it is q. Required the probability that if the event A take place the event B will take place, or *vice versâ*, the probability that if B take place, A will take place.

Let us represent the event A by x, the event B by y, and the event E by z. Then the data are—

$$\begin{aligned} &\text{Prob. } x = a, && \text{Prob. } y = b. \\ &\text{Prob. } xz = ap, && \text{Prob. } yz = bq. \end{aligned}$$

Whence it is required to find

$$\frac{\text{Prob. } xy}{\text{Prob. } x} \text{ or } \frac{\text{Prob. } xy}{\text{Prob. } y}.$$

Let

$$xy = s, \quad yz = t, \quad xy = w.$$

Eliminating z, we have, on reduction,

$$\begin{aligned} s\bar{x} + t\bar{y} + s\bar{y}\bar{t} + xt\bar{s} + xy\bar{w} + (1 - xy)w = 0, \\ \therefore w = \frac{s\bar{x} + t\bar{y} + s\bar{y}\bar{t} + xt\bar{s} + xy}{2xy - 1} \end{aligned} \quad \begin{aligned} &= xyst + \frac{1}{0}x\bar{y}st + \frac{1}{0}\bar{x}yst + \frac{1}{0}\bar{x}\bar{y}st \\ &+ \frac{1}{0}xys\bar{t} + 0x\bar{y}s\bar{t} + \frac{1}{0}\bar{x}ys\bar{t} + \frac{1}{0}\bar{x}\bar{y}s\bar{t} \\ &+ \frac{1}{0}xy\bar{s}t + \frac{1}{0}x\bar{y}\bar{s}t + 0\bar{x}y\bar{s}t + \frac{1}{0}\bar{x}\bar{y}\bar{s}t \\ &+ xy\bar{s}\bar{t} + 0x\bar{y}\bar{s}\bar{t} + 0\bar{x}y\bar{s}\bar{t} + 0\bar{x}\bar{y}\bar{s}\bar{t}. \end{aligned} \quad (1)$$

wherein $a = c_1p_1$, $b = c_2p_2$, $a' = 1 - c_1(1 - p_1)$, $b' = 1 - c_2(1 - p_2)$.

The conditions of limitation are the following:—That value of u must be chosen which exceeds each of the three quantities

$$a, b, \text{ and } a' + b' - 1,$$

而同时又少于这三个量中的每一个

$$a', b', \text{ and } a + b.$$

完全如同前一问题的解中那样，可以证明二次方程(1)将有一个根，且只有一个根，满足这些条件。这些条件本身是按与以前相同的规则推导出来的，只是下限 a′ + b′ – 1 是通过寻求 1 – z 的上限而得到的。

还可以补充说，数据中的常数除了满足上面隐含的条件，即量 a′、b′和 a + b 必须分别超过 a、b 和 a′ + b′ – 1 之外，还必须满足条件= c 1 + c 2 > 1。这从规则的运用中也看得出来。

6. 问题 III.- 两个事件 A 和 B 的概率分别为 a 和 b，若事件 A 发生则事件 E 将伴随发生的概率为 p，若事件 B 发生则同一事件 E 将伴随发生的概率为 q。求若事件 A 发生则事件 B 将发生的概率，或反之亦然，即若 B 发生则 A 将发生的概率。

让我们用 x 表示事件 A，用 y 表示事件 B，用 z 表示事件 E。于是数据为--

$$\begin{aligned} &\text{Prob. } x = a, && \text{Prob. } y = b. \\ &\text{Prob. } xz = ap, && \text{Prob. } yz = bq. \end{aligned}$$

由此需要求出

$$\frac{\text{Prob. } xy}{\text{Prob. } x} \text{ or } \frac{\text{Prob. } xy}{\text{Prob. } y}.$$

让et

$$xy = s, \quad yz = t, \quad xy = w.$$

消去 z，化简后得，

$$\begin{aligned} s\bar{x} + t\bar{y} + s\bar{y}\bar{t} + xt\bar{s} + xy\bar{w} + (1 - xy)w = 0, \\ \therefore w = \frac{s\bar{x} + t\bar{y} + s\bar{y}\bar{t} + xt\bar{s} + xy}{2xy - 1} \end{aligned} \quad \begin{aligned} &= xyst + \frac{1}{0}x\bar{y}st + \frac{1}{0}\bar{x}yst + \frac{1}{0}\bar{x}\bar{y}st \\ &+ \frac{1}{0}xys\bar{t} + 0x\bar{y}s\bar{t} + \frac{1}{0}\bar{x}ys\bar{t} + \frac{1}{0}\bar{x}\bar{y}s\bar{t} \\ &+ \frac{1}{0}xy\bar{s}t + \frac{1}{0}x\bar{y}\bar{s}t + 0\bar{x}y\bar{s}t + \frac{1}{0}\bar{x}\bar{y}\bar{s}t \\ &+ xy\bar{s}\bar{t} + 0x\bar{y}\bar{s}\bar{t} + 0\bar{x}y\bar{s}\bar{t} + 0\bar{x}\bar{y}\bar{s}\bar{t}. \end{aligned} \quad (1)$$

Hence, passing from Logic to Algebra,

$$\text{Prob.}\, xy = \frac{xyst + xy\bar{s}\bar{t}}{V},$$

x, y, s, and t being determined by the system of equations

$$\frac{xyst + x\bar{y}s\bar{t} + xy\bar{s}\bar{t} + x\bar{y}\bar{s}\bar{t}}{a} = \frac{xyst + \bar{x}y\bar{s}t + xy\bar{s}\bar{t} + \bar{x}y\bar{s}\bar{t}}{b}$$
$$= \frac{xyst + x\bar{y}s\bar{t}}{ap} = \frac{xyst + \bar{x}y\bar{s}t}{bq}$$
$$= xyst + x\bar{y}s\bar{t} + \bar{x}y\bar{s}t + xy\bar{s}\bar{t} + x\bar{y}\bar{s}\bar{t} + \bar{x}y\bar{s}\bar{t} + \bar{x}\bar{y}\bar{s}\bar{t} = V.$$

To reduce the above system to a more convenient form, let every member be divided by $\bar{x}\bar{y}\bar{s}\bar{t}$, and in the result let

$$\frac{xs}{\bar{x}\bar{s}} = m, \quad \frac{yt}{\bar{y}\bar{t}} = m', \quad \frac{x}{\bar{x}} = n, \quad \frac{y}{\bar{y}} = n'.$$

We then find

$$\frac{mm' + m + nn' + n}{a} = \frac{mm' + m' + nn' + n'}{b}$$
$$= \frac{mm' + m}{ap} = \frac{mm' + m'}{bq}$$
$$= mm' + m + m' + nn' + n + n' + 1.$$

Also,

$$\text{Prob.}\, xy = \frac{mm' + nn'}{mm' + m + m' + nn' + n + n' + 1}.$$

These equations may be reduced to the form

$$\frac{mm' + m}{ap} = \frac{mm' + m'}{bq} = \frac{nn' + n}{a(1-p)} = \frac{nn' + n'}{b(1-q)}$$
$$= (m+1)(m'+1) + (n+1)(n'+1) - 1.$$
$$\text{Prob.}\, xy = \frac{mm' + nn'}{(m+1)(m'+1) + (n+1)(n'+1) - 1}.$$

Now assume

$$(m+1)(m'+1) = \frac{\mu}{\nu + \mu - 1}, \quad (n+1)(n'+1) = \frac{\nu}{\nu + \mu - 1}. \tag{2}$$

Then since $mm' + m = \dfrac{m(m'+1)(m+1)}{m+1} = \dfrac{m\mu}{(m+1)(\nu+\mu-1)}$, and so on for the other numerators of the system, we find, on substituting and multiplying each member of the system by $\nu + \mu - 1$, the following results:

$$\frac{m\mu}{(m+1)ap} = \frac{m'\mu}{(m'+1)bq} = \frac{n\nu}{(n+1)a(1-p)} = \frac{n'\nu}{(n'+1)b(1-q)} = 1.$$
$$\text{Prob.}\, xy = (mm' + nn')(\nu + \mu - 1). \tag{3}$$

因此，从逻辑过渡到代数，

$$\text{Prob.}\, xy = \frac{xyst + xy\bar{s}\bar{t}}{V},$$

x、y、s 和 t 由方程组确定

$$\frac{xyst + x\bar{y}s\bar{t} + xy\bar{s}\bar{t} + x\bar{y}\bar{s}\bar{t}}{a} = \frac{xyst + \bar{x}y\bar{s}t + xy\bar{s}\bar{t} + \bar{x}y\bar{s}\bar{t}}{b}$$
$$= \frac{xyst + x\bar{y}s\bar{t}}{ap} = \frac{xyst + \bar{x}y\bar{s}t}{bq}$$
$$= xyst + x\bar{y}s\bar{t} + \bar{x}y\bar{s}t + xy\bar{s}\bar{t} + x\bar{y}\bar{s}\bar{t} + \bar{x}y\bar{s}\bar{t} + \bar{x}\bar{y}\bar{s}\bar{t} = V.$$

为将上述系统化为更便利的形式，令每一项都除以 $\bar{x}\bar{y}\bar{s}\bar{t}$，并在结果中令

$$\frac{xs}{\bar{x}\bar{s}} = m, \quad \frac{yt}{\bar{y}\bar{t}} = m', \quad \frac{x}{\bar{x}} \quad -$$

We then find

$$\frac{mm' + m + nn' + n}{a} = \frac{mm' + m' + nn' + n'}{b}$$
$$= \frac{mm' + m}{ap} = \frac{mm' + m'}{bq}$$
$$= mm' + m + m' + nn' + n + n' + 1.$$

Also,

$$\text{Prob.}\, xy = \frac{mm' + nn'}{mm' + m + m' + nn' + n + n' + 1}.$$

These equations may be reduced to the form

$$\frac{mm' + m}{ap} = \frac{mm' + m'}{bq} = \frac{nn' + n}{a(1-p)} = \frac{nn' + n'}{b(1-q)}$$
$$= (m+1)(m'+1) + (n+1)(n'+1) - 1.$$
$$\text{Prob.}\, xy = \frac{mm' + nn'}{(m+1)(m'+1) + (n+1)(n'+1) - 1}.$$

Now assume

$$(m+1)(m'+1) = \frac{\mu}{\nu + \mu - 1}, \quad (n+1)(n'+1) = \frac{\nu}{\nu + \mu - 1}. \tag{2}$$

Then since $mm' + m = \dfrac{m(m'+1)(m+1)}{m+1} = \dfrac{m\mu}{(m+1)(\nu+\mu-1)}$, and so on for the other numerators of the system, we find, on substituting and multiplying each member of the system by $\nu + \mu - 1$, the following results:

$$\frac{m\mu}{(m+1)ap} = \frac{m'\mu}{(m'+1)bq} = \frac{n\nu}{(n+1)a(1-p)} = \frac{n'\nu}{(n'+1)b(1-q)} = 1.$$
$$\text{Prob.}\, xy = (mm' + nn')(\nu + \mu - 1). \tag{3}$$

From the above system we have

$$\frac{m}{m+1} = \frac{ap}{\mu}, \text{ whence } m = \frac{ap}{\mu - ap}.$$

Similarly

$$m' = \frac{bq}{\mu - bq}, \quad n = \frac{a(1-p)}{\nu - a(1-p)}, \quad n' = \frac{b(1-q)}{\nu - b(1-q)}.$$

Hence

$$m+1 = \frac{\mu}{\mu - ap}, \quad n+1 = \frac{\nu}{\nu - a(1-p), \text{ \&c.}}$$

Substitute these values in (2) reduced to the form

$$\nu + \mu - 1 = \frac{\mu}{(m+1)(m'+1)} = \frac{\nu}{(n+1)(n'+1)},$$

and we have

$$\nu + \mu - 1 = \frac{(\mu - ap)(\mu - bq)}{\mu} = \frac{\{\nu - a(1-p)\}\,\{\nu - b(1-q)\}}{\nu}, \qquad (4)$$

Substitute also for m, m', &c. their values in (3), and we have

$$\text{Prob. } xy$$
$$= \left[\frac{abpq}{(\mu - ap)(\mu - bq)} + \frac{ab(1-p)(1-q)}{\{\nu - a(1-p)\}\{\nu - b(1-q)\}}\right](\nu + \mu - 1)$$
$$= \frac{abpq}{\mu} + \frac{ab(1-p)(1-q)}{\nu} \text{ by (4).}$$

Now the first equation of the system (4) gives

$$\nu + \mu - 1 = \mu - ap - bq + \frac{apbq}{\mu}, \qquad (5)$$
$$\therefore \frac{apbq}{\mu} = \nu - 1 + ap + bq.$$

Similarly,

$$\frac{ab(1-p)(1-q)}{\nu} = \mu - 1 + a(1-p) + b(1-q).$$

Adding these equations together, and observing that the first member of the result becomes identical with the expression just found for Prob. xy, we have

$$\text{Prob. } xy = \nu + \mu + a + b - 2.$$

Let us represent Prob. xy by u, and let $a + b - 2 = m$, then

$$\mu + \nu = u - m. \qquad (6)$$

由上述系统我们得到

$$\frac{m}{m+1} = \frac{ap}{\mu}, \text{ whence } m = \frac{ap}{\mu - ap}.$$

类似地

$$m' = \frac{bq}{\mu - bq}, \quad n = \frac{a(1-p)}{\nu - a(1-p)}, \quad n' = \frac{b(1-q)}{\nu - b(1-q)}.$$

Hence

$$m+1 = \frac{\mu}{\mu - ap}, \quad n+1 = \frac{\nu}{\nu - a(1-p), \text{ \&c.}}$$

Substitute these values in (2) reduced to the form

$$\nu + \mu - 1 = \frac{\mu}{(m+1)(m'+1)} = \frac{\nu}{(n+1)(n'+1)},$$

并且我们有

$$\nu + \mu - 1 = \frac{(\mu - ap)(\mu - bq)}{\mu} = \frac{\{\nu - a(1-p)\}\,\{\nu - b(1-q)\}}{\nu}, \qquad (4)$$

又用(3)中的值代替 m、m′等等，我们便有

$$\text{Prob. } xy$$
$$= \left[\frac{abpq}{(\mu - ap)(\mu - bq)} + \frac{ab(1-p)(1-q)}{\{\nu - a(1-p)\}\{\nu - b(1-q)\}}\right](\nu + \mu - 1)$$
$$= \frac{abpq}{\mu} + \frac{ab(1-p)(1-q)}{\nu} \text{ by (4).}$$

现在系统(4)的第一个方程给出

$$\nu + \mu - 1 = \mu - ap - bq + \frac{apbq}{\mu}, \qquad (5)$$
$$\therefore \frac{apbq}{\mu} = \nu - 1 + ap + bq.$$

Similarly,

$$\frac{ab(1-p)(1-q)}{\nu} = \mu - 1 + a(1-p) + b(1-q).$$

Adding these equations together, and observing that the first member of the result becomes identical with the expression just found for Prob. xy, we have

$$\text{Prob. } xy = \nu + \mu + a + b - 2.$$

Let us represent Prob. xy by u, and let $a + b - 2 = m$, then

$$\mu + \nu = u - m. \qquad (6)$$

Again, from (5) we have

$$\mu\nu = abpq - (ap + bq - 1)\mu. \tag{7}$$

Similarly from the first and third members of (4) equated we have

$$\mu\nu = ab(1-p)(1-q) - \{a(1-p) + b(1-q) - 1\}\nu.$$

Let us represent $ap + bq - 1$ by h, and $a(1-p) + b(1-q) - 1$ by h'. We find on equating the above values of $\mu\nu$,

$$\begin{aligned} h\mu - h'\nu &= ab\{pq + (1-p)(1-q)\} \\ &= ab(p + q - 1). \end{aligned}$$

Let $ab(p + q - 1) = l$, then

$$h\mu - h'\nu = l. \tag{8}$$

Now from (6) and (8) we get

$$\mu = \frac{h'(u-m) + l}{m}, \quad \nu = \frac{h(u-m) - l}{m}.$$

Substitute these values in (7) reduced to the form

$$\mu(\nu + h) = abpq,$$

and we have

$$(hu - l)\{h'(u-m) + l\} = abpqm^2, \tag{9}$$

a quadratic equation, the solution of which determines u, the value of Prob. xy sought.

The solution may readily be put in the form

$$h = \frac{lh' + h(h'm - l) \pm \sqrt{[\{lh' - h(h'm - l)\}^2 + 4hh'abpqm^2]}}{2hh'}.$$

But if we further observe that

$$lh' - h(h'm - l) = l(h + h') - hh'm = (1 - hh')m,$$

since

$$h = ap + bq - 1, \quad h' = a(1-p) + b(1-q) - 1,$$

whence

$$h + h' = a + b - 2 = m,$$

we find

$$\text{Prob. } xy = \frac{lh' + h(h'm - l) \pm m\sqrt{\{(l - hh')^2 + 4hh'abpq\}}}{2hh'}. \tag{10}$$

It remains to determine which sign must be given to the radical. We might ascertain this by the general method exemplified in the last problem, but it is

再次，由(5)我们得到

$$\mu\nu = abpq - (ap + bq - 1)\mu. \tag{7}$$

同样地，由(4)的第一和第三项相等，我们得到

$$\mu\nu = ab(1-p)(1-q) - \{a(1-p) + b(1-q) - 1\}\nu.$$

Let us represent $ap + bq - 1$ by h, and $a(1-p) + b(1-q) - 1$ by h'. We find on equating the above values of $\mu\nu$,

$$\begin{aligned} h\mu - h'\nu &= ab\{pq + (1-p)(1-q)\} \\ &= ab(p + q - 1). \end{aligned}$$

Let $ab(p + q - 1) = l$, then

$$h\mu - h'\nu = l. \tag{8}$$

Now from (6) and (8) we get

$$\mu = \frac{h'(u-m) + l}{m}, \quad \nu = \frac{h(u-m) - l}{m}.$$

将这些值代入化简为(7)式的形式

$$\mu(\nu + h) = abpq,$$

and we have

$$(hu - l)\{h'(u-m) + l\} = abpqm^2, \tag{9}$$

a quadratic equation, the solution of which determines u, the value of Prob. xy sought.

The solution may readily be put in the form

$$h = \frac{lh' + h(h'm - l) \pm \sqrt{[\{lh' - h(h'm - l)\}^2 + 4hh'abpqm^2]}}{2hh'}.$$

但如果我们进一步观察到

$$lh' - h(h'm - l) = l(h + h') - hh'm = (1 - hh')m,$$

因为 since

$$h = ap + bq - 1, \quad h' = a(1-p) + b(1-q) - 1,$$

由此 whence

$$h + h' = a + b - 2 = m,$$

我们求得 we find

$$\text{Prob. } xy = \frac{lh' + h(h'm - l) \pm m\sqrt{\{(l - hh')^2 + 4hh'abpq\}}}{2hh'}. \tag{10}$$

尚待确定必须给根号以何种符号。我们可以通过上一问题中所例示的一般方法来确定这一点，但这是

far easier, and it fully suffices in the present instance, to determine the sign by a comparison of the above formula with the result proper to some known case. For instance, if it were certain that the event A is *always*, and the event B *never*, associated with the event E, then it is certain that the events A and B are never conjoined. Hence if $p = 1$, $q = 0$, we ought to have $u = 0$. Now the assumptions $p = 1$, $q = 0$, give

$$h = a - 1, \quad h' = b - 1, \quad l = 0, \quad m = a + b - 2.$$

Substituting in (10) we have

$$\text{Prob. } xy = \frac{(a-1)(b-1)(a+b-2) \pm (a+b-2)(a-1)(b-1)}{2(a-1)(b-1)},$$

and[3] this expression vanishes when the lower sign is taken. Hence the final solution of the general problem will be expressed in the form

$$\frac{\text{Prob. } xy}{\text{Prob. } x} = \frac{lh' + h(h'm - l) - m\sqrt{\{(l - hh')^2 + 4hh'abpq\}}}{2ahh'},$$

wherein $h = ap + bq - 1, \quad h' = a(1-p) + b(1-q) - 1,$

$$l = ab(p + q - 1), \quad m = a + b - 2.$$

As the terms in the final logical solution affected by the coefficient $\frac{1}{0}$ are the same as in the first problem of this chapter, the conditions among the constants will be the same, viz.,

$$ap \overline{=}< 1 - b(1-q), \quad bq \overline{=}< 1 - a(1-p).$$

7. It is a confirmation of the correctness of the above solution that the expression obtained is symmetrical with respect to the two sets of quantities p, q, and $1-p$, $1-q$, i.e. that on changing p into $1-p$, and q into $1-q$, the expression is unaltered This is apparent from the equation

$$\text{Prob. } xy = ab\left\{\frac{pq}{\mu} + \frac{(1-p)(1-q)}{\nu}\right\}$$

employed in deducing the final result. Now if there exist probabilities p, q of the event E, as consequent upon a knowledge of the occurrences of A and B, there exist probabilities $1-p$, $1-q$ of the contrary event, that is, of the non-occurrence of E under the same circumstances. As then the data are unchanged in form, whether we take account in them of the occurrence or of the non-occurrence of E, it is evident that the solution ought to be, as it is, a symmetrical function of p, q and $1-p$, $1-q$.

[3]The numerator was originally "$(a-1)b-1)(a+b-2)\ldots$", and was fixed in 2004 by Distributed Proofreaders.

容易得多，而且在当前情形下完全足够，即通过将上述公式与某个已知情形所特有的结果进行比较来确定符号。例如，如果确定事件 A 总是发生，而事件 B 从不与事件 E 相关联，那么确定事件 A 和 B 从不合取。因此，如果 p = 1,q = 0，我们应当有 u = 0。现在假设 p = 1,q = 0，给出

$$h = a - 1, \quad h' = b - 1, \quad l = 0, \quad m = a + b - 2.$$

代入(10)得

$$\text{Prob. } xy = \frac{(a-1)(b-1)(a+b-2) \pm (a+b-2)(a-1)(b-1)}{2(a-1)(b-1)},$$

并且[3]当取下方符号时此表达式消失。因此一般问题的最终解将表示为如下形式

$$\frac{\text{Prob. } xy}{\text{Prob. } x} = \frac{lh' + h(h'm - l) - m\sqrt{\{(l - hh')^2 + 4hh'abpq\}}}{2ahh'},$$

$$\text{wherein } h = ap + bq - 1, \quad h' = a(1-p) + b(1-q) - 1,$$

$$l = ab(p + q - 1), \quad m = a + b - 2.$$

由于最终逻辑解中受系数[10]arethe影响的项与本章第一问题中的相同，常数之间的条件也将相同，即，

$$ap \overline{=}< 1 - b(1-q), \quad bq \overline{=}< 1 - a(1-p).$$

7. 上述解之正确性的一个确证是，所得表达式关于两组量 p、q 与 1 – p、1 – q 是对称的，也就是说，将 p 变为 1 – p，将 q 变为 1 – q 时，该表达式不变。这从方程

$$\text{Prob. } xy = ab\left\{\frac{pq}{\mu} + \frac{(1-p)(1-q)}{\nu}\right\}$$

用于推导最终结果。现在，如果存在事件 E 的概率 p、q，作为已知 A 和 B 发生的结果，那么就存在相反事件的概率 1 – p、1 – q，即在同样情况下 E 不发生。那么，由于无论我们在数据中考虑 E 的发生还是不发生，数据在形式上都不变，显然解应当是，正如它确实是，p、q 和 1 – p、1 – q 的对称函数。

3 分子原为"(a – 1) b – 1)(a + b – 2) ..."，于2004年由 Distributed Proofreaders 修正。

Let us examine the particular case in which $p = 1$, $q = 1$. We find

$$h = a + b - 1, \quad h' = -1, \quad l = ab, \quad m = a + b - 2,$$

and substituting

$$\frac{\text{Prob. } xy}{\text{Prob. } x} = \frac{-ab + (a+b-1)(2-a-b-ab) - (a+b-2)(ab-a-b+1)}{-2a(a+b-1)}$$
$$= \frac{-2ab(a+b-1)}{-2a(a+b-1)} = b.$$

It would appear, then, that in this case the events A and B are virtually independent of each other. The supposition of their invariable association with some other event E, of the frequency of whose occurrence, except as it may be inferred from this particular connexion, absolutely nothing is known, does not establish any dependence between the events A and B themselves. I apprehend that this conclusion is agreeable to reason, though particular examples may appear at first sight to indicate a different result. For instance, if the probabilities of the casting up, 1st, of a particular species of weed, 2ndly, of a certain description of zoophytes upon the sea-shore, had been separately determined, and if it had also been ascertained that neither of these events could happen except during the agitation of the waves caused by a tempest, it would, I think, justly be concluded that the events in question were not independent. The picking up of a piece of seaweed of the kind supposed would, it is presumed, render more probable the discovery of the zoophytes than it would otherwise have been. But I apprehend that this fact is due to our knowledge of another circumstance not implied in the actual conditions of the problem, viz., that the occurrence of a tempest is but an *occasional* phenomenon. Let the range of observation be confined to a sea *always* vexed with storm. It would then, I suppose, be seen that the casting up of the weeds and of the zoophytes ought to be regarded as independent events. Now, to speak more generally, there are conditions common to all phænomena,—conditions which, it is felt, do not affect their mutual independence. I apprehend therefore that the solution indicates, that when a particular condition has prevailed through the whole of our *recorded experience*, it assumes the above character with reference to the class of phænomena over which that experience has extended.

8. PROBLEM IV.—To illustrate in some degree the above observations, let there be given, in addition to the data of the last problem, the absolute probability of the event E, the completed system of data being

$$\text{Prob. } x = a, \quad \text{Prob. } y = b, \quad \text{Prob. } z = c,$$
$$\text{Prob. } xz = ap, \quad \text{Prob. } yz = bq,$$

and let it be required to find Prob. xy.

让我们考察 p = 1,q = 1 这一特例。我们得到

$$h = a + b - 1, \quad h' = -1, \quad l = ab, \quad m = a + b - 2,$$

并代入

$$\frac{\text{Prob. } xy}{\text{Prob. } x} = \frac{-ab + (a+b-1)(2-a-b-ab) - (a+b-2)(ab-a-b+1)}{-2a(a+b-1)}$$
$$= \frac{-2ab(a+b-1)}{-2a(a+b-1)} = b.$$

那么，在这种情况下，事件A和B似乎是彼此独立的。假设它们与某个其他事件E不变地相关联，而关于该事件发生的频率，除了可能从这种特定联系中推断出来的以外，绝对一无所知，这并不在事件A和B本身之间建立任何依赖关系。我认为这个结论是合乎理性的，尽管具体的例子乍看之下可能表明不同的结果。例如，如果抛上岸的、第一，某种特定杂草，第二，海滨某种类型的植虫的概率，已经被分别确定，并且如果也已经查明，除非在暴风雨引起的波浪激荡期间，这些事件中的任何一个都不可能发生，那么我认为，可以正当地得出结论说，所讨论的事件不是独立的。据推测，捡到一块所设想的那种海藻，会使发现植虫比否则更为可能。但我认为，这一事实是由于我们对另一情况的知识，而该情况并未隐含在问题的实际条件中，即暴风雨的发生只是一种偶发现象。让观察的范围局限于一个总是受风暴困扰的海洋。那么我设想，就会看出，杂草和植虫的抛上岸应当被视为独立的事件。现在，更一般地说，存在一切现象所共有的条件--人们感到这些条件并不影响它们的相互独立。因此我认为，该解表明，当一个特定条件贯穿于我们的有记录的经验之全部时，它就参照该经验所延伸于其上的那类现象而具有上述性质。

8. 问题 IV。--为了在某种程度上说明上述观察，除上一问题的数据之外，再给出事件 E 的绝对概率，完整的数据系统为

$$\text{Prob. } x = a, \quad \text{Prob. } y = b, \quad \text{Prob. } z = c,$$
$$\text{Prob. } xz = ap, \quad \text{Prob. } yz = bq,$$

并设要求求出 Prob .xy。

Assuming, as before, $xz = s$, $yz = t$, $xy = w$, the final logical equation is

$$w = xystz + xy\bar{s}\bar{t}\bar{z} + 0(x\bar{y}s\bar{t}z + \bar{x}ys\bar{t}z + x\bar{y}\bar{z}s\bar{t} + xy\bar{z}s\bar{t} + \bar{x}\bar{y}zs\bar{t} + \bar{x}\bar{y}\bar{z}s\bar{t}) + \text{terms whose coefficients are } \frac{1}{0}.$$

The algebraic system having been formed, the subsequent eliminations may be simplified by the transformations adopted in the previous problem. The final result is

$$\text{Prob.}\, xy = ab\left\{\frac{pq}{c} + \frac{(1-p)(1-q)}{1-c}\right\}. \tag{2}$$

The conditions among the constants are

$$c \overline{=}> ap, \quad c \overline{=}> bq, \quad c \overline{=}< 1 - a(1-p), \quad c \overline{=}< 1 - b(1-q).$$

Now if $p = 1$, $q = 1$, we find

$$\text{Prob.}\, xy = \frac{ab}{c},$$

c not admitting of any value less than a or b. It follows hence that if the event E is known to be an *occasional* one, its invariable attendance on the events x and y *increases* the probability of their conjunction in the inverse ratio of its own frequency. The formula (2) may be verified in a large number of cases. As a particular instance, let $q = c$, we find

$$\text{Prob.}\, xy = ab. \tag{3}$$

Now the assumption $q = c$ involves, by Definition (Chap. XVI.) the independence of the events B and E. If then B and E are independent, no relation which may exist between A and E can establish a relation between A and B; wherefore A and B are also independent, as the above equation (3) implies.

It may readily be shown from (2) that the value of Prob. z, which renders Prob. xy a minimum, is

$$\text{Prob.}\, z = \frac{\sqrt{(pq)}}{\sqrt{(pq)} + \sqrt{(1-p)(1-q)}}.$$

If $p = q$, this gives

$$\text{Prob.}\, z = p;$$

a result, the correctness of which may be shown by the same considerations which have been applied to (3).

PROBLEM V.—Given the probabilities of any three events, and the probability of their conjunction; required the probability of the conjunction of any two of them.

Suppose the data to be

$$\text{Prob.}\, x = p, \quad \text{Prob.}\, y = q, \quad \text{Prob.}\, z = r, \quad \text{Prob.}\, xyz = m,$$

Assuming, as before, $xz = s$, $yz = t$, $xy = w$, the final logical equation is

$$w = xystz + xy\bar{s}\bar{t}\bar{z} + 0(x\bar{y}s\bar{t}z + \bar{x}ys\bar{t}z + x\bar{y}\bar{z}s\bar{t} + xy\bar{z}s\bar{t} + \bar{x}\bar{y}zs\bar{t} + \bar{x}\bar{y}\bar{z}s\bar{t})$$

+ 系数为[1]的项 0.

代数系统一经形成，随后的消元便可借上一问题中所采用的变换而简化。最终结果为

result is

$$\text{Prob.}\, xy = ab\left\{\frac{pq}{c} + \frac{(1-p)(1-q)}{1-c}\right\}. \tag{2}$$

The conditions among the constants are

$$c \overline{=}> ap, \quad c \overline{=}> bq, \quad c \overline{=}< 1 - a(1-p), \quad c \overline{=}< 1 - b(1-q).$$

现在若 p = 1,q = 1，我们求得

$$\text{Prob.}\, xy = \frac{ab}{c},$$

c 不容有任何小于 a 或 b 的值。由此可知，若已知事件 E 是*occasiona*的，则它伴随事件 x 和 y 的恒定出现*increases*它们合取的概率，其比例为其自身频率的倒数。公式(2)可在大量情形中得到验证。作为一个特例，令 q = c，我们得到

$$\text{Prob.}\, xy = ab. \tag{3}$$

现在，假设 q = c 依定义(第十六章)蕴含事件 B 与 E 的独立性。那么，若 B 与 E 独立，则 A 与 E 之间可能存在的任何关系都不能建立 A 与 B 之间的关系；因此 A 与 B 也是独立的，正如上述方程(3)所蕴含的。

It may readily be shown from (2) that the value of Prob. z, which renders Prob. xy a minimum, is

$$\text{Prob.}\, z = \frac{\sqrt{(pq)}}{\sqrt{(pq)} + \sqrt{(1-p)(1-q)}}.$$

If $p = q$, this gives

$$\text{Prob.}\, z = p;$$

一个结果，其正确性可以通过应用于(3)的同样考虑来表明。

问题V。--给定任意三个事件的概率，以及它们合取的概率；要求其中任意两个的合取的概率。

假设数据为

$$\text{Prob.}\, x = p, \quad \text{Prob.}\, y = q, \quad \text{Prob.}\, z = r, \quad \text{Prob.}\, xyz = m,$$

and the quæsitum to be Prob. xy.

Assuming $xyz = s$, $xy = t$, we find as the final logical equation,

$$t = xyzs + xy\bar{z}\bar{s} + 0(x\bar{y}\bar{s} + \bar{x}\bar{s}) + \frac{1}{0}(\text{sum of all other constituents});$$

whence, finally,

$$\text{Prob. } xy = \frac{H - \sqrt{(H^2 - 4pq\bar{r}^2 - 4\bar{p}\bar{q}\bar{r}m)}}{2\bar{r}},$$

wherein

$$\bar{p} = 1 - p, \text{ \&c.} \quad H = \bar{p}\bar{q} + (p + q)\bar{r}.$$

This admits of verification when $p = 1$, when $q = 1$, when $r = 0$, and therefore $m = 0$, &c.

Had the condition, Prob. $z = r$, been omitted, the solution would still have been definite. We should have had

$$\text{Prob. } xy = \frac{pq(1 - m) + (1 - p)(1 - q)m}{1 - m};$$

and it may be added, as a final confirmation of their correctness, that the above results become identical when $m = pqr$.

9. The following problem is a generalization of Problem I., and its solution, though necessarily more complex, is obtained by a similar analysis.

PROBLEM VI.—If an event can only happen as a consequence of one or more of certain causes $A_1, A_2, \ldots A_n$, and if generally c_i represent the probability of the cause A_i and p_i the probability that if the cause A_i exist, the event E will occur, then the series of values of c_i and p_i being given, required the probability of the event E.[4]

Let the causes $A_1, A_2, \ldots A_n$ be represented by $x_1, x_2, \ldots x_n$, and the event E by z.

Then we have generally,

$$\text{Prob. } x_i = c_i, \qquad \text{Prob. } x_i z = c_i p_i.$$

[4]It may be proper to remark, that the above problem was proposed to the notice of mathematicians by the author in the Cambridge and Dublin Mathematical Journal, Nov. 1851, accompanied by the subjoined observations:

"The motives which have led me, after much consideration, to adopt, with reference to this question, a course unusual in the present day, and not upon slight grounds to be revived, are the following:—First, I propose the question as a test of the sufficiency of received methods. Secondly, I anticipate that its discussion will in some measure add to our knowledge of an important branch of pure analysis. However, it is upon the former of these grounds alone that I desire to rest my apology.

"While hoping that some may be found who, without departing from the line of their previous studies, may deem this question worthy of their attention, I wholly disclaim the notion of its being offered as a trial of personal skill or knowledge, but desire that it may be viewed solely with reference to those public and scientific ends for the sake of which alone it is proposed."

The author thinks it right to add, that the publication of the above problem led to some interesting private correspondence, but did not elicit a solution.

而所求为 Prob .xy。

假设 xyz = s,xy = t，我们得到最终的逻辑方程，

$$t = xyzs + xy\bar{z}\bar{s} + 0(x\bar{y}\bar{s} + \bar{x}\bar{s}) + \frac{1}{0}(\text{sum of all other constituents});$$

由此，最终，

$$\text{Prob. } xy = \frac{H - \sqrt{(H^2 - 4pq\bar{r}^2 - 4\bar{p}\bar{q}\bar{r}m)}}{2\bar{r}},$$

wherein

$$\bar{p} = 1 - p, \text{ \&c.} \quad H = \bar{p}\bar{q} + (p + q)\bar{r}.$$

This admits of verification when $p = 1$, when $q = 1$, when $r = 0$, and therefore $m = 0$, &c.

Had the condition, Prob. $z = r$, been omitted, the solution would still have been definite. We should have had

$$\text{Prob. } xy = \frac{pq(1 - m) + (1 - p)(1 - q)m}{1 - m};$$

and it may be added, as a final confirmation of their correctness, that the above results become identical when $m = pqr$.

9. The following problem is a generalization of Problem I., and its solution, though necessarily more complex, is obtained by a similar analysis.

问题 VI.- 如果一个事件只能作为某些原因A [1] , A [2] , ...A n中的一个或多个的结果而发生，并且如果一般地c i表示原因A i的概率，而p i表示如果原因A i存在，事件E将发生的概率，那么，在c i和p i的一系列值已给定的情况下，求事件E的概率。[4]

Let the causes $A_1, A_2, \ldots A_n$ be represented by $x_1, x_2, \ldots x_n$, and the event E by z.

Then we have generally,

$$\text{Prob. } x_i = c_i, \qquad \text{Prob. } x_i z = c_i p_i.$$

4 或许应当指出，上述问题是由作者于1851年11月在《剑桥与都柏林数学杂志》上提请数学家们注意的，并附有以下评述：

"经过深思熟虑之后，使我针对这个问题采取一种在当今不寻常、且没有充分理由不应重新启用的做法的动机如下：--第一，我提出这个问题作为对公认方法之充分性的一种检验。第二，我预期它的讨论将在某种程度上增进我们对纯分析一个重要分支的知识。然而，我仅希望以第一条理由作为我的辩解依据。

"虽然我希望或许能找到一些人，他们不偏离先前研究的路线，却认为这个问题值得关注，但我完全否认它是作为对个人技巧或知识的考验而提出的想法，而是希望它仅参照那些公共的和科学的目的来加以看待，仅仅为了这些目的它才被提出。"

作者认为应当补充说明，上述问题的发表引起了一些有趣的私人通信，但并未引出一个解答。

Further, the condition that E can only happen in connexion with some one or more of the causes $A_1, A_2, \dots A_n$ establishes the logical condition,

$$z(1-x_1)(1-x_2)\dots(1-x_n)=0. \tag{1}$$

Now let us assume generally

$$x_i z = t_i,$$

which is reducible to the form

$$x_i z(1-t_i)+t_i(1-x_i z)=0,$$

forming the type of a system of n equations which, together with (1), express the logical conditions of the problem. Adding all these equations together, as after the previous reduction we are permitted to do, we have

$$\sum\{x_i z(1-t_i)+t_i(1-x_i z)\}+z(1-x_1)(1-x_2)\dots(1-x_n)=0, \tag{2}$$

(the summation implied by $\sum$ extending from $i=1$ to $i=n$), and this single and sufficient logical equation, together with the $2n$ data, represented by the general equations

$$\text{Prob. } x_i = c_i, \quad \text{Prob. } t_i = c_i p_i, \tag{3}$$

constitute the elements from which we are to determine Prob. z.

Let (2) be developed with respect to z. We have

$$\left[\sum\{x_i(1-t_i)+t_i(1-x_i)\}+(1-x_1)(1-x_2)\dots(1-x_n)\right]z + \sum t_i(1-z)=0,$$

whence

$$z=\frac{\sum t_i}{\sum t_i-\sum\{x_i(1-t_i)+t_i(1-x_i)\}-(1-x_1)(1-x_2)\dots(1-x_n)}. \tag{4}$$

Now any constituent in the expansion of the second member of the above equation will consist of $2n$ factors, of which n are taken out of the set $x_1, x_2, \dots x_n, 1-x_1, 1-x_2, \dots 1-x_n$, and n out of the set $t_1, t_2, \dots t_n, 1-t_1, 1-t_2 \dots 1-t_n$, no such combination as $x_1(1-x_1), t_1(1-t_1)$, being admissible. Let us consider first those constituents of which $(1-t_1),(1-t_2)\dots(1-t_n)$ forms the t-factor, that is the factor derived from the set $t_1, \dots 1-t_1$.

The coefficient of any such constituent will be found by changing $t_1, t_2, \dots t_n$ respectively into 0 in the second member of (4), and then assigning to $x_1, x_2, \dots x_n$ their values as dependent upon the nature of the x-factor of the constituent. Now simply substituting for $t_1, t_2, \dots t_n$ the value 0, the second member becomes

$$\frac{0}{-\sum x_i-(1-x_1)(1-x_2)\dots(1-x_n)},$$

此外，E 只能与原因 A 1 , A 2 , ...A n 中的某一个或多个相关联而发生这一条件，确立了逻辑条件，

$$z(1-x_1)(1-x_2)\dots(1-x_n)=0. \tag{1}$$

现在让我们一般地假设

$$x_i z = t_i,$$

可化简为以下形式

$$x_i z(1-t_i)+t_i(1-x_i z)=0,$$

构成一个 n 方程系统的类型，这些方程连同(1)一起表达了该问题的逻辑条件。将所有这些方程相加，正如在先前约化之后我们被允许做的那样，我们得到

$$\sum\{x_i z(1-t_i)+t_i(1-x_i z)\}+z(1-x_1)(1-x_2)\dots(1-x_n)=0, \tag{2}$$

(P所隐含的求和从 i = 1 延伸到 i = n)，以及这个单一且充分的逻辑方程，连同由一般方程所表示的 2 n 个数据，

$$\text{Prob. } x_i = c_i, \quad \text{Prob. } t_i = c_i p_i, \tag{3}$$

构成我们据以确定 Prob.z 的元素。

设(2)按z展开。我们有

$$\left[\sum\{x_i(1-t_i)+t_i(1-x_i)\}+(1-x_1)(1-x_2)\dots(1-x_n)\right]z + \sum t_i(1-z)=0,$$

由此nce

$$z=\frac{\sum t_i}{\sum t_i-\sum\{x_i(1-t_i)+t_i(1-x_i)\}-(1-x_1)(1-x_2)\dots(1-x_n)}. \tag{4}$$

Now any constituent in the expansion of the second member of the above equation will consist of $2n$ factors, of which n are taken out of the set $x_1, x_2, \dots x_n, 1-x_1, 1-x_2, \dots 1-x_n$, and n out of the set $t_1, t_2, \dots t_n, 1-t_1, 1-t_2 \dots 1-t_n$, no such combination as $x_1(1-x_1), t_1(1-t_1)$, being admissible. Let us consider first those constituents of which $(1-t_1),(1-t_2)\dots(1-t_n)$ forms the t-factor, that is the factor derived from the set $t_1, \dots 1-t_1$.

The coefficient of any such constituent will be found by changing $t_1, t_2, \dots t_n$ respectively into 0 in the second member of (4), and then assigning to $x_1, x_2, \dots x_n$ their values as dependent upon the nature of the x-factor of the constituent. Now simply substituting for $t_1, t_2, \dots t_n$ the value 0, the second member becomes

$$\frac{0}{-\sum x_i-(1-x_1)(1-x_2)\dots(1-x_n)},$$

and this vanishes whatever values, 0, 1, we subsequently assign to $x_1, x_2, \dots x_n$. For if those values are not all equal to 0, the term $\sum x_i$ does not vanish, and if they are all equal to 0, the term $-(1-x_1)\dots(1-x_n)$ becomes -1, so that in either case the denominator does not vanish, and therefore the fraction does. Hence the coefficients of all constituents of which $(1-t_1)\dots(1-t_n)$ is a factor will be 0, and as the sum of all possible x-constituents is unity, there will be an aggregate term $0(1-t_1)\dots(1-t_n)$ in the development of z.

Consider, in the next place, any constituent of which the t-factor is $t_1\, t_2 \dots t_r(1-t_{r+1})\dots(1-t_n)$, r being equal to or greater than unity. Making in the second member of (4), $t_1 = 1, \dots t_r = 1, t_{r+1} = 0, \dots t_n = 0$, we get the expression

$$\frac{r}{x_1 \dots + x_r - x_{r+1} \dots - x_n - (1-x_1)(1-x_2)\dots(1-x_n)}$$

Now the only admissible values of the symbols being 0 and 1, it is evident that the above expression will be equal to 1 when $x_1 = 1 \dots x_r = 1, x_{r+1} = 0, \dots x_n = 0$, and that for all other combinations of value that expression will assume a value greater than unity. Hence the coefficient 1 will be applied to all constituents of the final development which are of the form

$$x_1 \dots x_r(1-x_{r+1})\dots(1-x_n)\, t_1 \dots t_r(1-t_{r+1})\dots(1-t_n),$$

the x-factor being similar to the t-factor, while other constituents included under the present case will have the virtual coefficient $\frac{1}{0}$. Also, it is manifest that this reasoning is independent of the particular arrangement and succession of the individual symbols.

Hence the complete expansion of z will be of the form

$$z = \sum(XT) + 0(1-t_1)(1-t_2)\dots(1-t_n) + \text{ constituents whose coefficients are } \frac{1}{0}, \quad (5)$$

where T represents any t-constituent except $(1-t_1)\dots(1-t_n)$, and X the corresponding or similar constituent of $x_1 \dots x_n$. For instance, if $n = 2$, we shall have

$$\sum(XT) = x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1t_2,$$

$\bar{x}_1$, $\bar{x}_2$, &c. standing for $1-x_1$, $1-x_2$, &c.; whence

$$\begin{aligned} z = {} & x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1t_2 \\ & + 0(x_1x_2\bar{t}_1\bar{t}_2 + x_1\bar{x}_2\bar{t}_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1\bar{t}_2 + \bar{x}_1\bar{x}_2\bar{t}_1\bar{t}_2) \\ & + \text{ constituents whose coefficients are } \frac{1}{0}. \end{aligned} \quad (6)$$

This result agrees, difference of notation being allowed for, with the developed form of z in Problem I. of this chapter, as it evidently ought to do.

并且无论我们随后给x ^{1} , x ^{2} , ...x n指定什么值，0、1，它都消失。因为如果那些值不全都等于0，则项^{P} x i不消失，而如果它们全都等于0，则项–(1 – x ^{1}) ...(1 – x n)变为– 1，以致在任一情况下分母都不消失，因而该分数消失。因此，以(1 – t ^{1}) ...(1 – t n)为因子的所有成分的系数都将为0，并且由于一切可能的x - 成分之和为1，所以在z的展开式中将有一个总项0(1 – t ^{1}) ...(1 – t n)。

Consider, in the next place, any constituent of which the t-factor is $t_1\, t_2 \dots t_r(1-t_{r+1})\dots(1-t_n)$, r being equal to or greater than unity. Making in the second member of (4), $t_1 = 1, \dots t_r = 1, t_{r+1} = 0, \dots t_n = 0$, we get the expression

$$\frac{r}{x_1 \dots + x_r - x_{r+1} \dots - x_n - (1-x_1)(1-x_2)\dots(1-x_n)}$$

现在，既然这些记号唯一可容许的值是0和1，那么显然，当x ^{1} = 1 ...x r = 1 , x r $^{+1}$ = 0 , ...x n = 0时，上述表达式将等于1，而对于所有其他值组合，该表达式将取大于1的值。因此，系数1将被赋予最终展开式中所有具有以下形式的成分

$$x_1 \dots x_r(1-x_{r+1})\dots(1-x_n)\, t_1 \dots t_r(1-t_{r+1})\dots(1-t_n),$$

x - 因子类似于 t - 因子，而包含在当前情形下的其他成分将具有虚拟系数$^{10.}$此外，这是显然的,这个推理独立于各个记号的特定排列和顺序。

因此，z 的完全展开将具有如下形式

$$z = \sum(XT) + 0(1-t_1)(1-t_2)\dots(1-t_n) + \text{ constituents whose coefficients are } \frac{1}{0}, \quad (5)$$

其中 T 表示除(1 – t ^{1}) ...(1 – t n) 之外的任一 t - 成分，而 X 表示 x ^{1} ...x n 的相应或类似成分。例如，若 n = 2，我们将有

$$\sum(XT) = x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1t_2,$$

$\bar{x}_1$, $\bar{x}_2$, &c. standing for $1-x_1$, $1-x_2$, &c.; whence

$$\begin{aligned} z = {} & x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1t_2 \\ & + 0(x_1x_2\bar{t}_1\bar{t}_2 + x_1\bar{x}_2\bar{t}_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1\bar{t}_2 + \bar{x}_1\bar{x}_2\bar{t}_1\bar{t}_2) \\ & + \text{ constituents whose coefficients are } \frac{1}{0}. \end{aligned} \quad (6)$$

这个结果，在允许记号差异的情况下，与本章问题I中 z 的展开形式一致，正如它显然应当如此。

10. To avoid complexity, I purpose to deduce from the above equation (6) the necessary conditions for the determination of Prob. z for the particular case in which $n = 2$, in such a form as may enable us, by pursuing in thought the same line of investigation, to assign the corresponding conditions for the more general case in which n possesses any integral value whatever.

Supposing then $n = 2$, we have

$$V = x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1t_2 + x_1x_2\bar{t}_1\bar{t}_2 + x_1\bar{x}_2\bar{t}_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1\bar{t}_2 + \bar{x}_1\bar{x}_2\bar{t}_1\bar{t}_2.$$

$$\text{Prob. } z = \frac{x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1t_2}{V},$$

the conditions for the determination of x_1, t_1, &c., being

$$\frac{x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + x_1x_2\bar{t}_1\bar{t}_2 + x_1\bar{x}_2\bar{t}_1\bar{t}_2}{c_1}$$
$$= \frac{x_1x_2t_1t_2 + \bar{x}_1x_2\bar{t}_1t_2 + x_1x_2\bar{t}_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1\bar{t}_2}{c_2}$$
$$= \frac{x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2}{c_1p_1} = \frac{x_1x_2t_1t_2 + \bar{x}_1x_2\bar{t}_1t_2}{c_2p_2} = V.$$

Divide the members of this system of equations by $\bar{x}_1\bar{x}_2\bar{t}_1\bar{t}_2$, and the numerator and denominator of Prob. z by the same quantity, and in the results assume

$$\frac{x_1t_1}{\bar{x}_1\bar{t}_1} = m_1, \quad \frac{x_2t_2}{\bar{x}_2\bar{t}_2} = m_2, \quad \frac{x_1}{\bar{x}_1} = n_1, \quad \frac{x_2}{\bar{x}_2} = n_2; \tag{7}$$

we find

$$\text{Prob. } z = \frac{m_1m_2 + m_1 + m_2}{m_1m_2 + m_1 + m_2 + n_1n_2 + n_1 + n_2 + 1},$$
$$\text{and } \frac{m_1m_2 + m_1 + n_1n_2 + n_1}{c_1} = \frac{m_1m_2 + m_2 + n_1n_2 + n_2}{c_2}$$
$$= \frac{m_1m_2 + m_1}{c_1p_1} + \frac{m_1m_2 + m_2}{c_2p_2} = m_1m_2 + m_1 + m_2 + n_1n_2 + n_1 + n_2 + 1, \tag{8}$$

whence, if we assume,

$$(m_1 + 1)(m_2 + 1) = M, \quad (n_1 + 1)(n_2 + 1) = N, \tag{9}$$

we have, after a slight reduction,

$$\text{Prob. } z = \frac{M - 1}{M + N - 1},$$
$$\frac{n_1(n_2 + 1)}{c_1(1 - p_1)} = \frac{n_2(n_1 + 1)}{c_2(1 - p_2)} = \frac{m_1(m_2 + 1)}{c_1p_1} = \frac{m_2(m_1 + 1)}{c_2p_2} = M + N - 1;$$

10. 为避免复杂化，我打算从上述方程(6)推导出确定 Prob .z 的必要条件，针对 n = 2 的特殊情形，以这样一种形式，使我们能够通过在思想上沿同一研究路线推进，为 n 具有任意整数值的更为一般的情形指定相应的条件。

假设 n = 2，则有

$$V = x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1t_2 + x_1x_2\bar{t}_1\bar{t}_2 + x_1\bar{x}_2\bar{t}_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1\bar{t}_2 + \bar{x}_1\bar{x}_2\bar{t}_1\bar{t}_2.$$

$$\text{Prob. } z = \frac{x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1t_2}{V},$$

确定 x ^{1}、t ^{1}等的条件为

$$\frac{x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2 + x_1x_2\bar{t}_1\bar{t}_2 + x_1\bar{x}_2\bar{t}_1\bar{t}_2}{c_1}$$
$$= \frac{x_1x_2t_1t_2 + \bar{x}_1x_2\bar{t}_1t_2 + x_1x_2\bar{t}_1\bar{t}_2 + \bar{x}_1x_2\bar{t}_1\bar{t}_2}{c_2}$$
$$= \frac{x_1x_2t_1t_2 + x_1\bar{x}_2t_1\bar{t}_2}{c_1p_1} = \frac{x_1x_2t_1t_2 + \bar{x}_1x_2\bar{t}_1t_2}{c_2p_2} = V.$$

将这个方程组的各项除以 x $^{1-}$x $^{2-}$t $^{1-}$t ^{2}，并将 Prob .z 的分子和分母除以同一量，且在结果中假设

$$\frac{x_1t_1}{\bar{x}_1\bar{t}_1} = m_1, \quad \frac{x_2t_2}{\bar{x}_2\bar{t}_2} = m_2, \quad \frac{x_1}{\bar{x}_1} = n_1, \quad \frac{x_2}{\bar{x}_2} = n_2; \tag{7}$$

我们求得

$$\text{Prob. } z = \frac{m_1m_2 + m_1 + m_2}{m_1m_2 + m_1 + m_2 + n_1n_2 + n_1 + n_2 + 1},$$
$$\text{and } \frac{m_1m_2 + m_1 + n_1n_2 + n_1}{c_1} = \frac{m_1m_2 + m_2 + n_1n_2 + n_2}{c_2}$$
$$= \frac{m_1m_2 + m_1}{c_1p_1} + \frac{m_1m_2 + m_2}{c_2p_2} = m_1m_2 + m_1 + m_2 + n_1n_2 + n_1 + n_2 + 1, \tag{8}$$

由此，如果我们假设，

$$(m_1 + 1)(m_2 + 1) = M, \quad (n_1 + 1)(n_2 + 1) = N, \tag{9}$$

经过略微化简，我们有，

$$\text{Prob. } z = \frac{M - 1}{M + N - 1},$$
$$\frac{n_1(n_2 + 1)}{c_1(1 - p_1)} = \frac{n_2(n_1 + 1)}{c_2(1 - p_2)} = \frac{m_1(m_2 + 1)}{c_1p_1} = \frac{m_2(m_1 + 1)}{c_2p_2} = M + N - 1;$$

or,

$$\frac{m_1M}{(m_1+1)c_1p_1}=\frac{m_2M}{(m_2+1)c_2p_2}=\frac{n_1N}{(n_1+1)c_1(1-p_1)}$$
$$=\frac{n_2N}{(n_2+1)c_2(1-p_2)}=M+N-1.$$

Now let a similar series of transformations and reductions be performed in thought upon the final logical equation (5). We shall obtain for the determination of Prob. z the following expression:

$$\text{Prob. } z=\frac{M-1}{M+N-1} \tag{10}$$

wherein

$$M=(m_1+1)(m_2+1)\dots(m_n+1),$$
$$N=(n_1+1)(n_2+1)\dots(n_n+1),$$

$m_1,\dots,m_n,n_1,\dots,n_n$, being given by the system of equations,

$$\frac{m_1M}{(m_1+1)c_1p_1}=\frac{m_2M}{(m_2+1)c_2p_2}\dots=\frac{m_nM}{(m_n+1)c_np_n}$$
$$=\frac{n_1N}{(n_1+1)c_1(1-p_1)}\dots=\frac{n_nN}{(n_n+1)c_n(1-p_n)}=M+N-1. \tag{11}$$

Still further to simplify the results, assume

$$\frac{M+N-1}{M}=\frac{1}{\mu},\quad \frac{M+N-1}{N}=\frac{1}{\nu};$$

whence

$$M=\frac{\mu}{\mu+\nu-1},\quad N=\frac{\nu}{\mu+\nu-1}.$$

We find

$$\frac{m_1}{(m_1+1)c_1p_1}=\frac{m_2}{(m_2+1)c_2p_2}\dots=\frac{m_n}{(m_n+1)c_np_n}=\frac{1}{\mu},$$
$$\frac{n_1}{(n_1+1)c_1(1-p_1)}=\frac{n_2}{(n_2+1)c_2(1-p_2)}\dots=\frac{n_n}{(n_n+1)c_n(1-p_n)}=\frac{1}{\nu};$$

whence

$$m_1=\frac{c_1p_1}{\mu-c_1p_1},\dots m_n=\frac{c_np_n}{\mu-c_np_n};$$

and finally,

$$m_1+1=\frac{\mu}{\mu-c_1p_1},\dots m_n+1=\frac{\mu}{\mu-c_np_n},$$
$$n_1+1=\frac{\nu}{\nu-c_1(1-p_1)},\dots n_n+1=\frac{\nu}{\nu-c_n(1-p_n)}.$$

或者，

$$\frac{m_1M}{(m_1+1)c_1p_1}=\frac{m_2M}{(m_2+1)c_2p_2}=\frac{n_1N}{(n_1+1)c_1(1-p_1)}$$
$$=\frac{n_2N}{(n_2+1)c_2(1-p_2)}=M+N-1.$$

现在让我们在思想上对最终逻辑方程(5)进行一系列类似的变换和约化。我们将得到确定 Prob .z 的如下表达式：

$$\text{Prob. } z=\frac{M-1}{M+N-1} \tag{10}$$

其中

$$M=(m_1+1)(m_2+1)\dots(m_n+1),$$
$$N=(n_1+1)(n_2+1)\dots(n_n+1),$$

$m_1,\dots,m_n,n_1,\dots,n_n$, being given by the system of equations,

$$\frac{m_1M}{(m_1+1)c_1p_1}=\frac{m_2M}{(m_2+1)c_2p_2}\dots=\frac{m_nM}{(m_n+1)c_np_n}$$
$$=\frac{n_1N}{(n_1+1)c_1(1-p_1)}\dots=\frac{n_nN}{(n_n+1)c_n(1-p_n)}=M+N-1. \tag{11}$$

为进一步简化结果，假设

$$\frac{M+N-1}{M}=\frac{1}{\mu},\quad \frac{M+N-1}{N}=\frac{1}{\nu};$$

whence

$$M=\frac{\mu}{\mu+\nu-1},\quad N=\frac{\nu}{\mu+\nu-1}.$$

We find

$$\frac{m_1}{(m_1+1)c_1p_1}=\frac{m_2}{(m_2+1)c_2p_2}\dots=\frac{m_n}{(m_n+1)c_np_n}=\frac{1}{\mu},$$
$$\frac{n_1}{(n_1+1)c_1(1-p_1)}=\frac{n_2}{(n_2+1)c_2(1-p_2)}\dots=\frac{n_n}{(n_n+1)c_n(1-p_n)}=\frac{1}{\nu};$$

whence

$$m_1=\frac{c_1p_1}{\mu-c_1p_1},\dots m_n=\frac{c_np_n}{\mu-c_np_n};$$

and finally,

$$m_1+1=\frac{\mu}{\mu-c_1p_1},\dots m_n+1=\frac{\mu}{\mu-c_np_n},$$
$$n_1+1=\frac{\nu}{\nu-c_1(1-p_1)},\dots n_n+1=\frac{\nu}{\nu-c_n(1-p_n)}.$$

Substitute these values with those of M and N in (9), and we have

$$\frac{\mu^n}{(\mu - c_1p_1)(\mu - c_2p_2)\dots(\mu - c_np_n)} = \frac{\mu}{\mu + \nu - 1},$$
$$\frac{\nu^n}{\{\nu - c_1(1-p_1)\}\{\nu - c_2(1-p_2)\}\dots\{\nu - c_n(1-p_n)\}} = \frac{\nu}{\mu + \nu - 1},$$

which may be reduced to the symmetrical form

$$\begin{aligned}\mu + \nu - 1 &= \frac{(\mu - c_1p_1)\dots(\mu - c_np_n)}{\mu^{n-1}} \\ &= \frac{\{\nu - c_1(1-p_1)\}\dots\{\nu - c_n(1-p_n)\}}{\nu^{n-1}}.\end{aligned} \tag{12}$$

Finally,

$$\text{Prob.}\, z = \frac{M-1}{M+N-1} = 1 - \nu. \tag{13}$$

Let us then assume $1 - \nu = u$, we have then

$$\begin{aligned}\mu - u &= \frac{(\mu - c_1p_1)\dots(\mu - c_np_n)}{\mu^{n-1}} \\ &= \frac{\{1 - c_1(1-p_1)\}\dots\{1 - c_n(1-p_n) - u\}}{(1-u)^{n-1}}.\end{aligned}$$

If we make for simplicity

$$c_1p_1 = a_1, \quad c_np_n = a_n, \quad 1 - c_1(1-p_1) = b_1, \text{ \&c.},$$

the above equations may be written as follows:

$$\mu - u = \frac{(\mu - a_1)\dots(\mu - a_n)}{\mu^{n-1}}, \tag{14}$$

wherein

$$\mu = u + \frac{(b_1 - u)\dots(b_n - u)}{(1-u)^{n-1}}. \tag{15}$$

This value of μ substituted in (14) will give an equation involving only u, the solution of which will determine Prob. z, since by (13) Prob. $z = u$. It remains to assign the limits of u.

11. Now the very same analysis by which the limits were determined in the particular case in which $n = 2$, (XIX. 12) conducts us in the present case to the following result. The quantity u, in order that it may represent the value of Prob. z, must must have for its inferior limits the quantities $a_1, a_2, \dots a_n$, and for its superior limits the quantities $b_1, b_2, \dots b_n$, $a_1 + a_2 \dots + a_n$. We may hence infer, *à priori*, that there will always exist one root, and only one root, of the equation (14) satisfying these conditions. I deem it sufficient, for practical verification, to show that there will exist one, and only one, root of the equation (14), between the limits $a_1, a_2, \dots a_n$, and $b_1, b_2, \dots b_n$.

将这些值用(9)中的M和N的值替换，我们得到

$$\frac{\mu^n}{(\mu - c_1p_1)(\mu - c_2p_2)\dots(\mu - c_np_n)} = \frac{\mu}{\mu + \nu - 1},$$
$$\frac{\nu^n}{\{\nu - c_1(1-p_1)\}\{\nu - c_2(1-p_2)\}\dots\{\nu - c_n(1-p_n)\}} = \frac{\nu}{\mu + \nu - 1},$$

可化简为对称形式

$$\begin{aligned}\mu + \nu - 1 &= \frac{(\mu - c_1p_1)\dots(\mu - c_np_n)}{\mu^{n-1}} \\ &= \frac{\{\nu - c_1(1-p_1)\}\dots\{\nu - c_n(1-p_n)\}}{\nu^{n-1}}.\end{aligned} \tag{12}$$

Finally,

$$\text{Prob.}\, z = \frac{M-1}{M+N-1} = 1 - \nu. \tag{13}$$

Let us then assume $1 - \nu = u$, we have then

$$\begin{aligned}\mu - u &= \frac{(\mu - c_1p_1)\dots(\mu - c_np_n)}{\mu^{n-1}} \\ &= \frac{\{1 - c_1(1-p_1)\}\dots\{1 - c_n(1-p_n) - u\}}{(1-u)^{n-1}}.\end{aligned}$$

If we make for simplicity

$$c_1p_1 = a_1, \quad c_np_n = a_n, \quad 1 - c_1(1-p_1) = b_1, \text{ \&c.},$$

上述方程可写为如下形式：

$$\mu - u = \frac{(\mu - a_1)\dots(\mu - a_n)}{\mu^{n-1}}, \tag{14}$$

其中

$$\mu = u + \frac{(b_1 - u)\dots(b_n - u)}{(1-u)^{n-1}}. \tag{15}$$

将这个μ的值代入(14)将给出一个只涉及u的方程，其解将确定Prob .z，因为由(13) Prob .z = u。剩下的是确定u的界限。

11. 现在，在 n = 2(XIX.12)的特殊情形中用以确定界限的同一分析，在目前情形下把我们引向以下结果。量 u，为了它能够表示 Prob .z 的值，必须以下列各量为它的下界：a ^{1} , a ^{2} , ...a n，并以下列各量为它的上界：b ^{1} , b ^{2} , ...b n,a ^{1} + a ^{2} ...+ a n。我们由此可以`*a priori*推断，方程(14)将始终存在一个根，而且只有一个根，满足这些条件。我认为，为了实际验证，只需证明在界限 a ^{1} , a ^{2} , ...a n 与 b ^{1} , b ^{2} , ...b n 之间，方程(14)将存在一个、而且只有一个根，便足够了。

First, let us consider the nature of the changes to which μ is subject in (15), as u varies from a_1, which we will suppose the greatest of its minor limits, to b_1, which we will suppose the least of its major limits. When $u = a_1$ it is evident that μ is positive and greater than a_1. When $u = b_1$, we have $\mu = b_1$, which is also positive. Between the limits $u = a_1$, $u = b_1$ it may be shown that μ increases with u. Thus we have

$$\frac{d\mu}{du} = 1 - \frac{(b_2-u)\dots(b_n-u)}{(1-u)^{n-1}} - \frac{(b_1-u)(b_3-u)\dots(b_n-u)}{(1-u)^{n-1}} \dots + (n-1)\frac{(b_1-u)(b_2-u)\dots(b_n-u)}{(1-u)^n}. \tag{16}$$

Now let

$$\frac{b_1-u}{1-u} = x_1 \dots \frac{b_n-u}{1-u} = x_n.$$

Evidently $x_1, x_2, \dots x_n$, will be proper fractions, and we have

$$\begin{aligned}\frac{d\mu}{du} &= 1 - x_2x_3\dots x_n - x_1x_3\dots x_n \dots - x_1x_2\dots x_{n-1} + (n-1)x_1x_2\dots x_n \\ &= 1 - (1-x_1)x_2x_3\dots x_n - x_1(1-x_2)x_3\dots x_n \dots \\ &\quad -x_1x_2\dots x_{n-1}(1-x_n) - x_1x_2\dots x_n.\end{aligned}$$

Now the negative terms in the second member are (if we may borrow the language of the logical developments) *constituents* formed from the fractional quantities $x_1, x_2, \dots x_n$. Their sum cannot therefore exceed unity; whence $\frac{d\mu}{du}$ is positive, and μ increases with u between the limits specified.

Now let (14) be written in the form

$$\frac{(\mu-a_1)\dots(\mu-a_n)}{\mu^{n-1}} - (\mu-u) = 0, \tag{17}$$

and assume $u = a_1$. The first member becomes

$$(\mu-a_1)\left\{\frac{(\mu-a_2)\dots(\mu-a_n)}{\mu^{n-1}} - 1\right\}, \tag{18}$$

and this expression is negative in value. For, making the same assumption in (15), we find

$$\mu - a_1 = \frac{(b_1-u)\dots(b_n-u)}{(1-u)^{n-1}} = \text{ a positive quantity.}$$

At the same time we have

$$\frac{(\mu-a_2)\dots(\mu-a_n)}{\mu^{n-1}} = \frac{\mu-a_2}{\mu}\dots\frac{\mu-a_n}{\mu},$$

and since the factors of the second member are positive fractions, that member is less than unity, whence (18) is negative. Wherefore *the assumption $u = a_1$ makes the first member of (17) negative.*

首先，让我们考虑在(15)中 μ 所经受的变化的性质，当 u 从 a ^{1}(我们将假定它是其较小界限中的最大者)变到 b ^{1}(我们将假定它是其较大界限中的最小者)时。当 u = a ^{1}时，显然 μ 为正且大于 a ^{1}。当 u = b ^{1}时，我们有 μ = b ^{1}，它也为正。在界限 u = a ^{1}、u = b ^{1}之间，可以证明 μ 随 u 增大。于是我们有

$$\frac{d\mu}{du} = 1 - \frac{(b_2-u)\dots(b_n-u)}{(1-u)^{n-1}} - \frac{(b_1-u)(b_3-u)\dots(b_n-u)}{(1-u)^{n-1}} \dots + (n-1)\frac{(b_1-u)(b_2-u)\dots(b_n-u)}{(1-u)^n}. \tag{16}$$

Now let

$$\frac{b_1-u}{1-u} = x_1 \dots \frac{b_n-u}{1-u} = x_n.$$

Evidently $x_1, x_2, \dots x_n$, will be proper fractions, and we have

$$\begin{aligned}\frac{d\mu}{du} &= 1 - x_2x_3\dots x_n - x_1x_3\dots x_n \dots - x_1x_2\dots x_{n-1} + (n-1)x_1x_2\dots x_n \\ &= 1 - (1-x_1)x_2x_3\dots x_n - x_1(1-x_2)x_3\dots x_n \dots \\ &\quad -x_1x_2\dots x_{n-1}(1-x_n) - x_1x_2\dots x_n.\end{aligned}$$

现在第二项中的否定项(如果我们可以借用逻辑展开式的语言)是由分数x ^{1} , x ^{2} , ...x n 构成的成分。因此它们的和不能超过1；由此$^{d\mu}$为正，且μ在指定界限之间随u增大而增大。

现在令(14)写成如下形式

$$\frac{(\mu-a_1)\dots(\mu-a_n)}{\mu^{n-1}} - (\mu-u) = 0, \tag{17}$$

并假设 u = a ^{1}。第一个成员变为

$$(\mu-a_1)\left\{\frac{(\mu-a_2)\dots(\mu-a_n)}{\mu^{n-1}} - 1\right\}, \tag{18}$$

且这个表达式的值为负。因为，在(15)中作同样的假设，我们得到

$$\mu - a_1 = \frac{(b_1-u)\dots(b_n-u)}{(1-u)^{n-1}} = \text{ a positive quantity.}$$

At the same time we have

$$\frac{(\mu-a_2)\dots(\mu-a_n)}{\mu^{n-1}} = \frac{\mu-a_2}{\mu}\dots\frac{\mu-a_n}{\mu},$$

且由于第二项的因子是正分数，该项小于1，因此(18)为负。所以假设 u = a ^{1}使*(17)*的第一项为负。

Secondly, let $u = b_1$, then by (15) $\mu = u = b_1$, and the *first member of (17) becomes positive.*

Lastly, between the limits $u = a_1$ and $u = b_1$ the first member of (17) continuously increases. For the first term of that expression written under the form

$$(\mu - a_1)\frac{\mu - a_1}{\mu} \dots \frac{\mu - a_n}{\mu}$$

increases, since μ increases, and, with it, every factor contained. Again, the negative term $\mu - u$ diminishes with the increase of u, as appears from its value deduced from (15), viz.,

$$\frac{(b_1 - u) \dots (b_n - u)}{(1 - u)^{n-1}}.$$

Hence then, between the limits $u = a_1$, $u = b_1$, the first member of (17) continuously increases, changing in so doing from a negative to a positive value. Wherefore, between the limits assigned, there exists one value of u, and only one, by which the said equation is satisfied.

12. Collecting these results together, we arrive at the following solution of the general problem.

The probability of the event E will be that value of u deduced from the equation

$$\mu - u = \frac{(\mu - c_1 p_1) \dots (\mu - c_n p_n)}{\mu^{n-1}}, \tag{19}$$

wherein

$$\mu = u + \frac{\{1 - c_1(1 - p_1) - u\} \dots \{1 - c_n(1 - p_n) - u\}}{(1 - u)^{n-1}},$$

which (value) lies between the two sets of quantities,

$$c_1 p_1, c_2 p_2, \dots, c_n p_n \text{ and } 1 - c_1(1 - p_1), 1 - c_2(1 - p_2) \dots 1 - c_n(1 - p_n),$$

the former set being its inferior, the latter its superior, limits.

And it may further be inferred in the general case, as it has been proved in the particular case of $n = 2$, that the value of u, determined as above, will not exceed the quantity

$$c_1 p_1 + c_2 p_2 \dots + c_n p_n.$$

13. Particular verifications are subjoined.

1st. Let $p_1 = 1, p_2 = 1, \dots p_n = 1$. This is to suppose it certain, that if any one of the events $A_1, A_2 \dots A_n$ happen, the event E will happen. In this case, then, the probability of the occurrence of E will simply be the probability that the events or causes $A_1, A_2 \dots A_n$ do not all fail of occurring, and its expression will therefore be $1 - (1 - c_1)(1 - c_2) \dots (1 - c_n)$.

Now the general solution (19) gives

$$\mu - u = \frac{(\mu - c_1) \dots (\mu - c_n)}{\mu^{n-1}},$$

其次，令u = b 1，则由(15) μ = u = b 1，且(17)的第一项变为正。

最后，在界限u = a 1和u = b 1之间，(17)的第一项连续增大。因为该表达式写成如下形式的第一项

$$(\mu - a_1)\frac{\mu - a_1}{\mu} \dots \frac{\mu - a_n}{\mu}$$

增大，因为μ增大，且随之所含的每个因子都增大。又，否定项μ – u随u的增大而减小，这从其由(15)推出的值可以看出，即，

$$\frac{(b_1 - u) \dots (b_n - u)}{(1 - u)^{n-1}}.$$

因此，在界限u = a 1,u = b 1之间，(17)的第一项连续增大，在此过程中从负值变为正值。所以，在指定的界限之间，存在一个u的值，且只有一个，使所述方程得到满足。

12. 将这些结果汇集在一起，我们得到一般问题的如下解。

事件E的概率将是从方程中推得的u的那个值

$$\mu - u = \frac{(\mu - c_1 p_1) \dots (\mu - c_n p_n)}{\mu^{n-1}}, \tag{19}$$

wherein

$$\mu = u + \frac{\{1 - c_1(1 - p_1) - u\} \dots \{1 - c_n(1 - p_n) - u\}}{(1 - u)^{n-1}},$$

which (value) lies between the two sets of quantities,

$$c_1 p_1, c_2 p_2, \dots, c_n p_n \text{ and } 1 - c_1(1 - p_1), 1 - c_2(1 - p_2) \dots 1 - c_n(1 - p_n),$$

前一组是它的下极限，后一组是它的上极限。

且在一般情形下还可以推断，正如在n = 2的特殊情形中已证明的那样，如上确定的u的值不会超过量

$$c_1 p_1 + c_2 p_2 \dots + c_n p_n.$$

13. 后附有具体的验证。

第一。设 p 1 = 1 , p 2 = 1 , ...p n = 1。这是要假定：若事件 A 1 , A 2 ...A n 中任何一个发生，则事件 E 必发生，这是确定的。于是，在此情形下，E 发生的概率就简单地是事件或原因 A 1 , A 2 ...A n 并非全部不发生这一情形的概率，因此其表达式将为 1 –(1 – c 1)(1 – c 2) ...(1 – c n)。

现在通解(19)给出

$$\mu - u = \frac{(\mu - c_1) \dots (\mu - c_n)}{\mu^{n-1}},$$

wherein

$$\mu = u + \frac{(1-u)^n}{(1-u)^{n-1}} = 1.$$

Hence,

$$1 - u = (1 - c_1) \dots (1 - c_n),$$
$$\therefore u = 1 - (1 - c_1) \dots (1 - c_n),$$

equivalent to the *à priori* determination above.

2nd. Let $p_1 = 0$, $p_2 = 0$, $p_n = 0$, then (19) gives

$$\mu - u = \mu,$$
$$\therefore u = 0,$$

as it evidently ought to be.

3rd. Let $c_1, c_2 \dots c_n$ be small quantities, so that their squares and products may be neglected. Then developing the second members of the equation (19),

$$\mu - u = \frac{\mu^n - (c_1p_1 + c_2p_2 \dots + c_np_n)\mu^{n-1}}{\mu^{n-1}}$$
$$= \mu - (c_1p_1 + c_2p_2 \dots + c_np_n),$$
$$\therefore u = c_1p_1 + c_2p_2 \dots + c_np_n.$$

Now this is what the solution would be were the causes $A_1, A_2 \dots A_n$ mutually exclusive. But the smaller the probabilities of those causes, the more do they approach the condition of being mutually exclusive, since the smaller is the probability of any concurrence among them. Hence the result above obtained will undoubtedly be the limiting form of the expression for the probability of E.

4th. In the particular case of $n = 2$, we may readily eliminate μ from the general solution. The result is

$$\frac{(u - c_1p_1)(u - c_2p_2)}{c_1p_1 + c_2p_2 - u} = \frac{\{1 - c_1(1 - p_1) - u\}\{1 - c_2(1 - p_2) - u\}}{1 - u},$$

which agrees with the particular solution before obtained for this case, Problem I.

Though by the system (19), the solution is in general made to depend upon the solution of an equation of a high order, its practical difficulty will not be great. For the conditions relating to the limits enable us to select at once a near value of u, and the forms of the system (19) are suitable for the processes of successive approximation.

14. PROBLEM 7.—The data being the same as in the last problem, required the probability, that if any definite and given combination of the causes $A_1, A_2, \dots A_n$, present itself, the event E will be realized.

The cases $A_1, A_2, \dots A_n$, being represented as before by $x_1, x_2, \dots x_n$ respectively, let the definite combination of them, referred to in the statement of

其中

$$\mu = u + \frac{(1-u)^n}{(1-u)^{n-1}} = 1.$$

因此，

$$1 - u = (1 - c_1) \dots (1 - c_n),$$
$$\therefore u = 1 - (1 - c_1) \dots (1 - c_n),$$

等价于上文`*a prior*的确定。

第二，令p ^{1} = 0,p ^{2} = 0,p n = 0，则(19)给出

$$\mu - u = \mu,$$
$$\therefore u = 0,$$

正如它显然应当如此。

第三，令c ^{1} , c ^{2} ...c n为小量，使得它们的平方和乘积可以忽略。然后展开方程(19)的第二项，

$$\mu - u = \frac{\mu^n - (c_1p_1 + c_2p_2 \dots + c_np_n)\mu^{n-1}}{\mu^{n-1}}$$
$$= \mu - (c_1p_1 + c_2p_2 \dots + c_np_n),$$
$$\therefore u = c_1p_1 + c_2p_2 \dots + c_np_n.$$

现在这就是如果原因A ^{1} , A ^{2} ...A n互斥时解将会是的样子。但这些原因的概率越小，它们就越接近互斥的条件，因为它们之间任何同时发生的概率越小。因此，上面得到的结果无疑将是E的概率表达式的极限形式。

第四，在n = 2的特殊情形中，我们可以容易地从一般解中消去μ。结果是

$$\frac{(u - c_1p_1)(u - c_2p_2)}{c_1p_1 + c_2p_2 - u} = \frac{\{1 - c_1(1 - p_1) - u\}\{1 - c_2(1 - p_2) - u\}}{1 - u},$$

which agrees with the particular solution before obtained for this case, Problem I.

虽然通过系统(19)，解一般依赖于一个高阶方程的解，其实际困难不会很大。因为关于界限的条件使我们能够立即选出一个接近的u值，且系统(19)的形式适合于逐次逼近的过程。

14. 问题7。--数据与上一问题相同，要求的是：如果原因A ^{1} , A ^{2} , ...A n的任何确定的给定组合出现，则事件E将实现的概率。

情形A ^{1} , A ^{2} , ...A n，如前分别用x ^{1} , x ^{2} , ...x n表示，令问题陈述中提到的它们的确定组合

the problem, be represented by the $\phi(x_1, x_2 \ldots x_n)$ so that the actual occurrence of that combination will be expressed by the logical equation,

$$\phi(x_1, x_2, \ldots x_n) = 1.$$

The data are

$$\begin{aligned} &\text{Prob. } x_1 = c_1, \ldots \quad \text{Prob. } x_n = c_n, \\ &\text{Prob. } x_1 z = c_1 p_1, \quad \text{Prob. } x_n z = c_n p_n; \end{aligned} \tag{1}$$

and the object of investigation is

$$\frac{\text{Prob. } \phi(x_1, x_2 \ldots x_n) z}{\text{Prob. } \phi(x_1, x_2 \ldots x_n)}. \tag{2}$$

We shall first seek the value of the numerator.

Let us assume,

$$x_1 z = t_1 \ldots x_n z = t_n, \tag{3}$$
$$\phi(x_1, x_2 \ldots x_n) z = w. \tag{4}$$

Or, if for simplicity, we represent $\phi(x_1, x_2 \ldots x_n)$ by ϕ, the last equation will be

$$\phi z = w, \tag{5}$$

to which must be added the equation

$$\bar{x}_1 \bar{x}_2 \ldots \bar{x}_n z = 0. \tag{6}$$

Now any equation $x_r z = t_r$ of the system (3) may be reduced to the form

$$x_r z \bar{t}_r + t_r (1 - x_r z) = 0.$$

Similarly reducing (5), and adding the different results together, we obtain the logical equation

$$\Sigma\{x_r z \bar{t}_r + t_r (1 - x_r z)\} + \bar{x}_1 \ldots \bar{x}_n z + \phi z \bar{w} + w(1 - \phi z) = 0, \tag{7}$$

from which z being eliminated, w must be determined as a developed logical function of $x_1, \ldots x_n, t_1, \ldots t_n$.

Now making successively $z = 1, z = 0$ in the above equation, and multiplying the results together, we have

$$\{\Sigma(x_r \bar{t}_r + \bar{x}_r t_r) + \bar{x}_1 \ldots \bar{x}_n + \phi \bar{w} + w \bar{\phi}\} \times (\Sigma t_r + w) = 0.$$

Developing this equation with reference to w, and replacing in the result $\sum t_r + 1$ by 1, in accordance with Prop. I. Chap. IX., we have

$$Ew + E'(1 - w) = 0;$$

用φ(x 1 , x 2 ...x n)表示，使得该组合的实际出现将由逻辑方程表达，

$$\phi(x_1, x_2, \ldots x_n) = 1.$$

数据是

$$\begin{aligned} &\text{Prob. } x_1 = c_1, \ldots \quad \text{Prob. } x_n = c_n, \\ &\text{Prob. } x_1 z = c_1 p_1, \quad \text{Prob. } x_n z = c_n p_n; \end{aligned} \tag{1}$$

而研究的对象是

$$\frac{\text{Prob. } \phi(x_1, x_2 \ldots x_n) z}{\text{Prob. } \phi(x_1, x_2 \ldots x_n)}. \tag{2}$$

我们先求分子的值。

让我们假设，

$$x_1 z = t_1 \ldots x_n z = t_n, \tag{3}$$
$$\phi(x_1, x_2 \ldots x_n) z = w. \tag{4}$$

或者，为简单起见，若将φ(x 1 , x 2 ...x n)表示为φ，则最后一个方程将是

$$\phi z = w, \tag{5}$$

必须加上方程

$$\bar{x}_1 \bar{x}_2 \ldots \bar{x}_n z = 0. \tag{6}$$

现在系统(3)的任何方程x r z = t r都可以化为形式

$$x_r z \bar{t}_r + t_r (1 - x_r z) = 0.$$

类似地化简(5)，并将不同结果相加，我们得到逻辑方程

$$\Sigma\{x_r z \bar{t}_r + t_r (1 - x_r z)\} + \bar{x}_1 \ldots \bar{x}_n z + \phi z \bar{w} + w(1 - \phi z) = 0, \tag{7}$$

from which z being eliminated, w must be determined as a developed logical function of $x_1, \ldots x_n, t_1, \ldots t_n$.

Now making successively $z = 1, z = 0$ in the above equation, and multiplying the results together, we have

$$\{\Sigma(x_r \bar{t}_r + \bar{x}_r t_r) + \bar{x}_1 \ldots \bar{x}_n + \phi \bar{w} + w \bar{\phi}\} \times (\Sigma t_r + w) = 0.$$

关于w展开这个方程，并在结果中按照第ix章命题i，将P t r +1替换为1，我们有

$$Ew + E'(1 - w) = 0;$$

wherein

$$E = \Sigma(x_r\bar{t}_r + t_r\bar{x}_r) + \bar{x}_1 \dots \bar{x}_n + \bar{\phi},$$
$$E' = \Sigma t_r\{\Sigma(x_r\bar{t}_r + t_r\bar{x}_r) + \bar{x}_1 \dots \bar{x}_n + \phi\}.$$

And hence

$$w = \frac{E'}{E' - E}. \tag{8}$$

The second member of this equation we must now develop with respect to the double series of symbols $x_1, x_2, \dots x_n, t_1, t_2, \dots t_n$. In effecting this object, it will be most convenient to arrange the constituents of the resulting development in three distinct classes, and to determine the coefficients proper to those classes separately.

First, let us consider those constituents of which $\bar{t}_1 \dots \bar{t}_n$ is a factor. Making $t_1 = 0 \dots t_n = 0$, we find

$$E' = 0, \quad E = \Sigma x_r + \bar{x}_1 \dots \bar{x}_n + \bar{\phi}.$$

It is evident, that whatever values $(0, 1)$ are given to the x-symbols, E does not vanish. Hence the coefficients of all constituents involving $\bar{t}_1 \dots \bar{t}_n$ are 0.

Consider secondly, those constituents which do not involve the factor $\bar{t}_1 \dots \bar{t}_n$, and which are symmetrical with reference to the two sets of symbols $x_1 \dots x_n$ and $t_1 \dots t_n$. By symmetrical constituents is here meant those which would remain unchanged if x_1 were converted into t_1, x_2 into t_2, &c., and *vice versâ*. The constituents $x_1 \dots x_n \;\; t_1 \dots t_n, \bar{x}_1 \dots \bar{x}_n \;\; \bar{t}_1 \dots \bar{t}_n$, &c., are in this sense symmetrical. For all symmetrical constituents it is evident that

$$\sum(x_r\bar{t}_r + t_r\bar{x}_r)$$

vanishes. For those which do not involve $\bar{t}_1 \dots \bar{t}_n$, it is further evident that $\bar{x}_1 \dots \bar{x}_n$ also vanishes, whence

$$E = \bar{\phi} \qquad E' = \Sigma t_r(\phi),$$
$$w = \frac{\sum t_r(\phi)}{\sum t_r(\phi) - \bar{\phi}}.$$

For those constituents of which the x-factor is found in ϕ the second member of the above equation becomes 1; for those of which the x-factor is found in $\bar{\phi}$ it becomes 0. Hence *the coefficients of symmetrical constituents not involving* $\bar{t}_1 \dots \bar{t}_n$, *of which the* x*-factor is found in* ϕ *will be 1; of those of which the* x*-factor is not found in* ϕ *it will be 0.*

Consider lastly, those constituents which are unsymmetrical with reference to the two sets of symbols, and which at the same time do not involve $\bar{t}_1 \dots \bar{t}_n$.

Here it is evident, that neither E nor E' can vanish, whence the numerator of the fractional value of w in (8) must exceed the denominator. That value cannot therefore be represented by 1, 0, or $\frac{0}{0}$. It must then, in the logical

其中

$$E = \Sigma(x_r\bar{t}_r + t_r\bar{x}_r) + \bar{x}_1 \dots \bar{x}_n + \bar{\phi},$$
$$E' = \Sigma t_r\{\Sigma(x_r\bar{t}_r + t_r\bar{x}_r) + \bar{x}_1 \dots \bar{x}_n + \phi\}.$$

因此

$$w = \frac{E'}{E' - E}. \tag{8}$$

我们现在必须就记号 x 1 , x 2 , ...x n , t 1 , t 2 , ...t n 的双重级数展开此方程的第二边。为实现这一目标，最方便的做法是把所得展开式的成分分为三个不同的类，并分别确定适于这些类的系数。

首先，让我们考虑那些以 t 1 ... t n为因子的成分。令t 1 = 0 ...t n = 0，我们得到

$$E' = 0, \quad E = \Sigma x_r + \bar{x}_1 \dots \bar{x}_n + \bar{\phi}.$$

显然，无论给x-记号什么值(0 , 1),E都不消失。因此所有涉及 t 1 ... t n的成分的系数都是0。

其次考虑那些不包含因子 t 1 ... t n 的 constituent，它们关于两组记号 x 1 ...x n 与 t 1 ...t n 是对称的。这里所谓对称的 constituent，是指那些在把 x 1换成 t 1、把 x 2换成 t 2等等，以及*vice versˆ a*时保持不变者。constituent x 1 ...x n t 1 ...t n , x 1 ... x n t 1 ... t n 等等，在这个意义上是对称的。对于所有对称的 constituent，显然有

$$\sum(x_r\bar{t}_r + t_r\bar{x}_r)$$

消失。对于那些不涉及 t 1 ... t n的成分，进一步显然 x 1 ... x n也消失，因此

$$E = \bar{\phi} \qquad E' = \Sigma t_r(\phi),$$
$$w = \frac{\sum t_r(\phi)}{\sum t_r(\phi) - \bar{\phi}}.$$

对于那些其 x 因子出现在 φ 中的组成部分，上述方程的第二项变为 1；对于那些其 x 因子出现在 φ 中的组成部分，它变为 0。因此不涉及 t 1 ... t n 的对称组成部分的系数，其 - 因子出现在其中的将为 *1*；其 - 因子不出现在其中的将为 *0*。

最后考虑那些关于两组记号不对称、同时又不涉及 t 1 ... t n的成分。

这里显然，E和E ′都不能消失，因此(8)中w的分数值的分子必须超过分母。所以该值不能用1、0或00表示。那么它在逻辑

development, be represented by $\frac{1}{0}$. Such then will be the coefficient of this class of constituents.

15. Hence the final logical equation by which w is expressed as a developed logical function of $x_1, \dots x_n$, $t_1, \dots t_n$, will be of the form

$$w = \sum\nolimits_1 (XT) + 0\{\sum\nolimits_2 (XT) + \bar{t}_1 \dots \bar{t}_n\} + \frac{1}{0}(\text{sum of other constituents}), \tag{9}$$

wherein $\sum_1(XT)$ represents the sum of all symmetrical constituents of which the factor X is found in ϕ, and $\sum_2(XT)$, the sum of all symmetrical constituents of which the factor X is not found in ϕ,—the constituent $\bar{x}_1 \dots \bar{x}_n\, \bar{t}_1 \dots \bar{t}_n$, should it appear, being in either case rejected.

Passing from Logic to Algebra, it may be observed, that here and in all similar instances, the function V, by the aid of which the algebraic system of equations for the determination of the values of $x_1, \dots x_n, t_1 \dots t_n$ is formed, is independent of the nature of any function ϕ involved, not in the expression of the *data*, but in that of the *quæsitum* of the problem proposed. Thus we have in the present example,

$$\text{Prob. } w = \frac{\sum_1(XT)}{V},$$

wherein
$$V = \sum\nolimits_1 (XT) + \sum\nolimits_2 (XT) + t_1 \dots \bar{t}_n$$
$$= \sum (XT) + \bar{t}_1 \dots t_n. \tag{10}$$

Here $\sum(XT)$ represents the sum of all symmetrical constituents of the x and t symbols, except the constituent $\bar{x}_1 \dots \bar{x}_n$, $\bar{t}_1 \dots \bar{t}_n$. This value of V is the same as that virtually employed in the solution of the preceding problem, and hence we may avail ourselves of the results there obtained.

If then, as in the solution referred to, we assume

$$\frac{x_1 t_1}{\bar{x}_1 \bar{t}_1} = m_1, \quad \frac{x_n t_n}{\bar{x}_n \bar{t}_n} = m_n, \quad \frac{x_1}{\bar{x}_1} = n_1, \text{ \&c.},$$

we shall obtain a result which may be thus written:

$$\text{Prob. } w = \frac{M_1}{M + N - 1}, \tag{11}$$

M_1 being formed by rejecting from the function ϕ the constituent $x_1 \dots \bar{x}_n$, if it is there found, dividing the result by the same constituent $\bar{x}_1 \dots \bar{x}_n$ and then changing $\frac{x_1}{\bar{x}_1}$ into m_1, $\frac{x_2}{\bar{x}_2}$ into m_2, and so on. The values of M and N are the same as in the preceding problem. Reverting to these and to the corresponding values of m_1, m_2, &c., we find

$$\text{Prob. } w = M_1(\mu + \nu - 1),$$

the general values of m_r, n_r being

$$m_r = \frac{c_r p_r}{\mu - c_r p_r}, \qquad n_r = \frac{c_r(1 - p_r)}{\mu - c_r(1 - p_r)},$$

展开式中必须用10表示。这样的那么将是这一类成分的系数。

15. 因此，将 w 表示为 x 1 , ...x n、t 1 , ...t n 的展开逻辑函数的最终逻辑方程，将具有如下形式

$$w = \sum\nolimits_1 (XT) + 0\{\sum\nolimits_2 (XT) + \bar{t}_1 \dots \bar{t}_n\} + \frac{1}{0}(\text{sum of other constituents}), \tag{9}$$

其中 P1(XT) 表示其中因子 X 出现于 φ 中，而 P2(XT) 其和的所有对称成分的其中因子 X 不出现于 φ 中，-- 成分 x 1 ... x n t 1 ... t n，若其出现，则在任一情形下均被舍弃。

从逻辑过渡到代数，可以观察到，在此处以及所有类似情形中，函数 V 借助它形成用于确定 x 1 , ...x n , t 1 ...t n 的值的代数方程组，它独立于所涉及的任何函数 φ 的性质，该函数不在数据的表达中，而在所提问题的所求的表达中。因此我们在本示例中有，

其中
$$\text{Prob. } w = \frac{\sum_1(XT)}{V},$$
$$V = \sum\nolimits_1 (XT) + \sum\nolimits_2 (XT) + t_1 \dots \bar{t}_n$$
$$= \sum (XT) + \bar{t}_1 \dots t_n. \tag{10}$$

这里P(XT) 表示 x 和 t 记号的所有对称组成部分之和，除了组成部分 x 1 ... x n , t 1 ... t n。这个 V 的值与前一问题的解中实际使用的值相同，因此我们可以利用那里获得的结果。

如果那么，正如所引用的解法中那样，我们假设

$$\frac{x_1 t_1}{\bar{x}_1 \bar{t}_1} = m_1, \quad \frac{x_n t_n}{\bar{x}_n \bar{t}_n} = m_n, \quad \frac{x_1}{\bar{x}_1} = n_1, \text{ \&c.},$$

我们将得到一个可如此表述的结果：

$$\text{Prob. } w = \frac{M_1}{M + N - 1}, \tag{11}$$

M 1由从函数 φ 中剔除成分 x 1 ...¯x n(若其中存在该成分)而形成，将结果除以同一成分 x 1 ...¯x n，然后将x1 x 1变为m1,x2¯x 2变为m2,并如此继续。M>与N的值与前一个问题中的相同。回到这些值以及 m 1、m 2等的对应值，我们求得 —

$$\text{Prob. } w = M_1(\mu + \nu - 1),$$

the general values of m_r, n_r being

$$m_r = \frac{c_r p_r}{\mu - c_r p_r}, \qquad n_r = \frac{c_r(1 - p_r)}{\mu - c_r(1 - p_r)},$$

and μ and ν being given by the solution of the system of equations,

$$\mu+\nu-1=\frac{(\mu-c_1p_1)\dots(\mu-c_np_n)}{\mu^{n-1}}=\frac{\{\nu-c_1(1-p_1)\}\dots\{\nu-c_n(1-p_n)\}}{\nu^{n-1}}.$$

The above value of Prob. w will be the numerator of the fraction (2). It now remains to determine its denominator.

For this purpose assume

$$\phi(x_1,x_2\dots x_n)=v,$$

or

$$\phi=v;$$

whence

$$\phi\bar{v}+v\bar{\phi}=0.$$

Substituting the first member of this equation in (7) in place of the corresponding form $\phi z\bar{w}+w(1-\phi z)$ we obtain as the primary logical equation,

$$\sum\{x_rz\bar{t}_r+t_r(1-x_rz)\}+\bar{x}_1\dots\bar{x}_nz+\phi\bar{v}+v\bar{\phi}=0,$$

whence eliminating z, and reducing by Prop. II. Chap. IX.,

$$\phi\bar{v}+v\bar{\phi}+\sum t_r\{\sum(x_r\bar{t}_r+t_r\bar{x}_r)+\bar{x}_1\dots\bar{x}_n\}=0.$$

Hence

$$v=\frac{\phi+\sum t_r\{\sum(x_r\bar{t}_r+t_r\bar{x}_r)+\bar{x}_1\dots\bar{x}_n\}}{2\phi-1}$$

and developing as before,

$$v=\sum\nolimits_1(XT)+\bar{t}_1\dots\bar{t}_n\sum\nolimits_1(X)+0\{\sum\nolimits_2(XT)+\bar{t}_1\dots\bar{t}_n\sum\nolimits_2(X)\}+\frac{1}{0}(\text{sum of other constituents}).\tag{12}$$

Here $\sum_1(X)$ indicates the sum of all constituents found in ϕ, $\sum_2(X)$ the sum of all constituents not found in ϕ. The expressions are indeed used in place of ϕ and $1-\phi$ to preserve symmetry.

It follows hence that $\sum_1(X)+\sum_2(X)=1$, and that, as before, $\sum_1(XT)+\sum_2(XT)=\sum(XT)$. Hence V will have the same value as before, and we shall have

$$\text{Prob. } v=\frac{\sum_1(XT)+\bar{t}_1\dots\bar{t}_n\sum_1(X)}{V},$$

Or transforming, as in the previous case,

$$\text{Prob. } v=\frac{M_1+N_1}{M+N-1},\tag{13}$$

wherein N_1 is formed by dividing ϕ by $\bar{x}_1\dots\bar{x}_n$, and changing in the result $\frac{x_1}{\bar{x}_1}$ into n_1, $\frac{x_2}{\bar{x}_2}$ into n_2, &c.

而 μ 与 ν 由该方程组的解给出，

$$\mu+\nu-1=\frac{(\mu-c_1p_1)\dots(\mu-c_np_n)}{\mu^{n-1}}=\frac{\{\nu-c_1(1-p_1)\}\dots\{\nu-c_n(1-p_n)\}}{\nu^{n-1}}.$$

上述 Prob.w 的值将是分数(2)的分子。现在尚需确定其分母。

为此，假设

$$\phi(x_1,x_2\dots x_n)=v,$$

或

$$\phi=v;$$

由此

$$\phi\bar{v}+v\bar{\phi}=0.$$

将本方程的第一个成员代入(7)中以代替相应的形式 φz¯w + w(1 – φz)，我们得到作为原始逻辑方程，

$$\sum\{x_rz\bar{t}_r+t_r(1-x_rz)\}+\bar{x}_1\dots\bar{x}_nz+\phi\bar{v}+v\bar{\phi}=0,$$

由此消去 z，并依第 IX 章命题 II 化简，

$$\phi\bar{v}+v\bar{\phi}+\sum t_r\{\sum(x_r\bar{t}_r+t_r\bar{x}_r)+\bar{x}_1\dots\bar{x}_n\}=0.$$

Hence

$$v=\frac{\phi+\sum t_r\{\sum(x_r\bar{t}_r+t_r\bar{x}_r)+\bar{x}_1\dots\bar{x}_n\}}{2\phi-1}$$

and developing as before,

$$v=\sum\nolimits_1(XT)+\bar{t}_1\dots\bar{t}_n\sum\nolimits_1(X)+0\{\sum\nolimits_2(XT)+\bar{t}_1\dots\bar{t}_n\sum\nolimits_2(X)\}+\frac{1}{0}(\text{sum of other constituents}).\tag{12}$$

所有不在φ中的成分之和。这些表达式确实在此处被使用 [illegible] 表示在φ中找到的所有成分之和，以及 1−φ 以保持对称性。

Here $\sum_1(X)$ indicates the sum of all constituents found in ϕ, $\sum_2(X)$ the sum of all constituents not found in ϕ. The expressions are indeed used in place of ϕ and $1-\phi$ to preserve symmetry.

It follows hence that $\sum_1(X)+\sum_2(X)=1$, and that, as before, $\sum_1(XT)+\sum_2(XT)=\sum(XT)$. Hence V will have the same value as before, and we shall have

$$\text{Prob. } v=\frac{\sum_1(XT)+\bar{t}_1\dots\bar{t}_n\sum_1(X)}{V},$$

Or transforming, as in the previous case,

$$\text{Prob. } v=\frac{M_1+N_1}{M+N-1},\tag{13}$$

wherein N_1 is formed by dividing ϕ by $\bar{x}_1\dots\bar{x}_n$, and changing in the result $\frac{x_1}{\bar{x}_1}$ into n_1, $\frac{x_2}{\bar{x}_2}$ into n_2, &c.

Now the final solution of the problem proposed will be given by assigning their determined values to the terms of the fraction

$$\frac{\text{Prob. } \phi(x_1, \dots x_n)z}{\text{Prob. } \phi(x_1, \dots x_n)}, \text{or} \frac{\text{Prob. } w}{\text{Prob. } v}.$$

Hence, therefore, by (11) and (13) we have

$$\text{Prob. sought} = \frac{M_1}{M_1 + N_1}$$

A very slight attention to the mode of formation of the functions M_1 and N_1 will show that the process may be greatly simplified. We may, indeed, exhibit the solution of the general problem in the form of a rule, as follows:

Reject from the function $\phi(x_1, x_2 \dots x_n)$ the constituent $\bar{x}_1 \dots \bar{x}_n$ if it is therein contained, suppress in all the remaining constituents the factors $\bar{x}_1$, $\bar{x}_2$, &c., and change generally in the result x_r into $\frac{c_r p_r}{\mu - c_r p_r}$. Call this result M_1.

Again, replace in the function $\phi(x_1, x_2 \dots x_n)$ the constituent $\bar{x}_1 \dots \bar{x}_n$ if is therein found, by unity; suppress in all the remaining constituents the factors $\bar{x}_1$, $\bar{x}_2$, &c., and change generally in the result x_r into $\frac{c_r(1-p_r)}{\nu - c_r(1-p_r)}$.

Then the solution required will be expressed by the formula

$$\frac{M_1}{M_1 + N_1}, \tag{14}$$

μ and ν being determined by the solution of the system of equations

$$\mu + \nu - 1 = \frac{(\mu - c_1 p_1) \dots (\mu - c_n p_n)}{\mu^{n-1}}$$
$$= \frac{\{\nu - c_1(1-p_1)\} \dots \{\mu - c_n(1-p_n)\}}{\nu^{n-1}}. \tag{15}$$

It may be added, that the limits of μ and ν are the same as in the previous problem. This might be inferred from the general principle of continuity; but conditions of limitation, which are probably sufficient, may also be established by other considerations.

Thus from the demonstration of the general method in probabilities, Chap. XVII. Prop,IV., it appears that the quantities $x_1, \dots x_n, t_1, \dots t_n$ in the primary system of algebraic equations, must be *positive proper fractions.* Now

$$\frac{x_r}{1 - x_r} = n_r = \frac{c_r(1-p_r)}{\nu - c_r(1-p_r)}.$$

Hence generally n_r must be a positive quantity, and therefore we must have

$$\nu \overset{=}{>} c_r(1-p_r).$$

In like manner since we have

$$\frac{x_r t_r}{(1-x_r)(1-t_r)} = m_r = \frac{c_r p_r}{\mu - c_r p_r},$$

现在，所提问题的最终解将由赋予分数各项以其确定的值而给出

$$\frac{\text{Prob. } \phi(x_1, \dots x_n)z}{\text{Prob. } \phi(x_1, \dots x_n)}, \text{or} \frac{\text{Prob. } w}{\text{Prob. } v}.$$

因此，由(11)和(13)可得

$$\text{Prob. sought} = \frac{M_1}{M_1 + N_1}$$

稍加注意函数 M 1与 N 1的构成方式，即可看出该过程可大大简化。事实上，我们可以将一般问题的解表述为如下规则的形式：

Reject from the function $\phi(x_1, x_2 \dots x_n)$ the constituent $\bar{x}_1 \dots \bar{x}_n$ if it is therein contained, suppress in all the remaining constituents the factors $\bar{x}_1$, $\bar{x}_2$, &c., and change generally in the result x_r into $\frac{c_r p_r}{\mu - c_r p_r}$. Call this result M_1.

再次，在函数 φ(x 1 , x 2 ...x n) 中替换成分 x 1 ...x n 若其中找到，则以单位替换；在所有剩余成分中消去因子 x 1, x 2， 等等，并在结果中一般地将 c r (1 – p r)

x_2, &c., and change generally in the result x_r into $\frac{c_r(1-p_r)}{\nu - c_r(1-p_r)}$.
Then the solution required will be expressed by the formula

$$\frac{M_1}{M_1 + N_1}, \tag{14}$$

μ 并由该方程组的解确定

$$\mu + \nu - 1 = \frac{(\mu - c_1 p_1) \dots (\mu - c_n p_n)}{\mu^{n-1}}$$
$$= \frac{\{\nu - c_1(1-p_1)\} \dots \{\mu - c_n(1-p_n)\}}{\nu^{n-1}}. \tag{15}$$

可以补充说，μ 与 v 的界限与前一问题中相同。这可以从连续性的一般原理推出；但那些很可能充分的限制条件，也可以通过其他考虑来建立。

因此，从概率中一般方法的论证(第十七章命题iv.)可以看出，原始代数方程组中的量 x 1 , ...x n , t 1 , ...t n 必须是正的真分数。现在

$$\frac{x_r}{1 - x_r} = n_r = \frac{c_r(1-p_r)}{\nu - c_r(1-p_r)}.$$

Hence generally n_r must be a positive quantity, and therefore we must have

$$\nu \overset{=}{>} c_r(1-p_r).$$

以同样的方式，既然我们有

$$\frac{x_r t_r}{(1-x_r)(1-t_r)} = m_r = \frac{c_r p_r}{\mu - c_r p_r},$$

we must have generally

$$\mu \overset{=}{>} c_r p_r.$$

16. It is probable that the two classes of conditions thus represented are together sufficient to determine generally which of the roots of the equations determining μ and ν are to be taken. Let us take in particular the case in which $n = 2$. Here we have

$$\mu + \nu - 1 = \frac{(\mu - c_1 p_1)(\mu - c_2\ p_2)}{\mu} = \mu - (c_1 p_1 + c_2 p_2) + \frac{c_1 p_1 c_2 p_2}{\mu},$$

$$\therefore \nu = 1 - c_1 p_1 - c_2 p_2 + \frac{c_1 p_1\ c_2 p_2}{\mu} = 1 - c_1 p_1 - \frac{(\mu - c_1 p_1) c_2 p_2}{\mu}.$$

Whence, since $\mu \overset{=}{>} c_1 p_1$ we have generally

$$\nu \overset{=}{<} 1 - c_1 p_1.$$

In like manner we have

$$\nu \overset{=}{<} 1 - c_2 p_2, \quad \mu \overset{=}{<} 1 - c_1(1 - p_1), \quad \mu \overset{=}{<} 1 - c_2(1 - p_2).$$

Now it has already been shown that there will exist but one value of μ satisfying the whole of the above conditions relative to that quantity, viz.

$$\mu \overset{=}{>} c_r p_r, \quad \mu \overset{=}{<} 1 - c_r(1 - p_r),$$

whence the solution for this case, at least, is determinate. And I apprehend that the same method is generally applicable and sufficient. But this is a question upon which a further degree of light is desirable.

To verify the above results, suppose $\phi(x_1, \dots x_n) = 1$, which is virtually the case considered in the previous problem. Now the development of 1 gives all possible constituents of the symbols $x_1, \dots x_n$. Proceeding then according to the Rule, we find

$$M_1 = \frac{\mu^n}{(\mu - c_1 p_1) \dots (\mu - c_n p_n)} - 1 = \frac{\mu}{\mu + \nu - 1} - 1 \text{ by (15).}$$

$$N_1 = \frac{\nu^n}{\{\nu - c_1(1 - p_1)\} \dots \{\nu - c_n(1 - p_n)\}} - 1 = \frac{\nu}{\mu + \nu - 1} - 1.$$

Substituting in (14) we find

$$\text{Prob. } z = 1 - \nu,$$

which agrees with the previous solution.

Again, let $\phi(x_1, \dots x_n) = x_1$, which, after development and suppression of the factors $\bar{x}_2, \dots \bar{x}_n$, gives $x_1(x_2 + 1) \dots (x_n + 1)$, whence we find

$$M_1 = \frac{c_1 p_1 \mu^{n-1}}{(\mu - c_1 p_1) \dots (\mu - c_n p_n)} = \frac{c_1 p_1}{\mu + \nu - 1} \text{ by (15).}$$

$$N_1 = \frac{c_1(1 - p_1)\nu^{n-1}}{\{\nu - c_1(1 - p_1)\} \dots \{\nu - c_n(1 - p_n)\}} = \frac{c_1(1 - p_1)}{\mu + \nu - 1}.$$

~~一般来说，~~我们必须有

$$\mu \overset{=}{>} c_r p_r.$$

16. 很可能这样表示的两类条件合在一起足以一般地确定在决定 μ 与 ν 的方程的根中应取哪一个。让我们特别考虑 n = 2 的情形。此处我们有

$$\mu + \nu - 1 = \frac{(\mu - c_1 p_1)(\mu - c_2\ p_2)}{\mu} = \mu - (c_1 p_1 + c_2 p_2) + \frac{c_1 p_1 c_2 p_2}{\mu},$$

$$\therefore \nu = 1 - c_1 p_1 - c_2 p_2 + \frac{c_1 p_1\ c_2 p_2}{\mu} = 1 - c_1 p_1 - \frac{(\mu - c_1 p_1) c_2 p_2}{\mu}.$$

Whence, since $\mu \overset{=}{>} c_1 p_1$ we have generally

$$\nu \overset{=}{<} 1 - c_1 p_1.$$

In like manner we have

$$\nu \overset{=}{<} 1 - c_2 p_2, \quad \mu \overset{=}{<} 1 - c_1(1 - p_1), \quad \mu \overset{=}{<} 1 - c_2(1 - p_2).$$

Now it has already been shown that there will exist but one value of μ satisfying the whole of the above conditions relative to that quantity, viz.

$$\mu \overset{=}{>} c_r p_r, \quad \mu \overset{=}{<} 1 - c_r(1 - p_r),$$

因此至少对于这一情形，解是确定的。并且我认为同样的方法一般地适用且充分。但这是一个尚需进一步阐明的问题。

为验证上述结果，设 φ(x ^{1} , ...x n) = 1，这实际上就是前一问题中所考虑的情形。现在 1 的展开式给出记号 x ^{1} , ...x n 的一切可能的成分。于是按照规则进行，我们得到

$$M_1 = \frac{\mu^n}{(\mu - c_1 p_1) \dots (\mu - c_n p_n)} - 1 = \frac{\mu}{\mu + \nu - 1} - 1 \text{ by (15).}$$

$$N_1 = \frac{\nu^n}{\{\nu - c_1(1 - p_1)\} \dots \{\nu - c_n(1 - p_n)\}} - 1 = \frac{\nu}{\mu + \nu - 1} - 1.$$

代入(14)我们得到

$$\text{Prob. } z = 1 - \nu,$$

which agrees with the previous solution.

Again, let $\phi(x_1, \dots x_n) = x_1$, which, after development and suppression of the factors $\bar{x}_2, \dots \bar{x}_n$, gives $x_1(x_2 + 1) \dots (x_n + 1)$, whence we find

$$M_1 = \frac{c_1 p_1 \mu^{n-1}}{(\mu - c_1 p_1) \dots (\mu - c_n p_n)} = \frac{c_1 p_1}{\mu + \nu - 1} \text{ by (15).}$$

$$N_1 = \frac{c_1(1 - p_1)\nu^{n-1}}{\{\nu - c_1(1 - p_1)\} \dots \{\nu - c_n(1 - p_n)\}} = \frac{c_1(1 - p_1)}{\mu + \nu - 1}.$$

Substituting, we have

Probability that if the event A_1, occur, E will occur $= p_1$.

And this result is verified by the data. Similar verifications might easily be added.

Let us examine the case in which

$$\phi(x_1, \dots x_n) = x_1\bar{x}_2 \dots \bar{x}_n + x_2\bar{x}_1\bar{x}_3 \dots \bar{x}_n \dots + x_n\bar{x}_1 \dots \bar{x}_{n-1}.$$

Here we find

$$M_1 = \frac{c_1p_1}{\mu - c_1p_1} \dots + \frac{c_np_n}{\mu - c_np_n},$$
$$N_1 = \frac{c_1(1-p_1)}{\nu - c_1(1-p_1)} \dots + \frac{c_n(1-p_n)}{\nu - c_n(1-p_n)};$$

whence we have the following result—

$$\left.\begin{array}{l}\text{Probability that if some one alone of the causes } A_1, A_2 \dots A_n \\ \text{present itself, the event } E \text{ will follow.}\end{array}\right\} = \frac{\sum \frac{c_rp_r}{\mu - c_rp_r}}{\sum \frac{c_rp_r}{\mu - c_rp_r} + \sum \frac{c_r(1-p_r)}{\nu - c_r(1-p_r)}}$$

Let it be observed that this case is quite different from the well-known one in which the mutually exclusive character of the causes $A_1, \dots A_n$ is one of the elements of the data, expressing a condition under which the very observations by which the probabilities of A_1, A_2, &c. are supposed to have been determined, were made.

Consider, lastly, the case in which $\phi(x_1, \dots x_n) = x_1x_2 \dots x_n$. Here

$$M_1 = \frac{c_1p_1 \dots c_np_n}{(\mu - c_1p_1) \dots (\mu - c_np_n)} = \frac{c_1p_1 \dots c_np_n}{\mu^{n-1}(\mu + \nu - 1)},$$
$$N_1 = \frac{c_1(1-p_1) \dots c_n(1-p_n)}{\{\nu - c_1(1-p_1)\} \dots \{\nu - c_n(1-p_n)\}} = \frac{c_1(1-p_1) \dots c_n(1-p_n)}{\nu^{n-1}(\mu + \nu - 1)}.$$

Hence the following result—

$$\left.\begin{array}{l}\text{Probability that if all the causes} \\ A_1, \dots A_n \text{ conspire, the event } E \\ \text{will follow.}\end{array}\right\} = \frac{p_1 \dots p_n\nu^{n-1}}{p_1 \dots p_n\nu^{n-1} + (1-p_1) \dots (1-p_n)\mu^{n-1}}.$$

This expression assumes, as it ought to do, the value 1 when any one of the quantities $p_1, \dots p_n$ is equal to 1.

17. PROBLEM VIII.—Certain causes $A_1, A_2, \dots A_n$ being so restricted that they cannot all fail, but still can only occur in certain definite combinations denoted by the equation

$$\phi(A_1, A_2 \dots A_n) = 1,$$

代入，我们得到

若事件 A 1发生，则 E 将发生的概率 = p 1 .

这一结果由数据得到验证。类似的验证可以很容易地补充。

让我们考察这样的情况：

$$\phi(x_1, \dots x_n) = x_1\bar{x}_2 \dots \bar{x}_n + x_2\bar{x}_1\bar{x}_3 \dots \bar{x}_n \dots + x_n\bar{x}_1 \dots \bar{x}_{n-1}.$$

此处我们发现

$$M_1 = \frac{c_1p_1}{\mu - c_1p_1} \dots + \frac{c_np_n}{\mu - c_np_n},$$
$$N_1 = \frac{c_1(1-p_1)}{\nu - c_1(1-p_1)} \dots + \frac{c_n(1-p_n)}{\nu - c_n(1-p_n)};$$

由此我们得到以下结果--

$$\left.\begin{array}{l}\text{Probability that if some one alone of the causes } A_1, A_2 \dots A_n \\ \text{present itself, the event } E \text{ will follow.}\end{array}\right\} = \frac{\sum \frac{c_rp_r}{\mu - c_rp_r}}{\sum \frac{c_rp_r}{\mu - c_rp_r} + \sum \frac{c_r(1-p_r)}{\nu - c_r(1-p_r)}}$$

请注意，这一情形与众所周知的情形大不相同，在后一情形中，原因 A 1 , ...A n 的互斥性质是数据的一个元素，表达了一种条件，据以进行那些据以确定 A 1 , A 2 ， 等等的概率的观察。

Consider, lastly, the case in which $\phi(x_1, \dots x_n) = x_1x_2 \dots x_n$. Here

$$M_1 = \frac{c_1p_1 \dots c_np_n}{(\mu - c_1p_1) \dots (\mu - c_np_n)} = \frac{c_1p_1 \dots c_np_n}{\mu^{n-1}(\mu + \nu - 1)},$$
$$N_1 = \frac{c_1(1-p_1) \dots c_n(1-p_n)}{\{\nu - c_1(1-p_1)\} \dots \{\nu - c_n(1-p_n)\}} = \frac{c_1(1-p_1) \dots c_n(1-p_n)}{\nu^{n-1}(\mu + \nu - 1)}.$$

由此得出以下结果--

$$\left.\begin{array}{l}\text{Probability that if all the causes} \\ A_1, \dots A_n \text{ conspire, the event } E \\ \text{will follow.}\end{array}\right\} = \frac{p_1 \dots p_n\nu^{n-1}}{p_1 \dots p_n\nu^{n-1} + (1-p_1) \dots (1-p_n)\mu^{n-1}}.$$

这一表达式如其所应然地，当量 p 1 , ...p n 中任何一个等于 1 时，取值为 1。

17. 问题 VIII.--某些原因 A 1 , A 2 , ...A n 受到如此限制，以致它们不能全部不成立，但仍然只能以由方程所表示的某些确定组合出现

$$\phi(A_1, A_2 \dots A_n) = 1,$$

and there being given the separate probabilities $c_1, \dots c_n$ of the said causes, and the corresponding probabilities $p_1, \dots p_n$ that an event E will follow if those respective causes are realized, required the probability of the event E.

This problem differs from the one last considered in several particulars, but chiefly in this, that the restriction denoted by the equation $\phi(A_1, \dots A_n) = 1$, forms one of the data, and is supposed to be furnished by or to be accordant with the very experience from which the knowledge of the numerical elements of the problem is derived.

Representing the events $A_1, \dots A_n$ by $x_1, \dots x_n$ respectively, and the event E by z, we have—

$$\text{Prob.}\, x_r = c_r, \quad \text{Prob.}\, x_r z = c_r p_r. \tag{1}$$

Let us assume, generally,

$$x_r z = t_r,$$

then combining the system of equations thus indicated with the equations

$$\bar{x}_1 \dots \bar{x}_n = 0, \quad \phi(x_1, \dots x_n) = 1, \quad \text{or } \phi = 1,$$

furnished in the data, we ultimately find, as the developed expression of z,

$$z = \sum(XT) + 0\bar{t}_1\bar{t}_2 \dots \bar{t}_n \sum(X), \tag{2}$$

where X represents in succession each constituent found in ϕ, and T a similar series of constituents of the symbols $t_1, \dots t_n$; $\sum(XT)$ including only *symmetrical* constituents with reference to the two sets of symbols.

The method of reduction to be employed in the present case is so similar to the one already exemplified in former problems, that I shall merely exhibit the results to which it leads. We find

$$\text{Prob.}\, z = \frac{M}{M+N} \tag{3}$$

with the relations

$$\frac{M_1}{c_1 p_1} \dots = \frac{M_n}{c_n p_n} = \frac{N_1}{c_1(1-p_1)} = \frac{N_n}{c_n(1-p_n)} = M + N. \tag{4}$$

Wherein M is formed by suppressing in $\phi(x_2, \dots x_n)$ all the factors $\bar{x}_1, \dots \bar{x}_n$, and changing in the result x_1 into m_1, x_n into m_n, while N is formed by substituting in M, n_1 for m_1, &c.; moreover M_1 consists of that portion of M of which m_1 is a factor, N_1 of that portion of N of which n_1 is a factor; and so on.

Let us take, in illustration, the particular case in which the causes $A_1 \dots A_n$ are mutually exclusive. Here we have

$$\phi(x_1, \dots x_n) = x_1\bar{x}_2 \dots \bar{x}_n \dots + x_n\bar{x}_1 \dots \bar{x}_{n-1}.$$

并且给出了上述原因的各自概率 c ^{1} , ...c n，以及相应的概率 p ^{1} , ...p n，即若那些各自的原因实现，事件 E 将随之发生，求事件 E 的概率。

这一问题在若干细节上与刚才考虑的问题不同，但主要在于：由方程 φ(A ^{1} , ...A n) = 1 所表示的限制构成数据之一，并被假定为由据以获得问题数值元素知识的那个经验所提供或与之相符。

Representing the events $A_1, \dots A_n$ by $x_1, \dots x_n$ respectively, and the event E by z, we have—

$$\text{Prob.}\, x_r = c_r, \quad \text{Prob.}\, x_r z = c_r p_r. \tag{1}$$

让我们一般地假设，

$$x_r z = t_r,$$

然后将由此指示的方程组与这些方程结合起来

$$\bar{x}_1 \dots \bar{x}_n = 0, \quad \phi(x_1, \dots x_n) = 1, \quad \text{or } \phi = 1,$$

在数据中给出，我们最终得到作为 z 的展开表达式，

$$z = \sum(XT) + 0\bar{t}_1\bar{t}_2 \dots \bar{t}_n \sum(X), \tag{2}$$

其中X依次代表在φ中发现的每个构成成分，而T代表记号t1、...、tn的类似构成成分序列；P(XT)仅包含关于两组记号的对称构成成分。本案例中要采用的归约方法与先前问题中已示例的方法如此相似，以至于我仅展示其所导出的结果。我们发现

$$\text{Prob.}\, z = \frac{M}{M+N} \tag{3}$$

与这些关系

$$\frac{M_1}{c_1 p_1} \dots = \frac{M_n}{c_n p_n} = \frac{N_1}{c_1(1-p_1)} = \frac{N_n}{c_n(1-p_n)} = M + N. \tag{4}$$

其中 M 是通过在 φ(x ^{2} , ...x n) 中消去所有因子 x ^{1} , ...¯x n 而形成的，并在结果中将 x ^{1} 变为 m ^{1} ，将 x n 变为 m n，而 N 是通过在 M 中用 n ^{1}替换 m ^{1}等而形成的；此外 M ^{1}由 M 中以 m ^{1}为因子的那部分构成，N ^{1}由 N 中以 n ^{1}为因子的那部分构成；如此

让我们举例说明原因 A ^{1} ...A n 互斥的特定情形。此处我们有

$$\phi(x_1, \dots x_n) = x_1\bar{x}_2 \dots \bar{x}_n \dots + x_n\bar{x}_1 \dots \bar{x}_{n-1}.$$

Whence

$$M = m_1 + m_2 \ldots + m_n,$$
$$N = n_1 + n_2 \ldots + n_n,$$
$$M_1 = m_1, N_1 = n_1, \text{ \&c.}$$

Substituting, we have

$$\frac{m_1}{c_1 p_1} \ldots = \frac{m_n}{c_n p_n} = \frac{n_1}{c_1(1-p_1)} \ldots = \frac{n_n}{c_n(1-p_n)} = M + N.$$

Hence we find

$$\frac{m_1 + m_2 \ldots + m_n}{c_1 p_1 + c_2 p_2 \ldots + c_n p_n} = M + N,$$

or

$$\frac{M}{c_1 p_1 \ldots + c_n p_n} = M + N.$$

Hence, by (3),

$$\text{Prob. } z = c_1 p_1 \ldots + c_n p_n,$$

a known result.

There are other particular cases in which the system (4) admits of ready solution. It is, however, obvious that in most instances it would lead to results of great complexity. Nor does it seem probable that the existence of a functional relation among causes, such as is assumed in the data of the general problem, will often be presented in actual experience; if we except only the particular cases above discussed.

Had the general problem been modified by the restriction that the event E cannot occur, all the causes $A_1 \ldots A_n$ being absent, instead of the restriction that the said causes cannot all fail, the remaining condition denoted by the equation $\phi(A_1, \ldots A_n) = 1$ being retained, we should have found for the final logical equation

$$z = \sum_1 (XT) + 0 \sum (X),$$

$\sum(X)$ being, as before, equal to $\phi(x_1, \ldots x_n)$, but $\sum_1(XT)$ formed by rejecting from ϕ the particular constituent $\bar{x_1} \ldots \bar{x_n}$ if therein contained, and then multiplying each x-constituent of the result by the corresponding t-constituent. It is obvious that in the particular case in which the causes are mutually exclusive the value of Prob. z hence deduced will be the same as before.

18. PROBLEM IX.—Assuming the data of any of the previous problems, let it be required to determine the probability that if the event E present itself, it will be associated with the particular cause A_r; in other words, to determine the *à posteriori* probability of the cause A_r when the event E has been observed to occur.

In this case we must seek the value of the fraction

$$\frac{\text{Prob. } x_r z}{\text{Prob. } z}, \text{ or } \frac{c_r p_r}{\text{Prob. } z}, \text{ by the data.} \qquad (1)$$

由此

$$M = m_1 + m_2 \ldots + m_n,$$
$$N = n_1 + n_2 \ldots + n_n,$$
$$M_1 = m_1, N_1 = n_1, \text{ \&c.}$$

代入，我们得到

$$\frac{m_1}{c_1 p_1} \ldots = \frac{m_n}{c_n p_n} = \frac{n_1}{c_1(1-p_1)} \ldots = \frac{n_n}{c_n(1-p_n)} = M + N.$$

Hence we find

$$\frac{m_1 + m_2 \ldots + m_n}{c_1 p_1 + c_2 p_2 \ldots + c_n p_n} = M + N,$$

or

$$\frac{M}{c_1 p_1 \ldots + c_n p_n} = M + N.$$

Hence, by (3),

$$\text{Prob. } z = c_1 p_1 \ldots + c_n p_n,$$

一个已知的结果。

还有其他特定情形使方程组(4)易于求解。然而显然，在大多数情况下它会导出极其复杂的结果。而且，原因之间存在函数关系(如一般问题的数据中所假定的那样)的情形，似乎也不太可能在实际经验中经常出现；除非我们只考虑上面讨论过的那些特定情形。

如果一般问题被修改为限制事件 E 不能发生，所有原因 A 1 ...A n 均不存在，而不是限制所述原因不能全部失败，保留由方程 φ(A 1 , ...A n) = 1 表示的剩余条件，我们本应求得最终逻辑方程为

$$z = \sum_1 (XT) + 0 \sum (X),$$

$\sum(X)$ being, as before, equal to $\phi(x_1, \ldots x_n)$, but $\sum_1(XT)$ formed by rejecting from ϕ the particular constituent $\bar{x_1} \ldots \bar{x_n}$ if therein contained, and then

将结果的每个 x -成分乘以相应的 t -成分。显然，在原因互斥的特定情形中，由此推出的 Prob.z 的值将与前面相同。

18. 问题 IX.--假定前述任一问题的数据，要求确定若事件 E 出现，它将与特定原因 A r 相关联的概率；换言之，确定当观察到事件 E 发生时原因 A r 的`后验概率。

在这种情况下，我们必须求该分数的值

$$\frac{\text{Prob. } x_r z}{\text{Prob. } z}, \text{ or } \frac{c_r p_r}{\text{Prob. } z}, \text{ by the data.} \qquad (1)$$

As in the previous problems, the value of Prob. z has been assigned upon different hypotheses relative to the connexion or want of connexion of the causes, it is evident that in all those cases the present problem is susceptible of a determinate solution by simply substituting in (1) the value of that element thus determined.

If the *à priori* probabilities of the causes are equal, we have $c_1 = c_2 \ldots = c_r$. Hence for the different causes the value (1) will vary directly as the quantity p_r. Wherefore *whatever the nature of the connexion among the causes*, the *à posteriori* probability of each cause will be proportional to the probability of the observed event E when that cause is known to exist. The particular case of this theorem, which presents itself when the causes are mutually exclusive, is well known. We have then

$$\frac{\text{Prob. } x_r z}{\text{Prob. } z} = \frac{c_r p_r}{\sum c_r p_r} = \frac{p_r}{p_1 + p_2 \ldots + p_n},$$

the values of $c_1, \ldots c_n$ being equal.

Although, for the demonstration of these and similar theorems in the particular case in which the causes are mutually exclusive, it is not necessary to introduce the functional symbol ϕ, which is, indeed, to claim for ourselves the choice of all possible and conceivable hypotheses of the connexion of the causes, yet, under every form, the solution by the method of this work of problems, in which the number of the data is indefinitely great, must always partake of a somewhat complex character. Whether the systematic evolution which it presents, first, of the logical, secondly, of the numerical relations of a problem, furnishes any compensation for the length and occasional tediousness of its processes, I do not presume to inquire. Its chief value undoubtedly consists in its power,—in the mastery which it gives us over questions which would apparently baffle the unassisted strength of human reason. For this cause it has not been deemed superfluous to exhibit in this chapter its application to problems, some of which may possibly be regarded as repulsive, from their difficulty, without being recommended by any prospect of immediate utility. Of the ulterior value of such speculations it is, I conceive, impossible for us, at present, to form any decided judgment.

19. The following problem is of a much easier description than the previous ones.

PROBLEM X.—*The probability of the occurrence of a certain natural phænomenon under given circumstances is p. Observation has also recorded a probability a of the existence of a permanent cause of that phænomenon, i.e. of a cause which would always produce the event under the circumstances supposed. What is the probability that if the phænomenon is observed to occur n times in succession under the given circumstances, it will occur the* $n + 1^{th}$ *time? What also is the probability, after such observation, of the existence of the permanent cause referred to?*

FIRST CASE.—Let t represent the existence of a permanent cause, and $x_1, x_2 \ldots x_{n+1}$ the successive occurrences of the natural phænomenon.

如前几个问题中那样，Prob .z 的值已就原因的联系或缺乏联系的不同假说而给定，显然在所有这些情形中，本问题都可通过简单地将如此确定的该元素的值代入(1)而得到确定的解。

如果原因的`先验概率相等，我们有 c 1 = c 2 ...= c r。因此对于不同原因，值(1)将直接随量 p r 变化。因此无论原因之间联系的性质如何，每个原因的`后验概率将与该原因已知存在时观察到事件 E 的概率成正比。这个定理的特殊情况，即当原因互斥时呈现的情况，是众所周知的。我们于是有

$$\frac{\text{Prob. } x_r z}{\text{Prob. } z} = \frac{c_r p_r}{\sum c_r p_r} = \frac{p_r}{p_1 + p_2 \ldots + p_n},$$

the values of $c_1, \ldots c_n$ being equal.

尽管在原因互斥的特殊情形下，为证明这些及类似定理，并不需要引入函数记号 φ--这实际上等于宣称我们有权选择关于原因联系的一切可能且可设想的假说--然而，无论以何种形式，用本著作的方法求解那些数据数目无限多的问题，总不免带有某种复杂的性质。它首先呈现逻辑关系、其次呈现数值关系的那种系统展开，是否足以补偿其过程的冗长与时而令人厌倦，我不敢妄加探究。其 chief value 无疑在于它的力量--在于它赋予我们对那些显然非人类理性独力所能及的问题的驾驭能力。因此，在本章中展示它对这些问题的应用，并不被认为是多余的，其中有些问题可能因其困难而被视为令人望而却步，而又无任何眼前效用的前景可资推荐。至于这类思辨的最终价值，我认为我们目前不可能作出任何确定的判断。

19. 下述问题比前几个问题容易得多。

问题 X.- 在给定情况下某一自然现象发生的概率为 。观察还记录到该现象存在一种永久原因的概率，即一种在假设的情况下总会产生该事件的原因的概率。如果在给定情况下观察到该现象连续发生次，那么它将在第 n + 1 次发生的概率是多少?在这样观察之后，所指的永久原因存在的概率又是多少？

第一种情形。--设 t 表示一个永久原因的存在，而 x 1 , x 2 ...x n +1表示自然现象的相继发生。

If the permanent cause exist, the events $x_1, x_2 \ldots x_{n+1}$ are necessary consequences. Hence

$$t = vx_1, \quad t = vx_2, \text{ \&c.},$$

and eliminating the indefinite symbols,

$$t(1 - x_1) = 0, \quad t(1 - x_2) = 0, \quad t(1 - x_{n+1}) = 0.$$

Now we are to seek the probability that if the combination $x_1 x_2 \ldots x_n$ happen, the event x_{n+1} will happen, i.e. we are to seek the value of the fraction

$$\frac{\text{Prob. } x_1 x_2 \ldots x_{n+1}}{\text{Prob. } x_1 x_2 \ldots x_n}.$$

We will first seek the value of Prob. $x_1 x_2 \ldots x_n$. Represent the combination $x_1\, x_2 \ldots x_n$ by w, then we have the following logical equations:

$$t(1 - x_1) = 0, \quad t(1 - x_2) = 0, \ldots t(1 - x_n) = 0,$$
$$x_1\, x_2 \ldots x_n = w.$$

Reducing the last to the form

$$(x_1\, x_2 \ldots x_n)(1 - w) + w(1 - x_1\, x_2 \ldots x_n) = 0,$$

and adding it to the former ones, we have

$$\sum t(1 - x_i) + x_1\, x_2 \ldots x_n(1 - w) + w(1 - x_1\, x_2 \ldots x_n) = 0, \qquad (1)$$

wherein $\sum$ extends to all values of i from 1 to n, for the one logical equation of the data. With this we must connect the numerical conditions,

$$\text{Prob. } x_1 = \text{ Prob. } x_2 \ldots = \text{ Prob. } x_n = p, \quad \text{Prob. } t = a;$$

and our object is to find Prob, w.

From (1) we have

$$w = \frac{\sum t(1 - x_i) + x_1\, x_2 \ldots x_n}{2x_1\, x_2 \ldots x_n - 1}$$
$$= \frac{\sum(1 - x_i) + x_1\, x_2 \ldots x_n}{2x_1\, x_2 \ldots x_n - 1} t + \frac{x_1\, x_2 \ldots x_n}{2x_1\, x_2 \ldots x_n - 1}(1 - t), \qquad (2)$$

on developing with respect to t. This result must further be developed with respect to $x_1, x_2, \ldots x_n$.

Now if we make $x_1 = 1, x_2 = 1, \ldots x_n = 1$, the coefficients both of t and of $1 - t$ become 1. If we give to the same symbols any other set of values formed by the interchange of 0 and 1, it is evident that the coefficient of t will become negative, while that of $1 - t$ will become 0. Hence the full development (2) will be

$$w = x_1 x_2 \ldots x_n t + x_1 x_2 \ldots x_n(1 - t) + 0(1 - x_1 x_2 \ldots x_n)(1 - t)$$
$$+ \text{ constituents whose coefficients are } \frac{1}{0}, \text{ or equivalent to} \frac{1}{0}.$$

If the permanent cause exist, the events $x_1, x_2 \ldots x_{n+1}$ are necessary consequences. Hence

$$t = vx_1, \quad t = vx_2, \text{ \&c.},$$

并消去不定符号，

$$t(1 - x_1) = 0, \quad t(1 - x_2) = 0, \quad t(1 - x_{n+1}) = 0.$$

Now we are to seek the probability that if the combination $x_1 x_2 \ldots x_n$ happen, the event x_{n+1} will happen, i.e. we are to seek the value of the fraction

$$\frac{\text{Prob. } x_1 x_2 \ldots x_{n+1}}{\text{Prob. } x_1 x_2 \ldots x_n}.$$

我们将首先寻求 Prob .x 1 x 2 ...x n 的值。用 w 表示组合 x 1 x 2 ...x n，则我们有以下逻辑方程：

$$t(1 - x_1) = 0, \quad t(1 - x_2) = 0, \ldots t(1 - x_n) = 0,$$
$$x_1\, x_2 \ldots x_n = w.$$

将后者化为以下形式

$$(x_1\, x_2 \ldots x_n)(1 - w) + w(1 - x_1\, x_2 \ldots x_n) = 0,$$

and adding it to the former ones, we have

$$\sum t(1 - x_i) + x_1\, x_2 \ldots x_n(1 - w) + w(1 - x_1\, x_2 \ldots x_n) = 0, \qquad (1)$$

其中P扩展到从 1 到 n 的 i 的所有值，对于数据的一个逻辑方程。与此我们必须连接数值条件，

$$\text{Prob. } x_1 = \text{ Prob. } x_2 \ldots = \text{ Prob. } x_n = p, \quad \text{Prob. } t = a;$$

而我们的目标是求 Prob, w。

由(1)可得

$$w = \frac{\sum t(1 - x_i) + x_1\, x_2 \ldots x_n}{2x_1\, x_2 \ldots x_n - 1}$$
$$= \frac{\sum(1 - x_i) + x_1\, x_2 \ldots x_n}{2x_1\, x_2 \ldots x_n - 1} t + \frac{x_1\, x_2 \ldots x_n}{2x_1\, x_2 \ldots x_n - 1}(1 - t), \qquad (2)$$

on developing with respect to t. This result must further be developed with respect to $x_1, x_2, \ldots x_n$.

现在如果我们令 x 1 = 1 , x 2 = 1 , ...x n = 1，则 t 和 1 – t 的系数都变为 1。如果我们给相同的记号任何其他由 0 和 1 互换形成的值集，显然 t 的系数将变为否定，而 1 – t 的系数将变为 0。因此完整的展开式(2)将

是

$$w = x_1 x_2 \ldots x_n t + x_1 x_2 \ldots x_n(1 - t) + 0(1 - x_1 x_2 \ldots x_n)(1 - t)$$
$$+ \text{ constituents whose coefficients are } \frac{1}{0}, \text{ or equivalent to} \frac{1}{0}.$$

Here we have

$$V = x_1x_2\ldots x_nt + x_1x_2\ldots x_n(1-t) + (1-x_1x_2\ldots x_n)(1-t)$$
$$= x_1x_2\ldots x_nt + 1 - t;$$

whence, passing from Logic to Algebra,

$$\frac{x_1x_2\ldots x_nt + x_1(1-t)}{p} = \frac{x_1x_2\ldots x_nt + x_2(1-t)}{p}\ldots$$
$$= \frac{x_1x_2\ldots x_nt + x_n(1-t)}{p} = \frac{x_1x_2\ldots x_nt}{a} = x_1x_2\ldots x_nt + 1 - t.$$
$$\text{Prob. } w = \frac{x_1x_2\ldots x_n}{x_1x_2\ldots x_nt + 1 - t}.$$

From the forms of the above equations it is evident that we have $x_1 = x_2\ldots = x_n$. Replace then each of these quantities by x, and the system becomes

$$\frac{x^nt + (1-t)x}{p} = \frac{x^nt}{a} = x^nt + 1 - t,$$
$$\text{Prob. } w = \frac{x^n}{x^nt + 1 - t};$$

from which we readily deduce

$$\text{Prob. } w = \text{ Prob. } x_1x_2\ldots x_n = a + (p-a)\left(\frac{p-a}{1-a}\right)^{n-1}$$

If in this result we change n into $n+1$, we get

$$\text{Prob. } x_1x_2\ldots x_{n+1} = a + (p-a)\left(\frac{p-a}{1-a}\right)^{n}$$

Hence we find—

$$\frac{\text{Prob. } x_1x_2\ldots x_{n+1}}{\text{Prob. } x_1x_2\ldots x_n} = \frac{a + (p-a)\left(\frac{p-a}{1-a}\right)^{n}}{a + (p-a)\left(\frac{p-a}{1-a}\right)^{n-1}} \qquad (3)$$

as the expression of the probability that if the phænomenon be n times repeated, it will also present itself the $n+1^{th}$ time. By the method of Chapter XIX. it is found that a cannot exceed p in value.

The following verifications are obvious:—

1st. If $a = 0$, the expression reduces to p, as it ought to do. For when it is certain that no permanent cause exists, the successive occurrences of the phænomenon are independent.

2nd. If $p = 1$, the expression becomes 1, as it ought to do.

3rd. If $p = a$, the expression becomes 1, unless $a = 0$. If the probability of a phænomenon is equal to the probability that there exists a cause which

这里我们有

$$V = x_1x_2\ldots x_nt + x_1x_2\ldots x_n(1-t) + (1-x_1x_2\ldots x_n)(1-t)$$
$$= x_1x_2\ldots x_nt + 1 - t;$$

由此，从逻辑学过渡到代数学，

$$\frac{x_1x_2\ldots x_nt + x_1(1-t)}{p} = \frac{x_1x_2\ldots x_nt + x_2(1-t)}{p}\ldots$$
$$= \frac{x_1x_2\ldots x_nt + x_n(1-t)}{p} = \frac{x_1x_2\ldots x_nt}{a} = x_1x_2\ldots x_nt + 1 - t.$$
$$\text{Prob. } w = \frac{x_1x_2\ldots x_n}{x_1x_2\ldots x_nt + 1 - t}.$$

从上述方程的形式可以明显看出，我们有 x 1 = x 2 ...= x n。然后将这些量中的每一个替

$$\frac{x^nt + (1-t)x}{p} = \frac{x^nt}{a} = x^nt + 1 - t,$$
$$\text{Prob. } w = \frac{x^n}{x^nt + 1 - t};$$

由此我们容易推得

$$\text{Prob. } w = \text{ Prob. } x_1x_2\ldots x_n = a + (p-a)\left(\frac{p-a}{1-a}\right)^{n-1}$$

若在此结果中我们将 n 改为 n + 1，则得到

$$\text{Prob. } x_1x_2\ldots x_{n+1} = a + (p-a)\left(\frac{p-a}{1-a}\right)^{n}$$

因此我们得到--

$$\frac{\text{Prob. } x_1x_2\ldots x_{n+1}}{\text{Prob. } x_1x_2\ldots x_n} = \frac{a + (p-a)\left(\frac{p-a}{1-a}\right)^{n}}{a + (p-a)\left(\frac{p-a}{1-a}\right)^{n-1}} \qquad (3)$$

作为如果现象重复 n 次，它也将出现第 n + 1 th次的概率的表达式。通过第十九章的方法发现 a 的值不能超过 p。

以下验证是明显的：- 第一。如果 a = 0，表达式简化为 p，正如它应该的那样。因为当确定不存在永久原因时，现象的连续出现是独立的。

第二。若 p = 1，则该表达式变为 1，正如它应当如此。

第三。如果 p = a，表达式变为 1，除非 a = 0。如果一个现象的概率等于存在一个原因的概率，该原因

under given circumstances would always produce it, then the fact that that phænomenon has ever been noticed under those circumstances, renders certain its re-appearance under the same.[5]

4th. As n increases, the expression approaches in value to unity. This indicates that the probability of the recurrence of the event increases with the frequency of its successive appearances,—a result agreeable to the natural laws of expectation.

SECOND CASE.—We are now to seek the probability *à posteriori* of the existence of a permanent cause of the phænomenon. This requires that we ascertain the value of the fraction

$$\frac{\text{Prob. } tx_1x_2\ldots x_n}{\text{Prob. } x_1x_2\ldots x_n}$$

the denominator of which has already been determined.

To determine the numerator assume

$$tx_1x_2\ldots x_n = w,$$

then proceeding as before, we obtain for the logical development,

$$w = tx_1x_2\ldots x_n + 0(1-t).$$

Whence, passing from Logic to Algebra, we have at once

$$\text{Prob. } w = a,$$

a result which might have been anticipated. Substituting then for the numerator and denominator of the above fraction their values, we have for the *à posteriori* probability of a permanent cause, the expression

$$\frac{a}{a + (p-a)\left(\frac{p-a}{1-a}\right)^{n-1}}.$$

It is obvious that the value of this expression increases with the value of n.

I am indebted to a learned correspondent,[6] whose original contributions to the theory of probabilities have already been referred to, for the following verification of the first of the above results (3).

[5] As we can neither re-enter nor recall the state of infancy, we are unable to say how far such results as the above serve to explain the confidence with which young children connect events whose association they have once perceived. But we may conjecture, generally, that the strength of their expectations is due to the necessity of inferring (as a part of their rational nature), and the narrow but impressive experience upon which the faculty is exercised. Hence the reference of every kind of sequence to that of cause and effect. A little friend of the author's, on being put to bed, was heard to ask his brother the pertinent question,—"Why does going to sleep at night make it light in the morning?" The brother, who was a year older, was able to reply, that it would be light in the morning even if little boys did not go to sleep at night.

[6] Professor Donkin.

在给定情况下总会产生它，那么该现象曾在那些情况下被注意到这一事实，就使其在相同情况下必定重新出现。[5]

第四，随着 n 增大，该表达式的值趋近于 1。这表明事件再次发生的概率随其连续出现次数的增加而增大--这一结果符合期望的自然规律。

第二种情形。--我们现在要寻求该现象存在一种永久原因的后验概率`*a posteriori*。这要求我们确定该分数的值

$$\frac{\text{Prob. } tx_1x_2\ldots x_n}{\text{Prob. } x_1x_2\ldots x_n}$$

其分母已经确定。

为确定分子，假设

$$tx_1x_2\ldots x_n = w,$$

然后按照前面的方法进行，我们得到逻辑展开式，

$$w = tx_1x_2\ldots x_n + 0(1-t).$$

由此，从逻辑学过渡到代数学，我们立刻就有

$$\text{Prob. } w = a,$$

这一结果本可预料。于是，将上述分数的分子和分母代以其值，我们得到永久原因的后验`*a posterior*概率的表达式

$$\frac{a}{a + (p-a)\left(\frac{p-a}{1-a}\right)^{n-1}}.$$

It is obvious that the value of this expression increases with the value of n.

我感谢一位博学的通信者[6]，他对概率论的原创性贡献前已提及，提供了对上述第一个结果(3)的如下验证。

5 我们既不能重新进入也不能回忆婴儿期的状态，因此无法说明上述结果在多大程度上有助于解释幼童为何如此确信地连接他们一旦感知到其关联的事件。但我们可以一般地推测，他们期望的强度源于推理的必然性(作为其理性本性的一部分)，以及该能力所运用的狭窄却深刻的印象经验。因此，一切序列都被归因于因果序列。作者的一位小朋友在被放上床睡觉时，有人听到他向他哥哥提出那个切中要害的问题--"为什么晚上睡觉会使早晨天亮?"比他大一岁的哥哥能够回答说，即使小男孩们晚上不睡觉，早晨也会天亮。

6 多金教授。

"The whole *à priori* probability of the event (under the circumstances) being p, and the probability of some cause C which would necessarily produce it, a, let x be the probability that it will happen if no such cause as C exist. Then we have the equation

$$p = a + (1-a)x,$$

whence

$$x = \frac{p-a}{1-a}.$$

Now the phænomenon observed is the occurrence of the event n times. The *à priori* probability of this would be—

$$\begin{aligned} &1 \text{ supposing } C \text{ to exist,} \\ &x^n \text{ supposing } C \text{ not to exist;} \end{aligned}$$

whence the *à posteriori* probability that C exists is

$$\frac{a}{a+(1-a)x^n},$$

that C does not exist is

$$\frac{(1-a)x^n}{a+(1-a)x^n}.$$

Consequently the probability of another occurrence is

$$\frac{a}{a+(1-a)x^n} \times 1 + \frac{(1-a)x^n}{a+(1-a)x^n} \times a,$$

or

$$\frac{a+(1-a)x^{n+1}}{a+(1-a)x^n},$$

which, on replacing n by its value $\frac{p-a}{1-a}$, will be found to agree with (3)."

Similar verifications might, it is probable, also be found for the following results, obtained by the direct application of the general method.

The probability, under the same circumstances, that if, out of n occasions, the event happen r times, and fail $n-r$ times, it will happen on the $n+1^{th}$ time is

$$\frac{a + m(p-a)\left(\frac{p-la}{1-a}\right)^r}{a + m(p-la)\left(\frac{p-la}{1-a}\right)^{r-1}}$$

wherein $m = \frac{n(n-1)\ldots n-r+1}{1\,.\,2\ldots r}$ and $l = \frac{r}{n}$.

The probability of a permanent cause (r being less than n) is 0. This is easily verified.

"该事件(在此情况下)的整个先验`*a priori*概率为p，某个必然产生它的原因C的概率为a，设x为若不存在C这样的原因时它将会发生的概率。于是我们有方程

$$p = a + (1-a)x,$$

whence

$$x = \frac{p-a}{1-a}.$$

Now the phænomenon observed is the occurrence of the event n times. The *à priori* probability of this would be—

1 假设存在，
x ⁿ假设 C 不存在；

由此，C 存在的`*a posteriori*概率是

$$\frac{a}{a+(1-a)x^n},$$

不存在,这是

$$\frac{(1-a)x^n}{a+(1-a)x^n}.$$

因此，再次发生的概率是

$$\frac{a}{a+(1-a)x^n} \times 1 + \frac{(1-a)x^n}{a+(1-a)x^n} \times a,$$

或

$$\frac{a+(1-a)x^{n+1}}{a+(1-a)x^n},$$

将n代以其值$^{p-a}$后，

$\overline{1-a}$,将被发现与(3) 相符 ."

对于通过直接应用一般方法所得到的下列结果，很可能也可以找到类似的验证。

在相同情况下，若在n次场合中事件发生r次、未发生n – r次，则它在第n + 1次发生的概率为

time is

$$\frac{a + m(p-a)\left(\frac{p-la}{1-a}\right)^r}{a + m(p-la)\left(\frac{p-la}{1-a}\right)^{r-1}}$$

wherein $m = \frac{n(n-1)\ldots n-r+1}{1\,.\,2\ldots r}$ and $l = \frac{r}{n}$.

The probability of a permanent cause (r being less than n) is 0. This is easily verified.

If p be the probability of an event, and c the probability that if it occur it will be due to a permanent cause; the probability after n successive observed occurrences that it will recur on the $n+1^{th}$ similar occasion is

$$\frac{c+(1-c)x^n}{c+(1-c)x^{n-1}},$$

wherein $x=\dfrac{p(1-c)}{1-cp}$.

20. It is remarkable that the solutions of the previous problems are void of any arbitrary element. We should scarcely, from the appearance of the data, have anticipated such a circumstance. It is, however, to be observed, that in all those problems the probabilities of the *causes* involved are supposed to be known *à priori*. In the absence of this assumed element of knowledge, it seems probable that arbitrary constants would *necessarily* appear in the final solution. Some confirmation of this remark is afforded by a class of problems to which considerable attention has been directed, and which, in conclusion, I shall briefly consider. It has been observed that there exists in the heavens a large number of double stars of extreme closeness. Either these apparent instances of connexion have some physical ground or they have not. If they have not, we may regard the phenomenon of a double star as the accidental result of a "random distribution" of stars over the celestial vault, i.e. of a distribution which would render it just as probable that either member of the binary system should appear in one spot as in another. If this hypothesis be assumed, and if the number of stars of a requisite brightness be known, we can determine what is the probability that two of them should be found within such limits of mutual distance as to constitute the observed phenomenon. Thus Mitchell,[7] estimating that there are 230 stars in the heavens equal in brightness to β Capricorni, determines that it is 80 to 1 against such a combination being presented were those stars distributed at random. The probability, when such a combination has been observed, that there exists between its members a physical ground of connexion, is then required.

Again, the sum of the inclinations of the orbits of the ten known planets to the plane of the ecliptic in the year 1801 was 91°·4187, according to the French measures. Were all inclinations equally probable, Laplace[8] determines, that there would be only the excessively small probability .00000011235 that the mean of the inclinations should fall within the limit thus assigned. And he hence concludes, that there is a very high probability in favour of a disposing cause, by which the inclinations of the planetary orbits have been confined within such narrow bounds. Professor De Morgan,[9] taking the sum of the inclinations at 92°, gives to the above probability the value .00000012, and infers that "it is 1: .00000012, that there was a necessary cause in the formation of the solar system for the inclinations being what they are." An equally determinate

[7]Phil. Transactions, An. 1767.
[8]Théorie Analytique des Probabilités, p. 276.
[9]Encyclopedia Metropolitana. Art. Probabilities.

若p为某事件的概率，c为若该事件发生则它将归因于永久原因的概率；则在观察到n次连续发生后，它在第n + 1次类似场合重现的概率为

$$\frac{c+(1-c)x^n}{c+(1-c)x^{n-1}},$$

wherein $x=\dfrac{p(1-c)}{1-cp}$.

20. 值得注意的是，前述问题的解答不含任何任意元素。仅从数据的外观来看，我们几乎不会预料到这种情况。然而应当指出，在所有这些题目中，所涉及的原因的概率都被假定为先验地已知。在缺乏这一被假定的知识元素的情况下，似乎很可能任意常数会必然地出现在最终解答中。有一类问题为这一评述提供了一些佐证，这类问题曾引起相当多的关注，最后我将简要地加以考察。人们已经注意到，天空中存在大量极其靠近的双星。这些表面上的联系实例要么有某种物理根据，要么没有。如果没有，我们可以把双星这一现象视为恒星在天穹上"随机分布"的偶然结果，也就是说，这样一种分布会使双星系统中任一成员出现在某一位置与出现在另一位置具有同等的概率。如果采用这一假说，并且如果已知具有所需亮度的恒星数目，我们就能确定其中两颗恒星彼此距离近到足以构成所观察到的现象的概率。于是，米切尔[7]估计天上有230颗亮度与β摩羯座星相等的恒星，并断定：若这些恒星随机分布，则出现这样一种组合的概率为80比1。于是，在观察到这样一种组合之后，就需要求出其成员之间存在物理联系根据的概率。

再者，根据法国度量，1801 年十颗已知行星轨道对黄道面的倾角之和为 91 ° · 4187。拉普拉斯[8]断定，若所有倾角都同等可能，则倾角平均值落在如此指定范围内的概率仅为极其微小的 .00000011235。他由此得出结论：有很高的概率支持存在某种安排性的原因，使行星轨道的倾角被限制在如此狭窄的界限之内。德摩根教授[9]取倾角之和为 92 °，赋予上述概率值 .00000012，并推断"太阳系形成时存在一个必然原因使倾角成为现在这样，其概率为 1: .00000012。"一个同样确定的

7《哲学汇刊》，1767年。
8《概率的分析理论》，第276页。
9《大都会百科全书》。条目"概率"。

conclusion has been drawn from observed coincidences between the direction of circular polarization in rock-crystal, and that of certain oblique faces in its crystalline structure.[10]

These problems are all of a similar character. A certain hypothesis is framed, of the various possible consequences of which we are able to assign the probabilities with perfect rigour. Now some actual result of observation being found among those consequences, and its *hypothetical* probability being therefore known, it is required thence to determine the probability of the hypothesis assumed, or its contrary. In Mitchell's problem, the hypothesis is that of a "random distribution of the stars,"—the possible and observed consequence, the appearance of a close double star. The very small probability of such a result is held to imply that the probability of the hypothesis is equally small, or, at least, of the same order of smallness. And hence the high and, and as some think, *determinate* probability of a disposing cause in the stellar arrangements is inferred. Similar remarks apply to the other examples adduced.

21. The general problem, in whatsoever form it may be presented, admits only of an *indefinite* solution. Let x represent the proposed hypothesis, y a phænomenon which might occur as one of its possible consequences, and whose calculated probability, on the assumption of the truth of the hypothesis, is p, and let it be required to determine the probability that if the phænomenon y is observed, the hypothesis x is true. The very data of this problem cannot be expressed without the introduction of an arbitrary element. We can only write

$$\text{Prob.}\, x = a, \quad \text{Prob.}\, xy = ap; \tag{1}$$

a being perfectly arbitrary, except that it must fall within the limits 0 and 1 inclusive. If then P represent the conditional probability sought, we have

$$P = \frac{\text{Prob.}\, xy}{\text{Prob.}\, y} = \frac{ap}{\text{Prob.}\, y}. \tag{2}$$

It remains then to determine Prob. y. Let $xy = t$, then

$$y = \frac{t}{x} = tx + \frac{1}{0}t(1-x) + 0(1-t)x + \frac{0}{0}(1-t)(1-x). \tag{3}$$

Hence observing that Prob. $x = a$, Prob. $t = ap$, and passing from Logic to Algebra, we have

$$\text{Prob.}\, y = \frac{tx + c(1-t)x}{tx + 1 - t},$$

with the relations

$$\frac{tx + (1-t)x}{a} = \frac{tx}{ap} = tx + 1 - t.$$

Hence we readily find

$$\text{Prob.}\, y = ap + c(1-a). \tag{4}$$

[10]Edinburgh Review, No. 185, p. 32. This article, though not entirely free from error, is well worthy of attention.

结论是从岩晶中圆偏振方向与其晶体结构中某些斜面的方向之间所观察到的巧合中得出的。[10]

这些题目都具有相似的性质。人们构设某个假说，对于它的各种可能后果，我们能够以完全的严格性赋予概率。现在，如果在那些后果中发现了某个实际的观察结果，因而其假定的概率已知，就需要由此确定所假定的假说或其反面的概率。在米切尔的题目中，假说是"恒星的随机分布"，可能且被观察到的后果是出现一颗靠近的双星。这种结果极小的概率被认为意味着该假说的概率同样小，或至少属于同一小量级。于是便推断出恒星安排中存在安排性原因的高概率，而且--如某些人所认为--是确定的概率。类似的评述适用于所举的其他例子。

21. 这个一般性问题，无论以何种形式提出，都只容许不确定的解答。设 x 表示所提出的假说，y 表示一个可能作为其可能后果之一而出现的现象，其在假定该假说为真的情况下的计算概率为 p，并要求确定：如果观察到现象 y，则假说 x 为真的概率。这个问题的数据本身若不引入一个任意元素就无法表达。我们只能写出

$$\text{Prob.}\, x = a, \quad \text{Prob.}\, xy = ap; \tag{1}$$

a完全任意，只是它必须落在0与1(含)的界限之内。于是若P表示所求的条件概率，我们有

$$P = \frac{\text{Prob.}\, xy}{\text{Prob.}\, y} = \frac{ap}{\text{Prob.}\, y}. \tag{2}$$

于是剩下要确定Prob .y。设xy = t，则

$$y = \frac{t}{x} = tx + \frac{1}{0}t(1-x) + 0(1-t)x + \frac{0}{0}(1-t)(1-x). \tag{3}$$

Hence observing that Prob. $x = a$, Prob. $t = ap$, and passing from Logic to Algebra, we have

$$\text{Prob.}\, y = \frac{tx + c(1-t)x}{tx + 1 - t},$$

with the relations

$$\frac{tx + (1-t)x}{a} = \frac{tx}{ap} = tx + 1 - t.$$

因此我们容易发现

$$\text{Prob.}\, y = ap + c(1-a). \tag{4}$$

[10]Edinburgh Review, No. 185, p. 32. This article, though not entirely free from error, is well worthy of attention.

Now recurring to (3), we find that c is the probability, that if the event $(1-t)(1-x)$ occur, the event y will occur. But

$$(1-t)(1-x) = (1-xy)(1-x) = 1-x.$$

Hence c is the probability *that if the event x do not occur, the event y will occur.*

Substituting the value of Prob. y in (2), we have the following theorem:

The calculated probability of any phænomenon y, upon an assumed physical hypothesis x, being p, the à posteriori probability P of the physical hypothesis, when the phænomenon has been observed, is expressed by the equation

$$P = \frac{ap}{ap + c(1-a)}, \tag{5}$$

where a and c are arbitrary constants, the former representing the à priori probability of the hypothesis, the latter the probability that if the hypothesis were false, the event y would present itself.

The principal conclusion deducible from the above theorem is that, other things being the same, the value of P increases and diminishes simultaneously with that of p. Hence the greater or less the probability of the phænomenon when the *hypothesis* is *assumed*, the greater or less is the probability of the hypothesis when the *phænomenon* has been *observed*. When p is very small, then generally P also is small, unless either a is large or c small. Hence, secondly, if the probability of the phænomenon is very small when the hypothesis is assumed, the probability of the hypothesis is very small when the phænomenon is observed, unless either the *à priori* probability a of the hypothesis is large, or the probability of the phænomenon upon any other hypothesis small.

The formula (5) admits of exact verification in various cases, as when $c = 0$, or $a = 1$, or $a = 0$. But it is evident that it does not, unless there be means for determining the values of a and c, yield a *definite* value of P. Any solutions which profess to accomplish this object, either are erroneous in principle, or involve a tacit assumption respecting the above arbitrary elements. Mr. De Morgan's solution of Laplace's problem concerning the existence of a determining cause of the narrow limits within which the inclinations of the planetary orbits to the plane of the ecliptic are confined, appears to me to be of the latter description. Having found a probability $p = .00000012$, that the sum of the inclinations would be less than 92° were all degrees of inclination equally probable in each orbit, this able writer remarks: "If there be a reason for the inclinations being as described, the probability of the event is 1. Consequently, it is $1 : .00000012$ (i.e. $1 : p$), that there was a necessary cause in the formation of the solar system for the inclinations being what they are." Now this result is what the equation (5) would really give, if, assigning to p the above value, we should assume $c = 1$, $a = \frac{1}{2}$. For we should thus find,

$$P = \frac{\frac{1}{2}p}{\frac{1}{2}p + \frac{1}{2}} = \frac{p}{1+p}$$
$$\therefore 1 - P : P :: 1 : p. \tag{6}$$

Now recurring to (3), we find that c is the probability, that if the event $(1-t)(1-x)$ occur, the event y will occur. But

$$(1-t)(1-x) = (1-xy)(1-x) = 1-x.$$

Hence c is the probability *that if the event x do not occur, the event y will occur.*

将Prob .y的值代入(2)，我们得到如下定理：在某一假定的物理假说下，任何现象的计算概率为，当该现象已被观察到时，该物理假说的后验概率由以下方程表示

$$P = \frac{ap}{ap + c(1-a)}, \tag{5}$$

其中和是任意常数，前者表示该假说的先验概率，后者表示若该假说为假时该事件将会出现的概率。

从上述定理可推出的主要结论是：在其他情况相同的条件下，P 的值与 p 的值同时增大和减小。因此，在假定假说时现象的概率越大或越小，在已观察到现象时假说的概率也就越大或越小。当 p 很小时，通常 P 也很小，除非 a 很大或 c 很小。因此，其次，如果在假定假说时现象的概率很小，那么在观察到现象时假说的概率就很小，除非假说的先验概率 a 很大，或者现象在任何其他假说下的概率很小。

公式(5)在多种情形下可得到精确验证，例如当c = 0，或a = 1，或a = 0时。但显然，除非有确定a与c之值的手段，它并不能给出P的确定值。任何声称能实现这一目标的解法，要么在原则上错误，要么包含关于上述任意元素的默认假定。德摩根先生对拉普拉斯问题的解法--该问题涉及行星轨道对黄道面倾角被限制在狭窄范围内的确定原因是否存在--在我看来属于后一类。在求得概率p = .00000012，即在每个轨道中所有倾角程度等可能时倾角之和小于92 °的概率之后，这位能干的作者评论道："如果倾角如所述那样有其原因，则该事件的概率为1。因此，太阳系形成时存在一个必然原因使倾角成为现在这样，其概率为1: .00000012(即1: p)。"现在，这个结果正是方程(5)在将上述值赋予p、并假定c = 1、a = 12.因为我们应当如此求得，

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$$P = \frac{\frac{1}{2}p}{\frac{1}{2}p + \frac{1}{2}} = \frac{p}{1+p}$$
$$\therefore 1 - P : P :: 1 : p. \tag{6}$$

But P representing the probability, *à posteriori*, that all inclinations are equally probable, $1-P$ is the probability, *à posteriori*, that such is not the case, or, adopting Mr. De Morgan's alternative, that a determining cause exists. The equation (6), therefore, agrees with Mr. De Morgan's result.

22. Are we, however, justified in assigning to a and c particular values? I am strongly disposed to think that we are not. The question is of less importance in the special instance than in its ulterior bearings. In the received applications of the theory of probabilities, arbitrary constants do not explicitly appear; but in the above, and in many other instances sanctioned by the highest authorities, some virtual determination of them has been attempted. And this circumstance has given to the results of the theory, especially in reference to questions of causation, a character of definite precision, which, while on the one hand it has seemed to exalt the dominion and extend the province of numbers, even beyond the measure of their ancient claim to rule the world;[11] on the other hand has called forth vigorous protests against their intrusion into realms in which conjecture is the only basis of inference. The very fact of the appearance of arbitrary constants in the solutions of problems like the above, treated by the method of this work, seems to imply, that definite solution is impossible, and to mark the point where inquiry ought to stop. We possess indeed the means of interpreting those constants, but the experience which is thus indicated is as much beyond our reach as the experience which would preclude the necessity of any attempt at solution whatever.

Another difficulty attendant upon these questions, and inherent, perhaps, in the very constitution of our faculties, is that of precisely defining what is meant by Order. The manifestations of that principle, except in very complex instances, we have no difficulty in detecting, nor do we hesitate to impute to it an almost necessary foundation in causes operating under Law. But to assign to it a standard of *numerical* value would be a vain, not to say a presumptuous, endeavour. Yet must the attempt be made, before we can aspire to weigh with accuracy the probabilities[12] [13] of different constitutions of the universe, so as to determine the elements upon which alone a definite solution of the problems in question can be established.

23. The most usual mode of endeavouring to evade the *necessary* arbitrariness of the solution of problems in the theory of probabilities which rest upon insufficient data, is to assign to some element whose real probability is unknown all possible degrees of probability; to suppose that these degrees of probability are themselves equally probable; and, regarding them as so many distinct causes of the phenomenon observed, to apply the theorems which represent the case of an effect due to some one of a number of equally probable but mutually exclusive causes (Problem 9). For instance, the rising of the sun after a certain interval

[11] Mundum regunt numeri.

[12] Original text was "probabibilities" and was fixed in 2004 by Distributed Proofreaders.

[13] The following footnote was in the original text but was not referenced in the text, so it is referenced here[14] in 2004 by Distributed Proofreaders.

[14] See an interesting paper by Prof. Forbes in the Philosophical Magazine, Dec. 1850; also Mill's Logic, chap, xviii.

但P表示所有倾角都同等可能的`*a posteriori*概率，1 – P则表示并非如此的`*a posteriori*概率，或者，采用德摩根先生的择一说法，即存在一个决定性的原因。因此，方程(6)与德摩根先生的结果一致。

22. 然而，我们有权将特定值赋予a与c吗?我强烈倾向于认为我们无权这样做。这一问题在特殊实例中的重要性不如在其深远影响中的重要性。在概率论已被接受的应用中，任意常数并不显式出现；但在上述情形以及许多其他得到最高权威认可的情形中，人们曾试图对它们作出某种实质性的确定。而这一情况赋予了该理论的结果--尤其是在涉及因果关系的问题上--一种确定精确的性质，这种性质一方面似乎抬高了数的统治地位并扩展了数的领域，甚至超出了其自古以来统治世界的权限；[11]另一方面则激起了强烈的抗议，反对它们侵入那些唯有猜测才是推论基础的领域。像上述这样用本书方法处理的问题，其解法中出现任意常数这一事实本身，似乎就意味着确定解是不可能的，并标志着探究应当停止之处。我们确实拥有解释这些常数的手段，但如此指示的经验，与那种将排除任何求解尝试之必要性的经验一样，远非我们所能企及。

伴随这些问题而来的另一困难--也许是我们官能构造本身所固有的--是精确界定所谓秩序意味着什么。除了在非常复杂的情形中，我们不难察觉该原则的表现，也不犹豫将其归因于在法则下起作用的原因中几乎必然的基础。但要给它指定一个数值价值的标准，则将是徒劳的，甚至可以说是僭妄的努力。然而，在我们能够期望准确权衡宇宙不同构成的概率[12][13]，从而确定唯有据此才能对所论问题建立确定解的元素之前，这一尝试必须作出。

23. 在概率论中，对于建立在不足数据之上的问题，要设法回避其解法之必然任意性，最常用的方式是将所有可能的概率程度赋予某个真实概率未知的元素；假定这些概率程度本身是等可能的；并把它们视为所观察现象的众多不同原因，进而应用那些代表由若干等可能但相互排斥的原因之一所致结果之情形的定理(问题9)。例如，太阳在一定间隔之后升起

11 数字统治世界。

12 原文为"probabibilities",2004年由Distributed Proofreaders修正。

13 以下脚注原在原文中但正文未引用，故此处引用[14],2004年由Distributed Proofreaders处理。

14 参见福布斯教授(Prof.Forbes)在《哲学杂志》(Philosophical Magazine)1850年12月号上的一篇有趣论文；另见密尔《逻辑学》(Mill's Logic)第十八章。

of darkness having been observed m times in succession, the probability of its again rising under the same circumstances is determined, on received principles, in the following manner. Let p be any unknown probability between 0 and 1, and c (infinitesimal and constant) the probability, that the probability of the sun's rising after an interval of darkness lies between the limits p and $p + dp$. Then the probability that the sun will rise m times in succession is

$$c \int_0^1 p^m dp;$$

and the probability that he will do this, and will rise again, or, which is the same thing, that he will rise $m + 1$ times in succession, is

$$c \int_0^1 p^{m+1} dp,$$

Hence the probability that if he rise m times in succession, he will rise the $m + 1^{th}$ time, is

$$\frac{c \int_0^1 p^{m+1} dp}{c \int_0^1 p^m dp} = \frac{m+1}{m+2},$$

the known and generally received solution.

The above solution is usually founded upon a supposed analogy of the problem with that of the drawing of balls from an urn containing a mixture of black and white balls, between which all possible numerical ratios are assumed to be equally probable. And it is remarkable, that there are two or three distinct hypotheses which lead to the same final result. For instance, if the balls are finite in number, and those which arc drawn are not replaced, or if they are infinite in number, whether those drawn are replaced or not, then, supposing that m successive drawings have yielded only white balls, the probability of the issue of a white ball at the $m + 1^{th}$ drawing is

$$\frac{m+1}{m+2}.$$[15]

It has been said, that the principle involved in the above and in similar applications is that of the equal distribution of our knowledge, or rather of our ignorance—the assigning to different states of things of which we know nothing, and upon the very ground that we know nothing, equal degrees of probability. I apprehend, however, that this is an arbitrary method of procedure. Instances may occur, and one such has been adduced, in which different hypotheses lead to the same final conclusion. But those instances are exceptional. With reference to the particular problem in question, it is shown in the memoir cited, that there is one hypothesis, viz., when the balls are finite in number and not replaced, which leads to a different conclusion, and it is easy to see that there are other hypotheses, as strictly involving the principle of the "equal distribution of knowledge or ignorance," which would also conduct to conflicting results.

[15]See a memoir by Bishop Terrot, Edinburgh Phil. Trans. vol. xx. Part iv.

连续 m 次观察到黑暗之后，按照公认的原理，太阳在同一情况下再次升起的概率按以下方式确定。设 p 为 0 与 1 之间的任一未知概率，而 c(无穷小且为常数)为这样的概率：太阳在经过一段黑暗间隔之后升起的概率介于 p 与 p + dp 两限之间。于是太阳连续升起 m 次的概率为

$$c \int_0^1 p^m dp;$$

而他做到这一点、并再次上升，或者，同一回事，他连续上升m + 1次的概率为

$$c \int_0^1 p^{m+1} dp,$$

Hence the probability that if he rise m times in succession, he will rise the $m + 1^{th}$ time, is

$$\frac{c \int_0^1 p^{m+1} dp}{c \int_0^1 p^m dp} = \frac{m+1}{m+2},$$

the known and generally received solution.

上述解法通常建立在一个假定的类比之上，即把该问题类比于从一个装有黑球与白球混合物的瓮中取球，并假定一切可能的数值比都同等概率。值得注意的是，有两三个不同的假说会导致同一最终结果。例如，若球数为有限，且取出的球不再放回，或者若球数为无限，无论取出的球放回与否，那么，假定 m 次连续抽取所得皆为白球，则在第 m + 1 th 次抽取中出白球的概率为

$$\frac{m+1}{m+2}.$$[15]

有人说，上述以及类似应用中所涉及的原理，是我们的知识的均等分配，或者毋宁说是我们的无知的均等分配--即对于那些我们一无所知的事物的不同状态，正是基于我们一无所知这一理由，赋予相等的概率程度。然而我认为，这是一种武断的做法。可能出现这样的情况，而且已经举出过一例，其中不同的假说导致同一最终结论。但那些情况是例外。就所讨论的这一特定问题而言，在所引的论文中已经表明，有一个假说，即当球数为有限且不再放回时，会导致不同的结论；而且不难看出，还有其他假说，同样严格地包含着"知识或无知的均等分配"这一原理，也会导致互相冲突的结果。

15 见特罗特主教的论文，爱丁堡哲学会刊，第xx卷，第iv部分。

24. For instance, let the case of sunrise be represented by the drawing of a white ball from a bag containing an infinite number of balls, which are all either black or white, and let the assumed principle be, that *all possible constitutions of the system of balls are equally probable.* By a constitution of the system, I mean an arrangement which assigns to every ball in the system a determinate colour, either black or white. Let us thence seek the probability, that if m white balls are drawn in m drawings, a white ball will be drawn in the $m+1^{th}$ drawing.

First, suppose the number of the balls to be μ, and let the symbols $x_1, x_2, \ldots x_\mu$ be appropriated to them in the following manner. Let x_i denote that event which consists in the i^{th} ball of the system being white, the proposition declaratory of such a state of things being $x_i = 1$. In like manner the compound symbol $1 - x_i$ will represent the circumstance of the i^{th} ball being black. It is evident that the several constituents formed of the entire set of symbols $x_1, x_2, \ldots x_\mu$ will represent in like manner the several possible constitutions of the system of balls with respect to blackness and whiteness, and the number of such constitutions being 2^μ, the probability of each will, in accordance with the hypothesis, be $\frac{1}{2^\mu}$. This is the value which we should find if we substituted in the expression of any constituent for

each of the symbols $x_1, x_2, \ldots x_\mu$, the value $\frac{1}{2}$. Hence, then, the probability of any event which can be expressed as a series of constituents of the above description, will be found by substituting in such expression the value $\frac{1}{2}$ for each of the above symbols.

Now the larger μ is, the less probable it is that any ball which has been drawn and replaced will be drawn again. As μ. approaches to infinity, this probability approaches to 0. And this being the case, the state of the balls actually drawn can be expressed as a logical function of m of the symbols $x_1, x_2, \ldots x_\mu$, and therefore, by development, as a series of constituents of the said m symbols. Hence, therefore, its probability will be found by substituting for each of the symbols, whether in the undeveloped or the developed form, the value $\frac{1}{2}$. But this is the very substitution which it would be necessary, and which it would suffice, to make, if the probability of a white ball at each drawing were known, *à priori*, to be $\frac{1}{2}$.

It follows, therefore, that if the number of balls be infinite, and all constitutions of the system equally probable, the probability of drawing m white balls in succession will be $\frac{1}{2^m}$, and the probability of drawing $m+1$ white balls in succession $\frac{1}{2^{m+1}}$;

whence the probability that after m white balls have been drawn, the next drawing will furnish a white one, will be $\frac{1}{2}$. In other words, past experience does not in this case affect future expectation.

25. It may be satisfactory to verify this result by ordinary methods. To

24. 例如，设日出之例由从一个装有无限多个球的袋中抽出一只白球来表示，袋中之球非黑即白，并设所假定的原理是，球系统的一切可能构成都同等概率。所谓球系统的一种构成，我指的是这样一种安排，它为系统中的每一个球指定一种确定的颜色，或黑或白。让我们由此寻求这样的概率：若在 m 次抽取中抽出 m 只白球，则在第 m + 1 th次抽取中将抽出一只白球。

首先，设球的数目为 μ，并令记号 x ^{1} , x ^{2} , ...x μ 按以下方式分派给它们。令 x i 表示这样的事件，即系统中第 i th个球为白，宣告这种事态的命题为 x i = 1。同样，复合记号 1 – x i 将表示第 i th个球为黑这一情况。显然，由全部记号 x ^{1} , x ^{2} , ...x μ 所构成的各个成分，将以同样方式表示球系统在黑白方面的各种可能构成，而这类构成的数目为 2 ^{μ}，按照假说，每一种构成的概率将为^{1}

2^{μ}.这是我们应当求得的值

如果我们在任何成分的表达式中代入

记号x ^{1} , x ^{2} , ...x μ中的每一个，其值^{1}

$2^{\cdot}$.因此，那么，该概率

任何可表述为上述成分序列的事件，其概率将通过在此类表达式中代入值^{1}而求得

$2^{\cdot}$for

上述每一个符号。

现在，μ 越大，任何已被抽出并放回的球再次被抽出的概率就越小。当 μ 趋于无穷大时，这个概率趋于 0。在这种情况下，实际抽出的球的状态可以表示为记号 x ^{1} , x ^{2} , ...x μ 的 m 的逻辑函数，因此，通过展开式，可以表示为所述 m 记号的成分的级数。因此，它的概率将通过为每个记号(无论是在未展开形式还是展开形式中)代入值^{1}来求得。

$2^{\cdot}$.但

这正是若每次抽取时白球概率先验*a priori*已知为^{1} 时所必需且充分作出的代入

$2^{\cdot}$.

因此可推出，若球数为无限，且系统的所有构成同等可能，则连续抽出m个白球的概率将为^{1}

$2^{\cdot m,}$和该概率的抽取m+1白

$\frac{1}{\quad}$

连续抽球 $2^{m+1;}$

由此，在已抽出m个白球之后，下一次抽取将给出白球的概率为^{1}

$2^{\cdot}$。换句话说，过去的

在这种情况下不影响未来的预期。

25. 用普通方法来验证这一结果或许令人满意。为了

accomplish this, we shall seek—

First: The probability of drawing r white balls, and $p-r$ black balls, in p trials, out of a bag containing μ balls, every ball being replaced after drawing, and all constitutions of the systems being equally probable, *à priori*.

Secondly: The value which this probability assumes when μ becomes infinite.

Thirdly: The probability hence derived, that if m white balls are drawn in succession, the $m+1^{th}$ ball drawn will be white also.

The probability that r white balls and $p-r$ black ones will be drawn in p trials out of an urn containing μ balls, each ball being replaced after trial, and all constitutions of the system as above defined being equally probable, is equal to the sum of the probabilities of the same result upon the separate hypotheses of there being no white balls, 1 white ball,—lastly μ white balls in the urn. Therefore, it is the sum of the probabilities of this result on the hypothesis of there being n white balls, n varying from 0 to μ.

Now supposing that there are n white balls, the probability of drawing a white ball in a single drawing is $\frac{n}{\mu}$, and the probability of drawing r white balls and $p-r$ black ones in a particular order in p drawings, is

$$\left(\frac{n}{\mu}\right)^r \left(1-\frac{n}{\mu}\right)^{p-r}$$

But there being as many such orders as there are combinations of r things in p things, the total probability of drawing r white balls in p drawings out of the system of μ balls of which n are white, is

$$\frac{p(p-1)\dots(p-r+1)}{1\cdot 2\dots r}\left(\frac{n}{\mu}\right)^r \left(1-\frac{n}{\mu}\right)^{p-r} \qquad (1)$$

Again, the number of constitutions of the system of μ balls, which admit of exactly n balls being white, is

$$\frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n},$$

and the number of possible constitutions of the system is 2^μ. Hence the probability that exactly n balls are white is

$$\frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n2^\mu},$$

Multiplying (1) by this expression, and taking the sum of the products from $n=0$ to $n=\mu$, we have

$$\frac{p(p-1)\dots p-r+1}{1\,.\,2\dots r}\sum_{n=0}^{n=\mu}\frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n2^\mu}\left(\frac{n}{\mu}\right)^r \left(1-\frac{n}{\mu}\right)^{p-r}, \qquad (2)$$

for the expression of the total probability, that out of a system of μ balls of which

要实现这一点，我们将寻求--

第一：从一只装有 μ 个球的袋中，在 p 次抽取中抽出 r 个白球和 p – r 个黑球的概率，每次抽后都将球放回，且系统的所有构成都同等可能，先验地。

第二：当 μ 变为无穷大时，这个概率所取的值。第三：由此推出的概率，即如果连续抽出 m 个白球，那么抽出的第 m + 1 th个球也将是白球。

从包含 μ 个球的瓮中，在 p 次试验中抽出 r 个白球和 p – r 个黑球，每次试验后将每个球放回，并且如上定义的系统所有构成具有相等的概率，其概率等于在瓮中没有白球、1 个白球--最后 μ 个白球的各个独立假说下同一结果的概率之和。因此，它是在有 n 个白球的假说下这一结果的概率之和，n 从 0 变化到 μ。

现在假设有 n 个白球，单次抽取中抽到白球的概率为^{n} μ ,且该概率为抽取r白球和 p – r 个黑球在 p 次抽取中按特定顺序出现的概率，为

$$\left(\frac{n}{\mu}\right)^r \left(1-\frac{n}{\mu}\right)^{p-r}$$

但是，这样的顺序的数目等于从 p 个东西中取 r 个东西的组合数，因此，从由 μ 个球组成、其中 n 个是白球的系统中，在 p 次抽取中抽出 r 个白球的总概率是

$$\frac{p(p-1)\dots(p-r+1)}{1\cdot 2\dots r}\left(\frac{n}{\mu}\right)^r \left(1-\frac{n}{\mu}\right)^{p-r} \qquad (1)$$

再者，由 μ 个球组成的系统中，恰好有 n 个球是白球的构成的数目是

$$\frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n},$$

而该系统可能的构成的数目是 2 ^{μ}。因此，恰好有 n 个球是白球的概率是

$$\frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n2^\mu},$$

将(1)乘以这个表达式，并对乘积从 n = 0 到 n = μ 求和，我们得到

$$\frac{p(p-1)\dots p-r+1}{1\,.\,2\dots r}\sum_{n=0}^{n=\mu}\frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n2^\mu}\left(\frac{n}{\mu}\right)^r \left(1-\frac{n}{\mu}\right)^{p-r}, \qquad (2)$$

作为总概率的表达式，即从由 μ 个球组成的系统中，其中

all constitutions are equally probable, r white balls will issue in p drawings. Now

$$\sum_{n=0}^{n=\mu} \frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n\,.\,2^{\mu}} \left(\frac{n}{\mu}\right)^{r} \left(1-\frac{n}{\mu}\right)^{p-r}$$
$$= \sum_{n=0}^{n=\mu} \frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n2^{\mu}} \left(\frac{n}{\mu}\right)^{r} \left(1-\frac{n}{\mu}\right)^{p-r} \varepsilon^{n\theta}\dots(\theta=0)$$
$$= \frac{1}{2^{\mu}} \left(\frac{D}{\mu}\right)^{r} \left(1-\frac{D}{\mu}\right)^{p-r} \sum_{n=0}^{n=\mu} \frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n} \varepsilon^{n\theta}$$
$$= \frac{1}{2^{\mu}} \left(\frac{D}{\mu}\right)^{r} \left(1-\frac{D}{\mu}\right)^{p-r} (1+\varepsilon^{\theta})^{\mu}, \qquad (3)$$

D standing for the symbol $\frac{d}{d\theta}$, so that $\phi(D)\varepsilon_{n\theta} = \phi(n)\varepsilon^{n\theta}$. But by a known theorem,

$$t^{m} = 1 + \Delta 0^{m} t + \frac{\Delta^{2} 0^{m}}{1\,.\,2} t(t-1) + \frac{\Delta^{3} 0^{m}}{1\,.\,2\,.\,3} t(t-1)(t-2).$$
$$\therefore D^{m}(1+\varepsilon^{\theta})^{\mu} = \{1 + \Delta 0^{m} D + \frac{\Delta^{2} 0^{m}}{1\,.\,2} D(D-1) + \ \&\text{c.}\}(1+\varepsilon^{\theta})^{\mu}.$$

In the second member let $\varepsilon^{\theta} = x$, then

$$D^{m}(1+\varepsilon^{\theta})^{\mu} = (1 + \Delta 0^{m} x \frac{d}{dx} + \frac{\Delta^{2} 0^{m}}{1\,.\,2} x^{2} \frac{d^{2}}{dx^{2}} + \ \&\text{c.})(1+x)^{\mu},$$

since

$$D(D-1)\dots(D-i+1) = x^{i} \left(\frac{d}{dx}\right)^{i}.$$

In the second member of the above equation, performing the differentiations and making $x = 1$ (since $\theta = 0$), we get

$$D^{m}(1+\varepsilon^{\theta})^{\mu} = \mu(\Delta 0^{m}) 2^{\mu-1} + \frac{\mu(\mu-1)}{1\,.\,2} (\Delta^{2} 0^{m}) 2^{\mu-2} + \ \&\text{c.}$$

The last term of the second member of this equation will be

$$\frac{\mu(\mu-1)\dots(\mu-m+1)\Delta^{m}0^{m}}{1\,.\,2\dots m} 2^{\mu-m} = \mu(\mu-1)\dots(\mu-m+1)2^{\mu-m};$$

since $\Delta^{m} 0^{m} = 1\,.\,2\dots m$. When μ is a large quantity this term exceeds all the others in value, and as μ approaches to infinity tends to become infinitely great in comparison with them. And as moreover it assumes the form $\mu^{m} 2^{\mu-m}$, we have, on passing to the limit,

$$D^{m}(1+\varepsilon^{\theta})^{\mu} = \mu^{m} 2^{\mu-m} = \left(\frac{\mu}{2}\right)^{m} 2^{\mu}.$$

所有构成都同等可能，在 p 次抽取中将抽出 r 个白球。现在

$$\sum_{n=0}^{n=\mu} \frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n\,.\,2^{\mu}} \left(\frac{n}{\mu}\right)^{r} \left(1-\frac{n}{\mu}\right)^{p-r}$$
$$= \sum_{n=0}^{n=\mu} \frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n2^{\mu}} \left(\frac{n}{\mu}\right)^{r} \left(1-\frac{n}{\mu}\right)^{p-r} \varepsilon^{n\theta}\dots(\theta=0)$$
$$= \frac{1}{2^{\mu}} \left(\frac{D}{\mu}\right)^{r} \left(1-\frac{D}{\mu}\right)^{p-r} \sum_{n=0}^{n=\mu} \frac{\mu(\mu-1)\dots(\mu-n+1)}{1\,.\,2\dots n} \varepsilon^{n\theta}$$
$$= \frac{1}{2^{\mu}} \left(\frac{D}{\mu}\right)^{r} \left(1-\frac{D}{\mu}\right)^{p-r} (1+\varepsilon^{\theta})^{\mu}, \qquad (3)$$

D standing for the symbol $\frac{d}{d\theta}$, so that $\phi(D)\varepsilon_{n\theta} = \phi(n)\varepsilon^{n\theta}$. But by a known theorem,

$$t^{m} = 1 + \Delta 0^{m} t + \frac{\Delta^{2} 0^{m}}{1\,.\,2} t(t-1) + \frac{\Delta^{3} 0^{m}}{1\,.\,2\,.\,3} t(t-1)(t-2).$$
$$\therefore D^{m}(1+\varepsilon^{\theta})^{\mu} = \{1 + \Delta 0^{m} D + \frac{\Delta^{2} 0^{m}}{1\,.\,2} D(D-1) + \ \&\text{c.}\}(1+\varepsilon^{\theta})^{\mu}.$$

在第二项中，令 ε θ = x，则

$$D^{m}(1+\varepsilon^{\theta})^{\mu} = (1 + \Delta 0^{m} x \frac{d}{dx} + \frac{\Delta^{2} 0^{m}}{1\,.\,2} x^{2} \frac{d^{2}}{dx^{2}} + \ \&\text{c.})(1+x)^{\mu},$$

因为

$$D(D-1)\dots(D-i+1) = x^{i} \left(\frac{d}{dx}\right)^{i}.$$

在上述方程的第二个成员中，进行微分并令 x = 1(因为 θ = 0)，我们得到

$$D^{m}(1+\varepsilon^{\theta})^{\mu} = \mu(\Delta 0^{m}) 2^{\mu-1} + \frac{\mu(\mu-1)}{1\,.\,2} (\Delta^{2} 0^{m}) 2^{\mu-2} + \ \&\text{c.}$$

本方程第二部分的最后一项将是

$$\frac{\mu(\mu-1)\dots(\mu-m+1)\Delta^{m}0^{m}}{1\,.\,2\dots m} 2^{\mu-m} = \mu(\mu-1)\dots(\mu-m+1)2^{\mu-m};$$

因为 Δ m 0 m = 1 . 2 ...m。当 μ 是一个大量时，这一项在数值上超过所有其他项，并且当 μ 趋近于无穷大时，与它们相比趋于变得无限大。此外，由于它取形式 μ m 2 μ-m，我们在取极限时得到

$$D^{m}(1+\varepsilon^{\theta})^{\mu} = \mu^{m} 2^{\mu-m} = \left(\frac{\mu}{2}\right)^{m} 2^{\mu}.$$

Hence if $\phi(D)$ represent any function of the symbol D, which is capable of being expanded in a series of ascending powers of D, we have

$$\phi(D)(1+\varepsilon^{\theta})^{\mu} = \phi\left(\frac{\mu}{2}\right)2^{\mu}, \tag{4}$$

if $\theta = 0$ and $\mu = \infty$. Strictly speaking, this implies that the ratio of the two members of the above equation approaches a state of equality, as μ increases towards infinity, θ being equal to 0.

By means of this theorem, the last member of (3) reduces to the form

$$\frac{1}{2^{\mu}}\left(\frac{1}{2}\right)^{r}\left(1-\frac{1}{2}\right)^{p-r}2^{\mu} = \left(\frac{1}{2}\right)^{p}.$$

Hence (2) gives

$$\frac{p(p-1)\dots(p-r+1)}{1\centerdot 2\dots r}\left(\frac{1}{2}\right)^{p},$$

as the expression for the probability that from an urn containing an infinite number of black and white balls, all constitutions of the system being equally probable, r white balls will issue in p drawings.

Hence, making $p = m$, $r = m$, the probability that in m drawings all the balls will be white is $\left(\frac{1}{2}\right)^{m}$, and the probability that this will be the case, and that moreover the $m+1^{th}$ drawing will yield a white ball is $\left(\frac{1}{2}\right)^{m+1}$, whence the probability, that if the first m drawings yield white balls only, the $m+1^{th}$ drawing will also yield a white ball, is

$$\left(\frac{1}{2}\right)^{m+1} \div \left(\frac{1}{2}\right)^{m} = \frac{1}{2};$$

and generally, any proposed result will have the same probability as if it were an even chance whether each particular drawing yielded a white or a black ball. This agrees with the conclusion before obtained.

26. These results only illustrate the fact, that when the defect of data is supplied by hypothesis, the solutions will, in general, vary with the nature of the hypotheses assumed; so that the question still remains, only more definite in form, whether the principles of the theory of probabilities serve to guide us in the election of such hypotheses. I have already expressed my conviction that they do not—a conviction strengthened by other reasons than those above stated. Thus, a definite solution of a problem having been found by the method of this work, an equally definite solution is sometimes attainable by the same method when one of the data, suppose Prob. $x = p_1$ is omitted. But I have not been able to discover any mode of deducing the second solution from the first by *integration*, with respect to p supposed variable within limits determined by Chap. XIX. This deduction would, however, I conceive, be possible, were the principle adverted to in Art. 23 valid. Still it is with diffidence that I express my dissent on these points from mathematicians generally, and more especially

因此，如果 φ(D) 表示记号 D 的任意函数，且该函数能够展开为 D 的升幂级数，我们有

$$\phi(D)(1+\varepsilon^{\theta})^{\mu} = \phi\left(\frac{\mu}{2}\right)2^{\mu}, \tag{4}$$

如果 θ = 0 且 μ = ∞。严格地说，这意味着上述方程两个成员的比值随着 μ 增大趋向无穷大而趋近于相等状态，θ 等于 0。

借助这一定理，(3)式的最后一项化为如下形式

$$\frac{1}{2^{\mu}}\left(\frac{1}{2}\right)^{r}\left(1-\frac{1}{2}\right)^{p-r}2^{\mu} = \left(\frac{1}{2}\right)^{p}.$$

因此(2)给出 gives

$$\frac{p(p-1)\dots(p-r+1)}{1\centerdot 2\dots r}\left(\frac{1}{2}\right)^{p},$$

作为概率的表达式，即从一只装有无限多个黑球和白球的瓮中，在系统的所有构成都同等可能的情况下，在 p 次抽取中将抽出 r 个白球。

因此，令 p = m,r = m，则在 m 次抽取中所有球都是白球的概率是

$$\frac{1}{2}$$

如果前 m 次抽取只产生白球，那么第 m + 1 th次抽取也将产生一个白球的概率是

$$\left(\frac{1}{2}\right)^{m+1} \div \left(\frac{1}{2}\right)^{m} = \frac{1}{2};$$

并且一般地，任何提出的结果都将具有与每次特定抽取产生白球或黑球的机会均等时相同的概率。这与先前得到的结论一致。

26. 这些结果只是说明了这样一个事实：当数据的缺陷由假说来补充时，解一般会随所假设的假说的性质而变化；因此问题仍然存在，只是形式上更明确了，即概率论的原则是否有助于我们在选择此类假说时提供指导。我已经表达了我的信念，即它们不能--这一信念得到了除上述理由之外的其他理由的加强。因此，通过本著作的方法找到了一个问题的确定解后，有时通过同样的方法，当其中一个数据，假设 Prob.x = p ^{1}被省略时，也能获得同样确定的解。但我未能发现任何通过关于 p 的积分从第一个解推导出第二个解的模式，假设 p 在第十九章确定的范围内变化。然而，我认为，如果第 23 条中提到的原则是有效的，这种推导将是可能的。尽管如此，我仍以谦逊的态度表达我在这些点上与一般数学家的不同意见，尤其是

from one who, of English writers, has most fully entered into the spirit and the methods of Laplace; and I venture to hope, that a question, second to none other in the Theory of Probabilities in importance, will receive the careful attention which it deserves.

来自一位在英国作家中最充分地领会了拉普拉斯的精神和方法的人；我冒昧希望，一个在概率论中重要性不亚于任何其他问题的问题，将得到它应得的仔细关注。

Chapter XXI

PARTICULAR APPLICATION OF THE PREVIOUS GENERAL METHOD TO THE QUESTION OF THE PROBABILITY OF JUDGMENTS.

1. On the presumption that the general method of this treatise for the solution of questions in the theory of probabilities, has been sufficiently elucidated in the previous chapters, it is proposed here to enter upon one of its practical applications selected out of the wide field of social statistics, viz., the estimation of the probability of judgments. Perhaps this application, if weighed by its immediate results, is not the best that could have been chosen. One of the first conclusions to which it leads is that of the *necessary* insufficiency of any data that experience alone can furnish, for the accomplishment of the most important object of the inquiry. But in setting clearly before us the *necessity* of hypotheses as supplementary to the data of experience, and in enabling us to deduce with rigour the consequences of *any* hypothesis which may be assumed, the method accomplishes all that properly lies within its scope. And it may be remarked, that in questions which relate to the conduct of our own species, hypotheses are more justifiable than in questions such as those referred to in the concluding sections of the previous chapter. Our general experience of human nature comes in aid of the scantiness and imperfection of statistical records.

2. The elements involved in problems relating to criminal assize are the following:—

1st. The probability that a particular member of the jury will form a correct opinion upon the case.

2nd. The probability that the accused party is guilty.

3rd. The probability that he will be condemned, or that he will be acquitted.

4th. The probability that his condemnation or acquittal will be just.

5th. The constitution of the jury.

6th. The data furnished by experience, such as the relative numbers of cases in which unanimous decisions have been arrived at, or particular majorities obtained; the number of cases in which decisions have been reversed by superior courts, &c.

Again, the class of questions under consideration may be regarded as either direct or inverse. The direct questions of probability are those in which the

第二十一章

前述一般方法在判决概率问题上的特称应用。

1. 假定本论著中用于解决概率论问题的一般方法已在前几章中得到充分阐明，这里建议进入其实际应用之一，从社会统计学的广阔领域中选出，即判断概率的估计。也许这一应用，如果以其直接结果来衡量，并不是所能选择的最佳应用。它导致的首要结论之一是，仅凭经验所能提供的数据对于完成探究的最重要目标而言是必然不充分的。但是，在向我们清楚地展示假说作为经验数据之补充的必然性，并使我们能够严格推导出可能假设的任何假说的后果方面，该方法完成了其范围内适当的一切。并且可以指出，在涉及我们自己物种行为的问题中，假说比在诸如前一章最后几节所提到的问题中更为合理。我们对人性的普遍经验有助于弥补统计记录的匮乏和不完善。

2. 与刑事审判相关的问题所涉及的元素如下：--

1. 陪审团某一特定成员对案件形成正确意见的概率。

第二，被告有罪的概率。

第三，他被判有罪或被宣告无罪的概率。

第四，他的定罪或无罪判决将是公正的概率。

第五，陪审团的构成。

6. 由经验提供的数据，例如一致裁决已经达成或获得特定多数的案件的相对数目；裁决被上级法院推翻的案件数目，等等。

再者，所考虑的这一类问题可以被视为要么是直接的，要么是反演的。概率的直接问题是那些其中

probability of correct decision for each member of the tribunal, or of guilt for the accused party, are supposed to be known *à priori*, and in which the probability of a decision of a particular kind, or with a definite majority, is sought. Inverse problems are those in which, from the data furnished by experience, it is required to determine some element which, though it stand to those data in the relation of cause to effect, cannot directly be made the subject of observation; as when from the records of the decisions of courts it is required to determine the probability that a member of a court will judge correctly. To this species of problems, the most difficult and the most important of the whole series, attention will chiefly be directed here.

3. There is no difficulty in solving the direct problems referred to in the above enumeration. Suppose there is but one juryman. Let k be the probability that the accused person is guilty; x the probability that the juryman will form a correct opinion; X the probability that the accused person will be condemned: then—

$kx =$ probability that the accused party is guilty, and that the juryman judges him to be guilty.

$(l-k)(l-x) =$ probability that the accused person is innocent, and that the juryman pronounces him guilty.

Now these being the only cases in which a verdict of condemnation can be given, and being moreover mutually exclusive, we have

$$X = kx + (1-k)(1-x). \tag{1}$$

In like manner, if there be n jurymen whose separate probabilities of correct judgment are $x_1, x_2, \ldots x_n$, the probability of an unanimous verdict of condemnation will be

$$X = kx_1x_2\ldots x_n + (1-k)(1-x_1)(1-x_2)\ldots(1-x_n).$$

Whence, if the several probabilities $x_1, x_2 \ldots x_n$ are equal, and are each represented by x, we have

$$X = kx^n + (1-k)(1-x)^n. \tag{2}$$

The probability in the latter case, that the accused person is guilty, will be

$$\frac{kx^n}{kx^n + (1-k)(1-x)^n}$$

All these results assume, that the events whose probabilities are denoted by k, x_1, x_2, &c., are independent, an assumption which, however, so far as we are concerned, is involved in the fact that those events are the only ones of which the probabilities are given.

The probability of condemnation by a given number of voices may be found on the same principles. If a jury is composed of three persons, whose several

法庭每一成员作出正确判决的概率，或被告有罪的概率，被假定为`*a priori*已知，而问题在于求某一特定种类的判决、或具有确定多数票的判决的概率。逆问题则是这样一类问题：从经验所提供的数据出发，要求确定某个元素，该元素虽然与那些数据处于原因与结果的关系之中，却不能直接成为观察的对象；例如，从法庭判决的记录中，要求确定法庭某一成员作出正确判断的概率。对于这一类问题--整个系列中最困难也最重要的问题--此处将主要予以关注。

3. 解决上述列举中所提到的直接问题没有困难。假设只有一个陪审员。令 k 为被告有罪的概率；x 为陪审员将形成正确意见的概率；X 为被告将被判罪的概率：那么--

kx = 被告有罪的概率，且该陪审员判定他有罪。

(l – k)(l – x) = 被告无分，且陪审员判定他有罪。

现在，既然这些是能够作出有罪裁决的唯一情形，并且此外它们相互排斥，我们有

$$X = kx + (1-k)(1-x). \tag{1}$$

同样地，如果有 n 个陪审员，他们各自作出正确判断的概率为 x 1 , x 2 , ...x n，则一致有罪裁决的概率将是

$$X = kx_1x_2\ldots x_n + (1-k)(1-x_1)(1-x_2)\ldots(1-x_n).$$

由此，如果各个概率 x 1 , x 2 ...x n 相等，并且每个都由 x 表示，我们有

$$X = kx^n + (1-k)(1-x)^n. \tag{2}$$

在后一种情况下，被告有罪的概率将是

$$\frac{kx^n}{kx^n + (1-k)(1-x)^n}$$

所有这些结果都假定，其概率由k、x1、x2等表示的事件是独立的，然而，就我们所关注的范围而言，这一假定已隐含在以下事实之中：这些事件是唯一给出了概率的事件。按给定票数被判有罪的概率可以依据同样的原理求得。如果陪审团由三人组成，他们各自的

probabilities of correct decision are x, x', x'', the probability X_2 that the accused person will be declared guilty by two of them will be

$$X_2 = k\{xx'(1-x'') + xx''(1-x') + x'x''(1-x)\}$$
$$+(1-k)\{(1-x)(1-x')x'' + (1-x)(1-x'')x' + (1-x')(1-x'')x\},$$

which if $x = x' = x''$ reduces to

$$3kx^2(1-x) + 3(1-k)x(1-x)^2.$$

And by the same mode of reasoning, it will appear that if X_i represent the probability that the accused person will be declared guilty by i voices out of a jury consisting of n persons, whose separate probabilities of correct judgment are equal, and represented by x, then

$$X_i = \frac{n(n-1)\dots(n-i+1)}{1\,.\,2\dots i}\{kx^i(1-x)^{n-i} + (1-k)x^{n-i}(1-x)^i\}. \qquad (3)$$

If the probability of condemnation by a determinate majority a is required, we have simply

$$i - a = n - i,$$

whence

$$i = \frac{n+a}{2},$$

which must be substituted in the above formula. Of course a admits only of such values as make i an integer. If n is even, those values are 0, 2, 4, &c.; if odd, 1, 3, 5, &c., as is otherwise obvious.

The probability of a condemnation by a majority of at least a given number of voices m, will be found by adding together the following several probabilities determined as above, viz.:

1st. The probability of a condemnation by an exact majority m;

2nd. The probability of condemnation by the next greater majority $m + 2$;

and so on; the last element of the series being the probability of unanimous condemnation. Thus the probability of condemnation by a majority of 4 at least out of 12 jurors, would be

$$X_8 + X_9 \dots + X_{12},$$

the values of the above terms being given by (3) after making therein $n = 12$.

4. When, instead of a jury, we are considering the case of a simple deliberative assembly consisting of n persons, whose separate probabilities of correct judgment are denoted by x, the above formulæ are replaced by others, made somewhat more simple by the omission of the quantity k.

正确判决的概率为 x、x′、x″，则被告被其中两人宣告有罪的概率 X 2将为

$$X_2 = k\{xx'(1-x'') + xx''(1-x') + x'x''(1-x)\}$$
$$+(1-k)\{(1-x)(1-x')x'' + (1-x)(1-x'')x' + (1-x')(1-x'')x\},$$

which if $x = x' = x''$ reduces to

$$3kx^2(1-x) + 3(1-k)x(1-x)^2.$$

并且按照同样的推理方式，可以看出，若 X i 表示在由 n 人组成的陪审团中，被告被 i 票宣告有罪的概率，这些人的各自正确判断的概率相等，并以 x 表示，则

$$X_i = \frac{n(n-1)\dots(n-i+1)}{1\,.\,2\dots i}\{kx^i(1-x)^{n-i} + (1-k)x^{n-i}(1-x)^i\}. \qquad (3)$$

If the probability of condemnation by a determinate majority a is required, we have simply

$$i - a = n - i,$$

whence

$$i = \frac{n+a}{2},$$

它必须代入上述公式中。当然，a 只容许使 i 成为整数的那些值。若 n 为偶数，则这些值为 0、2、4 等；若为奇数，则为 1、3、5 等，这在其他方面也是显然的。

以至少给定票数 m 的多数定罪的概率，将通过把以下按上述方式确定的若干概率相加而求得，即：

1st. The probability of a condemnation by an exact majority m;

第二，以下一个更大多数 m + 2 定罪的概率；

依此类推；该序列的最后一个元素是一致定罪的概率。因此，在 12 名陪审员中至少以 4 票多数定罪的概率，将为

$$X_8 + X_9 \dots + X_{12},$$

上述各项的值由(3)给出，在其中令 n = 12。

4. 当我们考虑的不是陪审团，而是由 n 人组成的简单审议会议的情形时，这些人的各自正确判断的概率以 x 表示，上述公式被其他公式所取代，这些公式由于省略了量 k 而变得稍微更简单一些。

The probability of unanimous decision is

$$X = x^n + (1-x)^n.$$

The probability of an agreement of i voices out of the whole number is

$$X_i = \frac{n(n-1)\dots(n-i+1)}{1\cdot 2\dots i}\{x^i(1-x)^{n-i} + x^{n-i}(1-x)^i\}. \qquad (4)$$

Of this class of investigations it is unnecessary to give any further account. They have been pursued to a considerable extent by Condorcet, Laplace, Poisson, and other writers, who have investigated in particular the modes of calculation and reduction which are necessary to be employed when n and i are large numbers. It is apparent that the whole inquiry is of a very speculative character. The values of x and k cannot be determined by direct observation. We can only presume that they must both in general exceed the value $\frac{1}{2}$; that the former, x, must increase with the progress of public intelligence; while the latter, k, must depend much upon those preliminary steps in the administration of the law by which persons suspected of crime are brought before the tribunal of their country. It has been remarked by Poisson, that in periods of revolution, as during the Reign of Terror in France, the value of k may fall, if account be taken of political offences, far below the limit $\frac{1}{2}$. The history of Europe in days nearer to our own would probably confirm this observation, and would show that it is not from the wild license of democracy alone, that the accusation of innocence is to be apprehended.

Laplace makes the assumption, that all values of x from

$$x = \frac{1}{2};\ \text{to}\ x = 1,$$

are equally probable. He thus excludes the supposition that a juryman is more likely to be deceived than not, but assumes that within the limits to which the probabilities of individual correctness of judgment are confined, we have no reason to give preference to one value of x over another. This hypothesis is entirely arbitrary, and it would be unavailing here to examine into its consequences.

Poisson seems first to have endeavoured to deduce the values of x and k, inferentially, from experience. In the six years from 1825 to 1830 inclusively, the number of individuals accused of crimes against the person before the tribunals of France was 11016, and the number of persons condemned was 5286. The juries consisted each of 12 persons, and the decision was pronounced by a simple majority. Assuming the above numbers to be sufficiently large for the estimation of probabilities, there would therefore be a probability measured by the fraction $\frac{5286}{11016}$ or .4782 that an accused person would be condemned by a simple majority. We should have the equation

$$X_7 + X_8 \dots + X_{12} = .4782, \qquad (5)$$

一致决定的概率是

$$X = x^n + (1-x)^n.$$

个声音在总数中一致的概率是

$$X_i = \frac{n(n-1)\dots(n-i+1)}{1\cdot 2\dots i}\{x^i(1-x)^{n-i} + x^{n-i}(1-x)^i\}. \qquad (4)$$

关于这一类研究，无需再作进一步说明。孔多塞、拉普拉斯、泊松以及其他作者已在相当程度上从事了这些研究，他们尤其考察了当 n 和 i 为大数时必须采用的计算与化简方式。显然，整个探究具有一种高度思辨的性质。x 和 k 的值无法通过直接观察来确定。我们只能假定它们一般在总体上都必须超过值[1]

2 ;即

前者 x 必定随着公众智识的进步而增大；而后者 k 则必定在很大程度上取决于法律执行中那些使犯罪嫌疑人被带到本国法庭面前的预备步骤。泊松曾指出，在革命时期，如法国恐怖统治期间，若将政治犯罪考虑在内，k 的值可能远低于极限[1]

2 。欧洲的历史在那些日子

更接近我们自己的那些日子，大概会证实这一观察，并且会表明，并非仅仅由于民主的狂放无度，才需要担心对无辜者的指控。

拉普拉斯作出假设，认为x的所有值都来自

$$x = \frac{1}{2};\ \text{to}\ x = 1,$$

是同等可能的。他由此排除了陪审员更可能被欺骗而非不被欺骗这一假设，但假定在个人正确判断的概率所被限制的界限之内，我们没有理由偏好 x 的某一个值而排斥另一个值。这一假说完全是任意的，在此考察其后果将是徒劳的。

泊松似乎首先致力于从经验中推断性地推出 x 和 k 的值。在1825年至1830年(含首尾)的六年中，在法国法庭上被指控犯有人身犯罪的个体数量为11016，而被判罪的人数为5286。每个陪审团由12人组成，判决以简单多数宣布。假定上述数字足够大，可用于估计概率，那么因此将有一个以分数$\frac{5286}{11016}$来度量的概率

或.4782即一名被告人将被判罪由一个

简单多数。我们应该有方程

$$X_7 + X_8 \dots + X_{12} = .4782, \qquad (5)$$

the general expression for X_i being given by (3) after making therein $n = 12$. In the year 1831 the law, having received alteration, required a majority of at least four persons for condemnation, and the number of persons tried for crimes against the person during that year being 2046, and the number condemned 743, the probability of the condemnation of an individual by the above majority was $\frac{743}{2046}$, or .3631. Hence we should have

$$X_8 + X_9 \ldots + X_{12} = .3631\,. \tag{6}$$

Assuming that the values of k and x were the same for the year 1831 as for the previous six years, the two equations (5) and (6) enable us to determine approximately their values. Poisson thus found,

$$k = .5354, \quad x = .6786\,.$$

For crimes against property during the same periods, he found by a similar analysis,

$$k = .6744, \quad x = .7771\,.$$

The solution of the system (5) (6) conducts in each case to two values of k, and to two values of x, the one value in each pair being greater, and the other less, than $\frac{1}{2}$. It was assumed, that in each case the larger value should be preferred, it being conceived more probable that a party accused should be guilty than innocent, and more probable that a juryman should form a correct than an erroneous opinion upon the evidence.

5. The data employed by Poisson, especially those which were furnished by the year 1831, are evidently too imperfect to permit us to attach much confidence to the above determinations of x and k; and it is chiefly for the sake of the method that they are here introduced. It would have been possible to record during the six years, 1825-30, or during any similar period, the number of condemnations pronounced with each possible majority of voices. The values of the several elements $X_8, X_9, \ldots X_{12}$ were there no reasons of policy to forbid, might have been accurately ascertained. Here then the conception of the general problem, of which Poisson's is a particular case, arises. How shall we, from this apparently supernumerary system of data, determine the values of x and k? If the hypothesis, adopted by Poisson and all other writers on the subject, of the absolute independence of the events whose probabilities are denoted by x and k be retained, we should be led to form a system of five equations of the type (3), and either select from these that particular pair of equations which might appear to be most advantageous, or combine together the equations of the system by the method of least squares. There might exist a doubt as to whether the latter method would be strictly applicable in such cases, especially if the values of x and k afforded by different selected pairs of the given equations were very different from each other. M. Cournot has considered a somewhat similar problem, in which, from the records of individual votes in a court consisting of four judges, it is proposed to investigate the separate probabilities of a correct verdict from each judge. For the determination of the elements x, x', x'', x''', he

在令其中 n = 12 之后，由(3)给出 X i 的一般表达式。1831年，该法律经修改后，要求至少四人多数方可定罪，而该年因针对人身的犯罪而受审的人数为2046，被定罪者为743，则一个人以上述多数被定罪的概率为7432046,或.3631.因此我们应当有

$$X_8 + X_9 \ldots + X_{12} = .3631\,. \tag{6}$$

假设k和x的值在1831年与之前六年相同，则方程(5)和(6)使我们能够近似确定它们的值。泊松由此发现，

$$k = .5354, \quad x = .6786\,.$$

For crimes against property during the same periods, he found by a similar analysis,

$$k = .6744, \quad x = .7771\,.$$

方程组(5)(6)的解在每种情况下都导出k的两个值和x的两个值，每对值中一个大于、另一个小于[1]

[2].它被假定，在每种情况下较大的值应

被优先采用，因为人们认为被告有罪比无罪更为可能，陪审员根据证据形成正确意见比错误意见更为可能。

5. 泊松所采用的数据，尤其是1831年所提供的那些数据，显然过于不完善，不足以使我们对上述 x 和 k 的确定给予很大信任；它们在此被引入，主要也是为了方法本身的缘故。本来有可能在1825-30这六年期间，或在任何类似的时期内，记录下以每一种可能的多数票宣告的定罪数目。各个元素 X_8 , X_9 , ...X_{12}的值，倘若没有政策上的理由加以禁止，本来是可以准确确定的。于是，一般问题的概念便由此产生，而泊松的问题只是它的一个特例。我们应当如何从这一看来是多余的数据系统出发，确定 x 和 k 的值呢?如果保留泊松以及所有其他论及此题的作者所采用的假说，即其概率由 x 和 k 表示的事件绝对独立，我们就会被引导去构造一个由(3)型五个方程组成的系统，并或者从中选出看来最为有利的那一对特殊方程，或者用最小二乘法把该系统的各方程结合起来。关于后一种方法在此类情形中是否严格适用，可能存在疑问，尤其是如果由给定的不同方程对所给出的 x 和 k 的值彼此相差很大。库尔诺先生曾考虑过一个有些类似的问题，其中根据一个由四名法官组成的法庭中各人投票的记录，试图研究每一法官作出正确裁断的各自概率。为了确定元素 x, x', x'', x'''，他

obtains eight equations, which he divides into two sets of four equations, and he remarks, that should any considerable discrepancy exist between the values of x, x', x'', x''', determined from those sets, it might be regarded as an indication that the hypothesis of the independence of the opinions of the judges was, in the particular case, untenable. The principle of this mode of investigation has been adverted to in (XVIII. 4).

6. I proceed to apply to the class of problems above indicated, the method of this treatise, and shall inquire, first, whether the records of courts and deliberative assemblies, *alone*, can furnish any information respecting the probabilities of correct judgment for their individual members, and, it appearing that they cannot, secondly, what kind and amount of necessary hypothesis will best comport with the actual data.

Proposition I.

From the mere records of the decisions of a court or deliberative assembly, it is not possible to deduce any definite conclusion respecting the correctness of the individual judgments of its members.

Though this Proposition may appear to express but the conviction of unassisted good sense, it will not be without interest to show that it admits of rigorous demonstration. Let us suppose the case of a deliberative assembly consisting of n members, no hypothesis whatever being made respecting the dependence or independence of their judgments. Let the logical symbols $x_1, x_2, \ldots x_n$ be employed according to the following definition, viz.: Let the generic symbol x_i denote that event which consists in the uttering of a correct opinion by the i^{th} member, A_i of the court. We shall consider the values of Prob. x_1, Prob. $x_2, \ldots$ Prob. x_n, as the *quæsita* of a problem, the expression of whose possible data we must in the next place investigate.

Now those data are the probabilities of events capable of being expressed by definite logical functions of the symbols $x_1, x_2, \ldots x_n$. Let $X_1, X_2, \ldots X_m$ represent the functions in question, and let the actual system of data be

$$\text{Prob. } X_1 = a_1, \quad \text{Prob. } X_2 = a_2, \quad \text{Prob. } X_m = a_m.$$

Then from the very nature of the case it may be shown that $X_1, X_2, \ldots X_m$, are functions which remain unchanged if $x_1, x_2, \ldots x_n$ are therein changed into $1 - x_1, 1 - x_2, \ldots 1 - x_n$ respectively. Thus, if it were recorded that in a certain proportion of instances the votes given were unanimous, the event whose probability, supposing the instances sufficiently numerous, is thence determined, is expressed by the logical function

$$x_1 x_2 \ldots x_n + (1 - x_1)(1 - x_2) \ldots (1 - x_n),$$

a function which satisfies the above condition. Again, let it be recorded, that in a certain proportion of instances, the vote of an individual, suppose A_1, differs from that of all the other members of the court. The event, whose probability is thus given, will be expressed by the function

$$x_1(1 - x_2) \ldots (1 - x_n) + (1 - x_1)x_2 \ldots x_n;$$

得到八个方程，他把它们分成两组各四个方程，并且指出，如果由这些组所确定的 x, x ′, x ″, x ‴的值之间存在任何相当大的差异，那就可以被视为一种迹象，表明法官们意见独立的假说在该特殊情形中是站不住脚的。这种研究方式的原理已在(XVIII.4)中提及。

6. 我接下来将本论著的方法应用于上述那类问题，并首先考察，法庭和审议会议的记录单独是否能提供任何关于其各个成员正确判断概率的信息；既然看来不能，其次考察，何种必要假说及其数量最能符合实际数据。

命题一。

仅从法庭或审议会议裁决的记录，不可能推演出关于其成员各个判断正确性的任何确定结论。

虽然这个命题看来只不过表达了未经辅助的良好常识的确信，但表明它容许严格的证明，将不会没有意义。让我们设想一个由 n 名成员组成的审议性集会的情形，对其判断的依赖或独立不作任何假说。设逻辑记号 x ^{1} , x ^{2} , ...x n 按如下定义使用，即：设一般记号 x i 表示这样的事件，它在于法庭的第 i th名成员 A i 发表一个正确意见。我们将把 Prob.x ^{1}, Prob.x ^{2} , ...Prob.x n 的值，看作一个问题的*quæsita*，而我们必须接着研究其可能数据的表达式。

现在那些数据是事件的概率，这些事件能够用记号x ^{1} , x ^{2} , ...x n的确定逻辑函数来表示。令X ^{1} , X ^{2} , ...X m表示所讨论的函数，并令实际的数据系统为

$$\text{Prob. } X_1 = a_1, \quad \text{Prob. } X_2 = a_2, \quad \text{Prob. } X_m = a_m.$$

于是，由情形的本性本身就可以表明，X ^{1} , X ^{2} , ...X m 是这样的函数，如果将其中的 x ^{1} , x ^{2} , ...x n 分别变为 1 – x ^{1} , 1 – x ^{2} , ...1 – x n，它们保持不变。因此，如果记录下在某一比例的情形中投票是一致的，那么由此(假定情形足够多)所确定其概率的事件，就由该逻辑函数表示

$$x_1 x_2 \ldots x_n + (1 - x_1)(1 - x_2) \ldots (1 - x_n),$$

一个满足上述条件的函数。再者，设记录表明，在某一比例的事例中，某个个人(假定为A ^{1})的投票与法庭所有其他成员的投票不同。其概率如此给出的事件，将由下述函数表示

$$x_1(1 - x_2) \ldots (1 - x_n) + (1 - x_1)x_2 \ldots x_n;$$

also satisfying the above conditions. Thus, as agreement in opinion may be an agreement in either truth or error; and as, when opinions are divided, either party may be right or wrong; it is manifest that the expression of any particular state, whether of agreement or difference of sentiment in the assembly, will depend upon a logical function of the symbols $x_1, x_2, \ldots x_n$, which similarly involves the privative symbols $1-x_1, 1-x_2, \ldots 1-x_n$. But in the records of assemblies, it is not presumed to declare which set of opinions is right or wrong. Hence the functions $X_1, X_2, \ldots X_m$ must be solely of the kind above described.

7. Now in proceeding, according to the general method, to determine the value of Prob. x_1, we should first equate the functions $X_1, \ldots X_m$ to a new set of symbols $t_1, \ldots t_m$. From the equations

$$X_1 = t_1, X_2 = t_2, \ldots X_m = t_m,$$

thus formed, we should eliminate the symbols $x_2, x_3, \ldots x_n$, and then determine x_1 as a developed logical function of the symbols $t_1, t_2, \ldots t_m$, expressive of events whose probabilities are given. Let the result of the above elimination be

$$Ex_1 + E'(1-x_1) = 0; \tag{1}$$

E and E' being function of $t_1, t_2, \ldots t_m$. Then

$$x_1 = \frac{E'}{E'-E}. \tag{2}$$

Now the functions $X_1, X_2, \ldots X_m$ are symmetrical with reference to the symbols $x_1, \ldots x_n$ and $1-x_1, \ldots 1-x_n$. It is evident, therefore, that in the equation E' must be identical with E. Hence (2) gives

$$x = \frac{E}{0},$$

and it is evident, that the only coefficients which can appear in the development of the second member of the above equation are $\frac{0}{0}$ and $\frac{1}{0}$. The former will present itself whenever the values assigned to $t_1, \ldots t_m$ in determining the coefficient of a constituent, are such as to make $E=0$, the latter, or an equivalent result, in every other case. Hence we may represent the development under the form

$$x_1 = \frac{0}{0}C + \frac{1}{0}D \tag{3}$$

C and D being constituents, or aggregates of constituents, of the symbols $t_1, t_2, \ldots t_m$. Passing then from Logic to Algebra, we have

$$\text{Prob. } x_1 = \frac{cC}{C} = c,$$

the function V of the general Rule (XVII. 17) reducing in the present case to C. The value of Prob. x_1 is therefore wholly arbitrary, if we except the condition that it must not transcend the limits 0 and 1. The individual values of Prob.

并且也满足上述条件。因此，由于意见的一致可能是真理上的一致，也可能是错误上的一致；并且，当意见分歧时，任何一方都可能对或错；显然，对于集会中任何特定状态--无论是意见的一致还是分歧--的表达，都将取决于记号 x 1 , x 2 , ...x n 的一个逻辑函数，该函数同样涉及私有记号 1 – x 1 , 1 – x 2 , ...1 – x n。但在集会的记录中，并不假定要宣布哪一组意见是对还是错。因此，函数 X 1 , X 2 , ...X m 必须仅仅是上述种类。

7. 现在按照一般方法进行，为确定Prob.x 1的值，我们应首先令函数X 1 , ...X m等于一组新记号t 1 , ...t m。由如此形成的方程

$$X_1 = t_1, X_2 = t_2, \ldots X_m = t_m,$$

我们应消去记号x 2 , x 3 , ...x n，然后将x 1确定为记号t 1 , t 2 , ...t m的一个展开的逻辑函数，用以表达其概率已给出的事件。令上述消元的结果为

$$Ex_1 + E'(1-x_1) = 0; \tag{1}$$

E and E' being function of $t_1, t_2, \ldots t_m$. Then

$$x_1 = \frac{E'}{E'-E}. \tag{2}$$

Now the functions $X_1, X_2, \ldots X_m$ are symmetrical with reference to the symbols $x_1, \ldots x_n$ and $1-x_1, \ldots 1-x_n$. It is evident, therefore, that in the equation E' must be identical with E. Hence (2) gives

$$x = \frac{E}{0},$$

而且很明显，在上述方程第二项的展开式中可能出现的唯一系数是00和10。前者在确定某一成分的系数时，只要赋予 t1 , ...t m 的值使得 E = 0，就会自行呈现；后者，或一个等价的结果，则在其他一切情形下呈现。因此我们可以把展开式表示为如下形式

$$x_1 = \frac{0}{0}C + \frac{1}{0}D \tag{3}$$

C和D是记号t 1 , t 2 , ...t m的构成成分，或构成成分的聚合。然后从逻辑过渡到代数，我们有

$$\text{Prob. } x_1 = \frac{cC}{C} = c,$$

一般规则(XVII.17)中的函数V在目前情形下归结为C。因此Prob.x 1的值完全是任意的，除非我们排除它不得超出界限0和1这一条件。Prob.的各个值

x_2, ... Prob. x_n, are in like manner arbitrary. It does not hence follow, that these arbitrary values are not connected with each other by necessary conditions dependent upon the data. The investigation of such conditions would, however, properly fall under the methods of Chap. XIX.

If, reverting to the final logical equation, we seek the interpretation of c, we obtain but a restatement of the original problem. For since C and D together include all possible constituents of $t_1, t_2, \dots t_m$, we have

$$C + D = 1;$$

and since D is affected by the coefficient $\frac{1}{0}$, it is evident that on substituting therein for $t_1, t_2, \dots t_m$, their expressions in terms of $x_1, x_2, \dots x_n$, we should have $D = 0$. Hence the same substitution would give $C = 1$. Now by the rule, c is the probability that if the event denoted by C take place, the event x_1 will take place. Hence C being equal to 1, and, therefore, embracing all possible contingencies, c must be interpreted as the absolute probability of the occurrence of the event x_1.

It may be interesting to determine in a particular case the actual form of the final logical equation. Suppose, then, that the elements from which the data are derived are the records of events distinct and mutually exclusive. For instance, let the numerical data $a_1, a_2, \dots a_m$, be the respective probabilities of distinct and definite majorities. Then the logical functions $X_1, X_2, \dots X_m$ being mutually exclusive, must satisfy the conditions

$$X_1X_2 = 0, \dots X_1X_m = 0, \quad X_2X_m = 0, \ \&c.$$

Whence we have,

$$t_1t_2 = 0, \quad t_1t_m = 0, \ \&c.$$

Under these circumstances it may easily be shown, that the developed logical value of x_1 will be

$$x_1 = \frac{0}{0}(\bar{t}_1\bar{t}_2 \dots \bar{t}_m + t_1\bar{t}_2 \dots \bar{t}_m \dots + t_m\bar{t}_1 \dots \bar{t}_{m-1}) + \text{ constitutents whose coefficients are } \frac{1}{0}.$$

In the above equation $\bar{t}_1$ stands for $1 - t_1$, &c.

These investigations are equally applicable to the case in which the probabilities of the verdicts of a jury, so far as agreement and disagreement of opinion are concerned, form the data of a problem. Let the logical symbol w denote that event or state of things which consists in the guilt of the accused person. Then the functions X_1, X_2 ... X_m of the present problem are such, that no change would therein ensue from simultaneously converting $w, x_1, x_2 \dots x_n$ into $\bar{w}, \bar{x}_1, \bar{x}_2, \dots \bar{x}_n$ respectively. Hence the final logical value of w, as well as those of $x_1, x_2, \dots x_n$ will be exhibited under the same form (3), and a like general conclusion thence deduced.

It is therefore established, that from mere statistical documents nothing can be inferred respecting either the individual correctness of opinion of a judge or

x 2, ...Prob.x n，同样也是任意的。由此并不能推出，这些任意值之间不存在依赖于数据的必然条件。然而，对此类条件的考察，理应属于第十九章的方法。

如果回到最终逻辑方程，寻求c的解释，我们得到的只是原问题的重述。因为既然C和D合起来包含t 1 , t 2 , ...t m的所有可能构成成分，我们有

$$C + D = 1;$$

并且由于 D 受系数10,的影响，显然,在其中代入以 t 1 ,t 2 , ...t m 用 x 1 , x 2 , ...x n 表示的表达式之后，我们应当得到 D = 0。因此同样的代入会给出 C = 1。现在按照规则，c 是这样一个概率：如果由 C 表示的事件发生，则事件 x 1将会发生。因此，既然 C 等于 1，从而囊括了一切可能的情形，c 就必须被解释为事件 x 1发生的绝对概率。

在特定情况下确定最终逻辑方程的实际形式可能是有趣的。那么，假设从中导出数据的元素是不同且互斥的事件的记录。例如，设数值数据 a 1 , a 2 , ...a m 分别是不同且确定的多数派的概率。那么逻辑函数 X 1 , X 2 , ...X m 由于互斥，必须满足条件

$$X_1X_2 = 0, \dots X_1X_m = 0, \quad X_2X_m = 0, \ \&c.$$

由此我们得到，

$$t_1t_2 = 0, \quad t_1t_m = 0, \ \&c.$$

在这些情况下，容易证明，x 1的展开逻辑值将是

$$x_1 = \frac{0}{0}(\bar{t}_1\bar{t}_2 \dots \bar{t}_m + t_1\bar{t}_2 \dots \bar{t}_m \dots + t_m\bar{t}_1 \dots \bar{t}_{m-1}) + \text{ 其系数为}1\text{的构成成分} \quad 0.$$

在上述方程中， t 1代表1 – t 1 ，等等。

这些研究同样适用于这样一种情况：陪审团裁决的概率，就意见的一致与分歧而言，构成一个问题的数据。设逻辑记号 w 表示由被告人有罪所构成的事件或事态。那么本问题的函数 X 1, X 2...X m 是这样的：同时将 w, x 1 , x 2 ...x n 分别转换为 w, ¯x 1 , ¯x 2 , ...¯x n 不会在其中引起任何变化。因此，w 的最终逻辑值，以及 x 1 , x 2 , ...x n 的最终逻辑值，将以相同的形式(3)展示，并由此推出一个同样的一般结论。

因此可以确定，仅从统计文献中，不能推断出任何关于法官个人意见正确性的东西，也不能推断出

counsellor, the guilt of an individual, or the merits of a disputed question. If the determination of such elements as the above can be reduced within the province of science at all, it must be by virtue either of some assumed criterion of truth furnishing us with new data, or of some hypothesis relative to the connexion or the independence of individual judgments, which may warrant a new form of the investigation. In the examination of the results of different hypotheses, the following general Proposition will be of importance.

PROPOSITION II.

8. *Given the probabilities of the n simple events* $x_1, x_2, \dots x_n$, *viz.:—*

$$\text{Prob.}\, x_1 = c_1, \quad \text{Prob.}\, x_2 = c_2, \dots \text{Prob.}\, x_n = c_n; \tag{1}$$

also the probabilities of the $m-1$ *compound events* $X_1, X_2, \dots X_{m-1}$, *viz.:—*

$$\text{Prob.}\, X_1 = a_1, \quad \text{Prob.}\, X_2 = a_2, \dots \text{Prob.}\, X_{m-1} = a_{m-1}; \tag{2}$$

the latter events $X_1 \dots X_{m-1}$ *being distinct and mutually exclusive; required the probability of any other compound event* X.

In this proposition it is supposed, that $X_1, X_2, \dots X_{m-1}$, as well as X, are functions of the symbols $x_1, x_2, \dots x_n$ alone. Moreover, the events $X_1, X_2, \dots X_{m-1}$ being mutually exclusive, we have

$$X_1 X_2 = 0, \dots X_1 X_{m-1} = 0, \quad X_2 X_3 = 0, \ \&\text{c.}; \tag{3}$$

the product of any two members of the system vanishing. Now assume

$$X_1 = t_1, \quad X_{m-1} = t_{m-1}, \quad X = t. \tag{4}$$

Then t must be determined as a logical function of $x_1, \dots x_n, t_1, \dots t_{m-1}$.

Now by (3),

$$t_1 t_2 = 0, \quad t_1 t_{m-1} = 0, \quad t_2 t_3 = 0, \ \&\text{c.}; \tag{5}$$

all binary products of $t_1, \dots \bar{t}_{m-1}$, vanishing. The developed expression for t can, therefore, only involve in the list of constituents which have 1, 0, or $\frac{0}{0}$ for their coefficients, such as contain some one of the following factors, viz.:—

$$\bar{t}_1 \bar{t}_2 \dots \bar{t}_{m-1}, \quad t_1 \bar{t}_2 \dots \bar{t}_{m-1}, \dots \bar{t}_1 \dots \bar{t}_{m-2} t_{m-1}; \tag{6}$$

$\bar{t}_1$ standing for $1 - t_1$, &c. It remains to assign that portion of each constituent which involves the symbols $x_1 \dots x_n$; together with the corresponding coefficients.

Since $X_i = t_i$ (i being any integer between 1 and $m-1$ inclusive), it is evident that

$$X_i \bar{t}_1 \dots \bar{t}_{m-1} = 0,$$

from the very constitution of the functions. Any constituent included in the first member of the above equation would, therefore, have $\frac{1}{0}$ for its coefficient.

顾问、个人的罪责，或一个争议问题的功过。如果对上述这类要素的判定能够被完全归入科学的领域，那必定是凭借某种被假定的真理标准为我们提供新的数据，或者凭借某种关于各个别判断的联系或独立的假说，而后者可能为一种新的研究形式提供依据。在考察不同假说的结果时，下述一般命题将具有重要意义。

命题二。

8. *Given the probabilities of the n simple events* $x_1, x_2, \dots x_n$, *viz.:—*

$$\text{Prob.}\, x_1 = c_1, \quad \text{Prob.}\, x_2 = c_2, \dots \text{Prob.}\, x_n = c_n; \tag{1}$$

also the probabilities of the $m-1$ *compound events* $X_1, X_2, \dots X_{m-1}$, *viz.:—*

$$\text{Prob.}\, X_1 = a_1, \quad \text{Prob.}\, X_2 = a_2, \dots \text{Prob.}\, X_{m-1} = a_{m-1}; \tag{2}$$

the latter events $X_1 \dots X_{m-1}$ *being distinct and mutually exclusive; required the probability of any other compound event* X.

In this proposition it is supposed, that $X_1, X_2, \dots X_{m-1}$, as well as X, are functions of the symbols $x_1, x_2, \dots x_n$ alone. Moreover, the events $X_1, X_2, \dots X_{m-1}$ being mutually exclusive, we have

$$X_1 X_2 = 0, \dots X_1 X_{m-1} = 0, \quad X_2 X_3 = 0, \ \&\text{c.}; \tag{3}$$

系统中任意两个成员的乘积为零。现在假设

$$X_1 = t_1, \quad X_{m-1} = t_{m-1}, \quad X = t. \tag{4}$$

那么t必须确定为x 1 , ...x n , t 1 , ...t m – 1的一个逻辑函数。

现在由(3),

$$t_1 t_2 = 0, \quad t_1 t_{m-1} = 0, \quad t_2 t_3 = 0, \ \&\text{c.}; \tag{5}$$

t 1 , ...¯t m – 1的所有二元乘积都消失。因此，t的展开表达式在具有1、0或0的构成成分列表中，只能涉及

它们的系数，例如包含以下诸因子之一，即：-

$$\bar{t}_1 \bar{t}_2 \dots \bar{t}_{m-1}, \quad t_1 \bar{t}_2 \dots \bar{t}_{m-1}, \dots \bar{t}_1 \dots \bar{t}_{m-2} t_{m-1}; \tag{6}$$

¯t 1代表1 – t 1，等等。剩下要指定每个构成成分中涉及记号x 1 ...x n的那一部分；连同相应的系数。

$$X_i \bar{t}_1 \dots \bar{t}_{m-1} = 0,$$

from the very constitution of the functions. Any constituent included in the first member of the above equation would, therefore, have $\frac{1}{0}$ for its coefficient.

Now let

$$X_m = 1 - X_1 \ldots - X_{m-1}; \tag{7}$$

and it is evident that such constituents as involve $\bar{t}_1 \ldots \bar{t}_{m-1}$, as a factor, and yet have coefficients of the form 1, 0, or $\frac{0}{0}$, must be included in the expression

$$X_m \bar{t}_1 \ldots \bar{t}_{m-1}.$$

Now X_m may be resolved into two portions, viz., $X\,X_m$ and $(1-X)X_m$, the former being the sum of those constituents of X_m which are found in X, the latter of those which are not found in X. It is evident that in the developed expression of t, which is equivalent to X, the coefficients of the constituents in the former portion $X\,X_m$ will be 1, while those of the latter portion $(1-X)X_m$ will be 0. Hence the elements we have now considered will contribute to the development of t the terms

$$XX_m\bar{t}_1 \ldots \bar{t}_{m-1} + 0(1-X)X_m\bar{t}_1 \ldots \bar{t}_{m-1}.$$

Again, since $X_1 = t_1$, while $X_2 t_1 = t_2 t_1 = 0$, &c., it is evident that the only constituents involving $t_1\bar{t}_2 \ldots \bar{t}_{m-1}$, as a factor which have coefficients of the form 1, 0, or $\frac{0}{0}$, will be included in the expression

$$X_1 t_1 \bar{t}_2 \ldots \bar{t}_{m-1};$$

and reasoning as before, we see that this will contribute to the development of t the terms

$$XX_1 t_1 \bar{t}_2 \ldots \bar{t}_{m-1} + 0(1-X)X_1 t_1 \bar{t}_2 \ldots \bar{t}_{m-1}.$$

Proceeding thus with the remaining terms of (6), we deduce for the final expression of t,

$$\begin{aligned} t = {} & XX_m\bar{t}_1 \ldots \bar{t}_{m-1} + XX_1 t_1\bar{t}_2 \ldots \bar{t}_{m-1} \ldots + XX_{m-1}\bar{t}_1 \ldots \bar{t}_{m-2}t_{m-1} \\ & + 0(1-X)X_m\bar{t}_1 \ldots \bar{t}_{m-1} + 0(1-X)X_1 t_1\bar{t}_2 \ldots \bar{t}_{m-1} + \text{ \&c.} \\ & + \text{terms whose coefficients are } \frac{1}{0}. \end{aligned} \tag{8}$$

In this expression it is to be noted that $X\,X_m$ denotes the sum of those constituents which are common to X and X_m, that sum being actually given by multiply ing X and X_m together, according to the rules of the calculus of Logic.

In passing from Logic to Algebra, we shall represent by $(X\,X_m)$ what the above product becomes, when, after effecting the multiplication, or selecting the common constituents, we give to the symbols $x_1, \ldots x_n$, a quantitative meaning. With this understanding we shall have, by the general Rule (XVII. 17),

$$\text{Prob. } t = \frac{(XX_m)\bar{t}_1 \ldots \bar{t}_{m-1} + (XX_1)t_1\bar{t}_2 \ldots \bar{t}_{m-1} + (XX_{m-1})\bar{t}_1 \ldots \bar{t}_{m-2}t_{m-1}}{V}, \tag{9}$$

$$V = X_m\bar{t}_1 \ldots \bar{t}_{m-1} + X_1 t_1\bar{t}_2 \ldots \bar{t}_{m-1} \ldots + X_{m-1}\bar{t}_1 \ldots \bar{t}_{m-2}t_{m-1} \tag{10}$$

Now let

$$X_m = 1 - X_1 \ldots - X_{m-1}; \tag{7}$$

and it is evident that such constituents as involve $\bar{t}_1 \ldots \bar{t}_{m-1}$, as a factor, and yet have coefficients of the form 1, 0, or $\frac{0}{0}$, must be included in the expression

$$X_m \bar{t}_1 \ldots \bar{t}_{m-1}.$$

现在 X m 可以分解为两部分，即 X X m 和(1 – X) X m，前者是 X m 中那些见于 X 的构成元素之和，后者是那些不见于 X 的构成元素之和。显然，在 t 的展开式中(它等价于 X)，前一部分 X X m 中各构成元素的系数将为 1，而后一部分(1 – X) X m 中各构成元素的系数将为 0。因此，我们刚才所考虑的这些元素将对 t 的展开式贡献如下各项

$$XX_m\bar{t}_1 \ldots \bar{t}_{m-1} + 0(1-X)X_m\bar{t}_1 \ldots \bar{t}_{m-1}.$$

再者，由于 X ^{1} = t ^{1}，而 X ^{2} t ^{1} = t ^{2} t ^{1} = 0，等等，显然，唯一以 t ^{1} ¯ t ^{2} ... ¯ t m – ^{1}为因子、且其系数具有 1、0 或$\frac{0}{0}$,将被包含于该表达式形式的构成元素

$$X_1 t_1 \bar{t}_2 \ldots \bar{t}_{m-1};$$

and reasoning as before, we see that this will contribute to the development of t the terms

$$XX_1 t_1 \bar{t}_2 \ldots \bar{t}_{m-1} + 0(1-X)X_1 t_1 \bar{t}_2 \ldots \bar{t}_{m-1}.$$

Proceeding thus with the remaining terms of (6), we deduce for the final expression of t,

$$\begin{aligned} t = {} & XX_m\bar{t}_1 \ldots \bar{t}_{m-1} + XX_1 t_1\bar{t}_2 \ldots \bar{t}_{m-1} \ldots + XX_{m-1}\bar{t}_1 \ldots \bar{t}_{m-2}t_{m-1} \\ & + 0(1-X)X_m\bar{t}_1 \ldots \bar{t}_{m-1} + 0(1-X)X_1 t_1\bar{t}_2 \ldots \bar{t}_{m-1} + \text{ \&c.} \\ & + \text{terms whose coefficients are } \frac{1}{0}. \end{aligned} \tag{8}$$

在此表达式中须注意，X X m表示那些为X和X m所共有的构成成分之和，该和实际上是通过按照逻辑演算的规则将X和X m相乘而给出的。

在从逻辑过渡到代数时，我们将用(X X m)表示上述乘积在完成乘法、或选出共同构成成分之后，当我们赋予记号x ^{1}，...x n以量的意义时所变成的东西。作此理解，根据一般规则(XVII.17)，我们将有

$$\text{Prob. } t = \frac{(XX_m)\bar{t}_1 \ldots \bar{t}_{m-1} + (XX_1)t_1\bar{t}_2 \ldots \bar{t}_{m-1} + (XX_{m-1})\bar{t}_1 \ldots \bar{t}_{m-2}t_{m-1}}{V}, \tag{9}$$

$$V = X_m\bar{t}_1 \ldots \bar{t}_{m-1} + X_1 t_1\bar{t}_2 \ldots \bar{t}_{m-1} \ldots + X_{m-1}\bar{t}_1 \ldots \bar{t}_{m-2}t_{m-1} \tag{10}$$

whence the relations determining $x_1, \dots x_n, t_1, \dots t_{m-1}$ will be of the following type (i varying from 1 to n),

$$\frac{(x_iX_m)\bar{t}_1 \dots \bar{t}_{m-1} + (x_iX_1)t_1\bar{t}_2 \dots \bar{t}_{m-1} + (x_iX_{m-1})\bar{t}_1 \dots \bar{t}_{m-2}t_{m-1}}{c_i}$$
$$= \frac{X_1t_1\bar{t}_2 \dots \bar{t}_{m-1}}{a_1} \dots = \frac{X_{m-1}\bar{t}_1 \dots \bar{t}_{m-2}t_{m-1}}{a_{m-1}} = V. \tag{11}$$

From the above system we shall next eliminate the symbols $t_1, \dots t_{m-1}$. We have

$$t_1\bar{t}_2 \dots \bar{t}_{m-1} = \frac{a_1V}{X_1}, \quad \bar{t}_1 \dots \bar{t}_{m-2}t_{m-1} = \frac{a_{m-1}V}{X_{m-1}}. \tag{12}$$

Substituting these values in (10), we find

$$V = X_m\bar{t}_1 \dots \bar{t}_{m-1} + a_1V \dots + a_{m-1}V.$$

Hence,

$$\bar{t}_1 \dots \bar{t}_{m-1} = \frac{(1 - a_1 \dots - a_{m-1})V}{X_m}.$$

Now let

$$a_m = 1 - a_1 \dots - a_{m-1}, \tag{13}$$

then we have

$$\bar{t}_1 \dots \bar{t}_{m-1} = \frac{a_mV}{X_m}. \tag{14}$$

Now reducing, by means of (12) and (14), the equation (9), and the equation formed by equating the first line of (11) to the symbol V; writing also Prob. X for Prob. t, we have

$$\text{Prob. } X = \frac{a_1(XX_1)}{X_1} + \frac{a_2(XX_2)}{X_2} \dots + \frac{a_m(XX_m)}{X_m}, \tag{15}$$
$$\frac{a_1(x_iX_1)}{X_1} + \frac{a_2(x_iX_2)}{X_2} \dots + \frac{a_m(x_iX_m)}{X_m} = c_i; \tag{16}$$

wherein X_m and a_m are given by (7) and (13). These equations involve the direct solution of the problem under consideration. In (16) we have the type of n equations (formed by giving to i the values $1, 2, \dots n$ successively), from which the values of $x_1, x_2, \dots x_n$, will be found, and those values substituted in (15) give the value of Prob. X as a function of the constants a_1, c_1, &c.

One conclusion deserving of notice, which is deducible from the above solution, is, that if the probabilities of the compound events $X_1, \dots X_{m-1}$, are the same as they would be were the events $x_1, \dots x_n$ entirely independent, and with given probabilities $c_1, \dots c_n$, then the probability of the event X will be the same as if calculated upon the same hypothesis of the absolute independence of the events $x_1, \dots x_n$. For upon the hypothesis supposed, the assumption

whence the relations determining $x_1, \dots x_n, t_1, \dots t_{m-1}$ will be of the following type (i varying from 1 to n),

$$\frac{(x_iX_m)\bar{t}_1 \dots \bar{t}_{m-1} + (x_iX_1)t_1\bar{t}_2 \dots \bar{t}_{m-1} + (x_iX_{m-1})\bar{t}_1 \dots \bar{t}_{m-2}t_{m-1}}{c_i}$$
$$= \frac{X_1t_1\bar{t}_2 \dots \bar{t}_{m-1}}{a_1} \dots = \frac{X_{m-1}\bar{t}_1 \dots \bar{t}_{m-2}t_{m-1}}{a_{m-1}} = V. \tag{11}$$

从上述系统中，我们接下来将消去记号t ^{1} , ...t m – ^{1}。我们有

$$t_1\bar{t}_2 \dots \bar{t}_{m-1} = \frac{a_1V}{X_1}, \quad \bar{t}_1 \dots \bar{t}_{m-2}t_{m-1} = \frac{a_{m-1}V}{X_{m-1}}. \tag{12}$$

将这些值代入(10)，我们得到

$$V = X_m\bar{t}_1 \dots \bar{t}_{m-1} + a_1V \dots + a_{m-1}V.$$

因此，

$$\bar{t}_1 \dots \bar{t}_{m-1} = \frac{(1 - a_1 \dots - a_{m-1})V}{X_m}.$$

现在让我们

$$a_m = 1 - a_1 \dots - a_{m-1}, \tag{13}$$

have

$$\bar{t}_1 \dots \bar{t}_{m-1} = \frac{a_mV}{X_m}. \tag{14}$$

现在借助(12)和(14)化简方程(9)，以及将(11)的第一行等于记号V所形成的方程；又用Prob.X代替Prob.t，我们有

$$\text{Prob. } X = \frac{a_1(XX_1)}{X_1} + \frac{a_2(XX_2)}{X_2} \dots + \frac{a_m(XX_m)}{X_m}, \tag{15}$$
$$\frac{a_1(x_iX_1)}{X_1} + \frac{a_2(x_iX_2)}{X_2} \dots + \frac{a_m(x_iX_m)}{X_m} = c_i; \tag{16}$$

其中 X m 和 a m 由(7)和(13)给出。这些方程涉及对所考虑问题的直接求解。在(16)中，我们得到 n 个方程的类型(通过依次给 i 取值 1 , 2 , ...n 而形成)，由此将求得 x ^{1} , x ^{2} , ...x n 的值，将这些值代入(15)即给出 Prob .X 作为常数 a ^{1}、c ^{1}等等的函数的值。

从上述解答可以推出一个值得注意的结论，即：如果复合事件 X ^{1} , ...X m – ^{1}的概率，与假定事件 x ^{1} , ...x n 完全独立且具有给定概率 c ^{1} , ...c n 时它们将会具有的概率相同，那么事件 X 的概率就将与按照事件 x ^{1} , ...x n 绝对独立的同一假说所计算出的概率相同。因为在所设的假说下，该假定

$x_1 = c_1$, $x_n = c_n$, in the quantitative system would give $X_1 = a_1$, $X_m = a_m$, whence (15) and (16) would give

$$\text{Prob. } X = (XX_1) + (XX_2) \ldots + (XX_m), \tag{17}$$

$$(x_i X_1) + (x_i X_2) \ldots + (x_i X_m) = c_i. \tag{18}$$

But since $X_1 + X_2 \ldots + X_m = 1$, it is evident that the second member of (17) will be formed by taking all the constituents that are contained in X, and giving them an algebraic significance. And a similar remark applies to (18). Whence those equations respectively give

$$\text{Prob. } X \text{ (logical)} = X \text{ (algebraic)},$$
$$x_i = c_i.$$

Wherefore, if $X = \phi(x_1, x_2, \ldots x_n)$, we have

$$\text{Prob. } X = \phi(c_1, c_2, \ldots c_n),$$

which is the result in question.

Hence too it would follow, that if the quantities $c_1, \ldots c_n$ were indeterminate, and no hypothesis were made as to the possession of a mean common value, the system (15) (16) would be satisfied by giving to those quantities any such values, $x_1, x_2, \ldots x_n$, as would satisfy the equations

$$X_1 = a_1 \ldots X_{m-1} = a_{m-1}, \quad X = a,$$

supposing the value of the element a, like the values of $a_1, \ldots a_{m-1}$, to be given by experience. 9. Before applying the general solution (15) (16), to the question of the probability of judgments, it will be convenient to make the following transformation. Let the data be

$$x_1 = c_1 \ldots x_n = c_n,$$
$$\text{Prob. } X_1 = a_1 \ldots \text{Prob. } X_{m-2} = a_{m-2};$$

and let it be required to determine Prob. X_{m-1}, the unknown value of which we will represent by a_{m-1}. Then in (15) and (16) we must change

$$\begin{array}{ll} X \text{ into } X_{m-1}, & \text{Prob. } X \text{ into } a_{m-1}, \\ X_{m-1} \text{ into } X_{m-2}, & a_{m-1} \text{ into } a_{m-2}, \\ X_m \text{ into } X_{m-1} + X_m, & a_m \text{ into } a_{m-1} + a_m; \end{array}$$

with these transformations, and observing that $(X_{m-1} X_r) = 0$, except when $r = m - 1$, and that it is then equal to X_{m-1}, the equations (15) (16) give

$$a_{m-1} = \frac{(a_{m-1} + a_m) X_{m-1}}{X_{m-1} + X_m}, \tag{19}$$

x 1 = c 1, x n = c n，在量的系统中将给出X 1 = a 1, X m = a m，由此(15)和(16)将给出

$$\text{Prob. } X = (XX_1) + (XX_2) \ldots + (XX_m), \tag{17}$$

$$(x_i X_1) + (x_i X_2) \ldots + (x_i X_m) = c_i. \tag{18}$$

但既然X 1 + X 2 ...+ X m = 1，显然(17)的第二项将由取X中所含的全部构成成分并赋予它们代数意义而形成。类似的评述也适用于(18)。由此那些方程分别给出

Prob .X(逻辑的)= X(代数的),

$$x_i = c_i.$$

Wherefore, if $X = \phi(x_1, x_2, \ldots x_n)$, we have

$$\text{Prob. } X = \phi(c_1, c_2, \ldots c_n),$$

这就是所讨论的结果。

由此还可推出，如果量c 1 , ...c n是不确定的，且未就具有某一共同平均值作出任何假说，那么系统(15)(16)将通过赋予这些量以任何满足方程的值x 1 , x 2 , ...x n而得到满足

$$X_1 = a_1 \ldots X_{m-1} = a_{m-1}, \quad X = a,$$

假定元素a的值，如同a 1 , ...a m – 1的值一样，是由经验给出的。9. 在将一般解(15)(16)应用于判断概率问题之前，作如下变换将是方便的。令数据为

$$x_1 = c_1 \ldots x_n = c_n,$$
$$\text{Prob. } X_1 = a_1 \ldots \text{Prob. } X_{m-2} = a_{m-2};$$

并设要求确定Prob .X m – 1，其未知值我们将用a m – 1表示。那么在(15)和(16)中我们必须改变

$$\begin{array}{ll} X \text{ into } X_{m-1}, & \text{Prob. } X \text{ into } a_{m-1}, \\ X_{m-1} \text{ into } X_{m-2}, & a_{m-1} \text{ into } a_{m-2}, \\ X_m \text{ into } X_{m-1} + X_m, & a_m \text{ into } a_{m-1} + a_m; \end{array}$$

作这些变换，并注意到(X m – 1 X r) = 0，除非r = m – 1，且此时它等于X m – 1，方程(15)(16)给出

$$a_{m-1} = \frac{(a_{m-1} + a_m) X_{m-1}}{X_{m-1} + X_m}, \tag{19}$$

$$\frac{a_1(x_iX_1)}{X_1}\ldots+\frac{a_{m-2}(x_iX_{m-2})}{X_{m-2}}+\frac{(a_{m-1}+a_m)(x_iX_{m-1}+x_iX_m}{X_{m-1}+X_m}. \tag{20}$$

Now from (19) we find

$$\frac{X_{m-1}}{a_{m-1}}=\frac{X_m}{a_m}=\frac{X_{m-1}+X_m}{a_{m-1}+a_m},$$

by virtue of which the last term of (20) may be reduced to the form

$$\frac{a_{m-1}(x_iX_{m-1})}{X_{m-1}}+\frac{a_m(x_iX_m)}{X_m}.$$

With these reductions the system (17) and (18) may be replaced by the following symmetrical one, viz.:

$$\frac{X_{m-1}}{a_{m-1}}=\frac{X_m}{a_m}, \tag{21}$$

$$\frac{a_1(x_iX_1)}{X_1}+\frac{a_2(x_iX_2)}{X_2}\ldots+\frac{a_m(x_iX_m)}{X_m}=c_i. \tag{22}$$

These equations, in connexion with (7) and (13), enable us to determine a_{m-1}, as a function of $c_1\ldots c_n, a_1\ldots a_{m-2}$, the numerical data supposed to be furnished by experience. We now proceed to their application.

PROPOSITION III.

10. *Given any system of probabilities drawn from recorded instances of unanimity, or of assigned numerical majority in the decisions of a deliberative assembly; required, upon a certain determinate hypothesis, the mean probability of correct judgment for a member of the assembly.*

In what way the probabilities of unanimous decision and of specific numerical majorities may be determined from experience, has been intimated in a former part of this chapter. Adopting the notation of Prop. I. we shall represent the events whose probabilities are given by the functions $X_1, X_2, \ldots X_{m-1}$. It has appeared from the very nature of the case that these events are mutually exclusive, and that the functions by which they are represented are symmetrical with reference to the symbols $x_1, x_2, \ldots x_n$. Those symbols we continue to use in the same sense as in Prop. I., viz., by x_i we understand that event which consists in the formation of a correct opinion by the i^{th} member of the assembly.

Now the immediate data of experience are—

$$\text{Prob. } X_1=a_1,\quad \text{Prob. } X_2=a_2\ldots\text{Prob. } X_{m-2}=a_{m-2}, \tag{1}$$

$$\text{Prob. } X_{m-1}=a_{m-1}. \tag{2}$$

$X_1\ldots X_{m-1}$ being functions of the logical symbols $x_1,\ldots x_n$ to the probabilities of the events denoted by which, we shall assign the indeterminate value c. Thus we shall have

$$\text{Prob. } x_1=\text{Prob. } x_2\cdots=\text{Prob. } x_n=c. \tag{3}$$

$$\frac{a_1(x_iX_1)}{X_1}\ldots+\frac{a_{m-2}(x_iX_{m-2})}{X_{m-2}}+\frac{(a_{m-1}+a_m)(x_iX_{m-1}+x_iX_m}{X_{m-1}+X_m}. \tag{20}$$

Now from (19) we find

$$\frac{X_{m-1}}{a_{m-1}}=\frac{X_m}{a_m}=\frac{X_{m-1}+X_m}{a_{m-1}+a_m},$$

凭借这一点，(20)式的最后一项可以化为以下形式

$$\frac{a_{m-1}(x_iX_{m-1})}{X_{m-1}}+\frac{a_m(x_iX_m)}{X_m}.$$

With these reductions the system (17) and (18) may be replaced by the following symmetrical one, viz.:

$$\frac{X_{m-1}}{a_{m-1}}=\frac{X_m}{a_m}, \tag{21}$$

$$\frac{a_1(x_iX_1)}{X_1}+\frac{a_2(x_iX_2)}{X_2}\ldots+\frac{a_m(x_iX_m)}{X_m}=c_i. \tag{22}$$

这些方程，连同(7)和(13)，使我们能够确定 a m – 1，作为 c 1 ...c n , a 1 ...a m – 2的函数，其中的数值数据被假定由经验提供。我们现在着手它们的应用。

命题三。

10. 给定由记录到的一致同意事例、或审议性集会决议中指定的数值多数所引出的任何概率系统；要求在某一确定的假说下，求出集会中某一成员作出正确判断的平均概率。

一致判决和特定数值多数派的概率可以以何种方式从经验中确定，已在本章前一部分中有所提示。采用命题 i.的记号，我们将用函数 X 1 , X 2 , ...X m – 1来表示其概率已给定的那些事件。从情况的本质来看，这些事件是互斥的，并且用以表示它们的函数关于记号 x 1 , x 2 , ...x n 是对称的。我们继续在与命题 i.相同的意义上使用这些记号，即，我们用 x i 来理解那个由集会中第 i 个成员形成正确意见所构成的事件。

经验的直接材料是--

$$\text{Prob. } X_1=a_1,\quad \text{Prob. } X_2=a_2\ldots\text{Prob. } X_{m-2}=a_{m-2}, \tag{1}$$

$$\text{Prob. } X_{m-1}=a_{m-1}. \tag{2}$$

X 1 ...X m – 1是逻辑记号 x 1 , ...x n 的函数，对于这些记号所表示的事件的概率，我们将赋予不确定值 c。于是我们将有

$$\text{Prob. } x_1=\text{Prob. } x_2\cdots=\text{Prob. } x_n=c. \tag{3}$$

Now it has been seen, Prop. I., that the immediate data (1) (2), unassisted by any hypothesis, merely conduct us to a restatement of the problem. On the other hand, it is manifest that if, adopting the methods of Laplace and Poisson, we employ the system (3) alone as the data for the application of the method of this work, finally comparing the results obtained with the experimental system (1) (2), we are relying *wholly* upon a doubtful hypothesis,—the independence of individual judgments. But though we ought not wholly to rely upon this hypothesis, we cannot wholly dispense with it, or with some equivalent substitute. Let us then examine the consequences of a *limited* independence of the individual judgments; the conditions of limitation being furnished by the apparently superfluous data. From the system (1) (3) let us, by the method of this work, determine Prob. X_{m-1}, and, comparing the result with (2), determine c. Even here an arbitrary power of selection is claimed. But it is manifest from Prop. I. that something of this kind is unavoidable, if we would obtain a definite solution at all. As to the principle of selection, I apprehend that the equation (2) reserved for final comparison should be that which, from the magnitude of its numerical element a_{m-1}, is esteemed the most important of the primary series furnished by experience.

Now, from the mutually exclusive character of the events denoted by the functions $X_1, X_2, \ldots X_{m-1}$, the concluding equations of the previous proposition become applicable. On account of the symmetry of the same functions, and the reduction of the system of values denoted by c_i, to a single value c, the equations represented by (22) become identical, the values of $x_1, x_2, \ldots x_n$ become equal, and may be replaced by a single value x, and we have simply,

$$\frac{X_{m-1}}{a_{m-1}} = \frac{X_m}{a_m}, \tag{4}$$

$$\frac{a_1(xX_1)}{X_1} + \frac{a_2(xX_2)}{X_2} \ldots + \frac{a_m(xX_m)}{X_m} = c. \tag{5}$$

The following is the nature of the solution thus indicated:

The functions $X_1, \ldots X_{m-1}$, and the values $a_1, \ldots a_{m-1}$, being given in the data, we have first,

$$X_m = 1 - X_1 \ldots - X_{m-1},$$
$$a_m = 1 - a_1 \ldots - a_{m-1}.$$

From each of the functions $X_1, X_2, \ldots X_m$ thus given or determined, we must select those constituents which contain a particular symbol, as x_1 for a factor. This will determine the functions (xX_1), (xX_2), &c., and then in all the functions we must change $x_1, x_2, \ldots x_n$ individually to x. Or we may regard any algebraic function X_i in the system (4) (5) as expressing the probability of the event denoted by the logical function X_i, on the supposition that the logical symbols $x_1, x_2, \ldots x_n$ denote independent events whose common probability is x. On the same supposition (xX_i) would denote the probability of the concurrence of any particular event of the series $x_1, x_2, \ldots x_n$ with X_i. The forms of X_i, (xX_i), &c. being determined, the equation (4) gives the value of x, and this,

现在我们已经看到，命题 i.表明，直接材料(1)(2)在没有任何假说辅助的情况下，仅仅引导我们重新陈述问题。另一方面，显而易见的是，如果我们采用拉普拉斯和泊松的方法，仅仅把系统(3)作为应用本工作方法的材料，最后将所得结果与实验系统(1)(2)进行比较，我们便完全依赖一个可疑的假说--个体判断的独立性。但是，尽管我们不应完全依赖这个假说，我们也不能完全抛弃它，或抛弃某种等效的替代物。那么，让我们考察个体判断的有限独立性的后果；限制的条件由那些表面上多余的材料提供。从系统(1)(3)出发，让我们用本工作的方法确定 Prob.X m – 1，并将结果与(2)比较，确定 c。即使在这里，也要求一种任意的选择权力。但从命题 i.可以明显看出，如果我们要得到一个确定的解，某种这类东西是不可避免的。至于选择的原则，我理解，留作最终比较的方程(2)应当是那个因其数值元素 a m – 1 的大小而被认为是由经验提供的主要系列中最重要的方程。

现在，由于函数 X ^{1} , X ^{2} , ...X m – ^{1}所表示的事件的互斥性质，前一命题的结论方程变得适用。由于这些函数的对称性，以及由 c i 所表示的数值系统归结为单一值 c，由(22)所表示的方程变得完全相同，x ^{1} , x ^{2} , ...x n 的值变得相等，并可由单一值 x 代替，于是我们简单地有，

$$\frac{X_{m-1}}{a_{m-1}} = \frac{X_m}{a_m}, \tag{4}$$

$$\frac{a_1(xX_1)}{X_1} + \frac{a_2(xX_2)}{X_2} \ldots + \frac{a_m(xX_m)}{X_m} = c. \tag{5}$$

The following is the nature of the solution thus indicated:

The functions $X_1, \ldots X_{m-1}$, and the values $a_1, \ldots a_{m-1}$, being given in the data, we have first,

$$X_m = 1 - X_1 \ldots - X_{m-1},$$
$$a_m = 1 - a_1 \ldots - a_{m-1}.$$

从如此给定或确定的每个函数 X ^{1} , X ^{2} , ...X m 中，我们必须选出那些含有一个特定记号作为因子的成分，如 x ^{1}。这将确定函数(xX ^{1})、(xX ^{2})等等，然后在所有函数中我们必须将 x ^{1} , x ^{2} , ...x n 逐一变为 x。或者我们可以将系统(4)(5)中的任何代数函数 X i 视为表示逻辑函数 X i 所表示的事件的概率，前提是逻辑记号 x ^{1} , x ^{2} , ...x n 表示其共同概率为 x 的独立事件。在同一前提下，(xX i)将表示系列 x ^{1} , x ^{2} , ...x n 中任何特定事件与 X i 同时发生的概率。在 X i、(xX i)等等的形式确定之后，方程(4)给出 x 的值，而这个值，

substituted in (5), determines the value of the element c required. Of the two values which its solution will offer, one being greater, and the other less, than $\frac{1}{2}$, the greater one must be chosen, whensoever, upon general considerations, it is thought more probable that a member of the assembly will judge correctly, than that he will judge incorrectly.

Here then, upon the assumed principle that the largest of the values a_{m-1} shall be reserved for final comparison in the equation (2), we possess a definite solution of the problem proposed. And the same form of solution remains applicable should any other equation of the system, upon any other ground, as that of superior accuracy, be similarly reserved in the place of (2).

11. Let us examine to what extent the above reservation has influenced the final solution. It is evident that the equation (5) is quite independent of the choice in question. So is likewise the second member of (4). Had we reserved the function X_1, instead of X_{m-1}, the equation for the determination of x would have been

$$\frac{X_1}{a_1} = \frac{X_m}{a_m}, \tag{6}$$

but the value of x thence determined would still have to be substituted in the same final equation (5). We know that were the events $x_1, x_2, \ldots x_n$ really independent, the equations (4), (6), and all others of which they are types, would prove equivalent, and that the value of x furnished by any one of them would be the true value of c. This affords a means of verifying (5). For if that equation be correct, it ought, under the above circumstances, to be satisfied by the assumption $c = x$. In other words, the equation

$$\frac{a_1(xX_1)}{X_1} + \frac{a_2(xX_2)}{X_2} \ldots + \frac{a_m(xX_m)}{X_m} = x \tag{7}$$

ought, on solution, to give the same value of x as the equation (4) or (6). Now this will be the case. For since, by hypothesis,

$$\frac{X_1}{a_1} = \frac{X_2}{a_2} \ldots = \frac{X_m}{a_m},$$

we have, by a known theorem,

$$\frac{X_1}{a_1} = \frac{X_2}{a_2} \ldots = \frac{X_m}{a_m} = \frac{X_1 + X_2 \ldots + X_m}{a_1 + a_2 \ldots + a_m} = 1.$$

Hence (7) becomes on substituting a_1 for X_1, &c.

$$(xX_1) + (xX_2) \ldots (xX_m) = x$$

a mere identity.

Whenever, therefore, the events $x_1, x_2, \ldots x_n$ are really independent, the system (4) (5) is a correct one, and is independent of the arbitrariness of the first step of the process by which it was obtained. When the said events are not independent, the final system of equations will possess, leaving in abeyance the

代入(5)后，确定所需的元素 c 的值。在其解将提供的两个值中，一个大于、另一个小于12,
当根据一般考虑认为集会成员正确判断比错误判断更为可能时，必须选择较大的那个值。
–

那么在这里，根据所假定的原则，即 a m – 1中的最大值应留待方程(2)中作最终比较，我们就对所提出的问题拥有一个确定的解。而且，如果系统中的任何其他方程，基于任何其他理由，例如更高的精确性，被同样地留出以代替(2)，同一形式的解仍然适用。

11. 让我们考察上述保留在多大程度上影响了最终的解。显然，方程(5)与所讨论的选择完全无关。(4)的第二项同样如此。如果我们保留函数 X 1，而不是 X m – 1，那么用于确定 x 的方程将是

$$\frac{X_1}{a_1} = \frac{X_m}{a_m}, \tag{6}$$

但由此确定的 x 的值仍须代入同一个最终方程(5)。我们知道，若事件 x 1 , x 2 , ...x n 确实独立，则方程(4)、(6)以及它们作为其类型的其他一切方程都将被证明是等价的，而由其中任何一个所提供的 x 的值都将是 c 的真值。这提供了一种验证(5)的手段。因为若该方程正确，则在上述情形下，它应当由假设 c = x 所满足。换言之，方程

$$\frac{a_1(xX_1)}{X_1} + \frac{a_2(xX_2)}{X_2} \ldots + \frac{a_m(xX_m)}{X_m} = x \tag{7}$$

在求解时，应当给出与方程(4)或(6)相同的 x 值。现在情况将会如此。因为根据假说，

$$\frac{X_1}{a_1} = \frac{X_2}{a_2} \ldots = \frac{X_m}{a_m},$$

由一定理可知，

$$\frac{X_1}{a_1} = \frac{X_2}{a_2} \ldots = \frac{X_m}{a_m} = \frac{X_1 + X_2 \ldots + X_m}{a_1 + a_2 \ldots + a_m} = 1.$$

因此，以 a 1代换 X 1等等之后，(7)变为

$$(xX_1) + (xX_2) \ldots (xX_m) = x$$

一个纯粹的同一式。

因此，只要事件x1, x2, . . . xn真正独立，系统(4)(5)就是正确的，并且独立于获得它的过程第一步的任意性。当所述事件不独立时，最终的方程系统将具有，暂时搁置

principle of selection above stated, an arbitrary element. But from the persistent form of the equation (5) it may be inferred that the solution is arbitrary in a less degree than the solutions to which the hypothesis of the absolute independence of the individual judgments would conduct us. The discussion of the limits of the value of c, as dependent upon the limits of the value of x, would determine such points.

These considerations suggest to us the question whether the equation (7), which is symmetrical with reference to the functions $X_1, X_2, \dots X_m$, free from any arbitrary elements, and rigorously exact when the events $x_1, x_2, \dots x_n$ are really independent, might not be accepted as a mean general solution of the problem. The proper mode of determining this point would, I conceive, be to ascertain whether the value of x which it would afford would, in general, fall within the limits of the value of c, as determined by the systems of equations of which the system (4), (5), presents the type. It seems probable that under ordinary circumstances this would be the case. Independently of such considerations, however, we may regard (7) as itself the expression of a certain principle of solution, viz., that regarding $X_1, X_2, \dots X_m$ as exclusive *causes* of the event whose probability is x, we accept the probabilities of those causes $a_1, a_2, \dots a_m$ from experience, but form the conditional probabilities of the event as dependent upon such causes,

$$\frac{(xX_1)}{x_1}, \quad \frac{(xX_2)}{X_2}, \text{ \&c. (XVII. Prop I.)}$$

on the hypothesis of the independence of individual judgments, and so deduce the equation (7). I conceive this, however, to be a less rigorous, though possibly, in practice a more convenient mode of procedure than that adopted in the general solution.

12. It now only remains to assign the particular forms which the algebraic functions X_i, (xX_i), &c. in the above equations assume when the logical function X_i represents that event which consists in r members of the assembly voting one way, and $n - r$ members the other way. It is evident that in this case the algebraic function X_i expresses what the probability of the supposed event would be were the events $x_1, x_2, \dots x_n$ independent, and their common probability measured by x. Hence we should have, by Art. 3,

$$X_i = \frac{n(n-1)\dots(n-r+1)}{1 \cdot 2 \dots r}\{x^r + (1-x)^{n-r}\}.$$

Under the same circumstances (xX_i) would represent the probability of the compound event, which consists in a particular member of the assembly forming a correct judgment, conjointly with the general state of voting recorded above. It would, therefore, be the probability that a particular member votes correctly, while of the remaining $n-1$ members, $r-1$ vote correctly; or that the same member votes correctly, while of the remaining $n-1$ members r vote incorrectly. Hence

$$(xX_i) = \frac{(n-1)(n-2)\dots(n-r+1)}{1 \cdot 2 \dots r-1}x^r + \frac{(n-1)(n-2)\dots(n-r)}{1 \cdot 2 \dots r}x^{n-r}.$$

上述选择原则，是一个任意的元素。但从方程(5)的持久形式可以推断，该解在任意性程度上低于各个别判断绝对独立的假说所会引导我们得出的那些解。关于c的值以x的值为转移的界限的讨论，将会确定这样的要点。

这些考虑向我们提出了这样一个问题：方程(7)关于函数 X 1 , X 2 , ...X m 是对称的，不含任何任意元素，并且当事件 x 1 , x 2 , ...x n 确实独立时是严格精确的，那么它是否可以被接受为该问题的一种平均的一般解。我认为，确定这一点的恰当方式将是查明：它所提供的 x 的值是否一般会落在 c 的值的界限之内，而后者是由以系统(4)、(5)为其类型的那些方程系统所确定的。在通常情形下，情况似乎很可能如此。然而，独立于这类考虑，我们可以把(7)本身视为某种解的原则的表述，即：把 X 1 , X 2 , ...X m 视为其概率为 x 的事件的排他性原因，我们从经验中接受那些原因的概率 a 1 , a 2 , ...a m，但把该事件依赖于这些原因的条件概率构造成

$$\frac{(xX_1)}{x_1}, \quad \frac{(xX_2)}{X_2}, \text{ \&c. (XVII. Prop I.)}$$

在个别判断独立的假说之下，并由此推出方程(7)。然而我认为，这是一种不如一般解法中所采用的那样严格、尽管在实践中可能更为方便的程序方式。

12. 现在只剩下指出上述方程中的代数函数 X i、(xX i) 等在逻辑函数 X i 表示由 r 名成员以一方投票、n – r 名成员以另一方投票所构成的那个事件时所采取的具体形式。显然，在此情形下，代数函数 X i 所表达的是：若事件 x 1 , x 2 , ...x n 独立，且其共同概率以 x 度量，则所设事件的概率将会是什么。因此，根据第3条，我们应当有

$$X_i = \frac{n(n-1)\dots(n-r+1)}{1 \cdot 2 \dots r}\{x^r + (1-x)^{n-r}\}.$$

在同一情形下，(xX i) 将表示复合事件的概率，该复合事件由议会中某一特定成员作出正确判断与上述记录的一般投票状态共同构成。因此，它将是如下情形的概率：某一特定成员投票正确，而在其余 n – 1 名成员中，有 r – 1 名投票正确；或者同一成员投票正确，而在其余 n – 1 名成员中有 r 名投票不正确。因此

$$(xX_i) = \frac{(n-1)(n-2)\dots(n-r+1)}{1 \cdot 2 \dots r-1}x^r + \frac{(n-1)(n-2)\dots(n-r)}{1 \cdot 2 \dots r}x^{n-r}.$$

PROPOSITION IV.

13. *Given any system of probabilities drawn from recorded instances of unanimity, or of assigned numerical majority in the decisions of a criminal court of justice, required upon hypotheses similar to those of the last proposition, the mean probability c of correct judgment for a member of the court, and the general probability k of guilt in an accused person.*

The solution of this problem differs in but a slight degree from that of the last, and may be referred to the same general formulæ, (4) and (5), or (7). It is to be observed, that as there are two elements, c and k, to be determined, it is necessary to reserve two of the functions $X_1, X_2, \dots X_{m-1}$, let us suppose X_1, and X_{m-1}, for final comparison, employing either the remaining $m-3$ functions in the expression of the data, or the two respective sets $X_2, X_3, \dots X_{m-1}$, and $X_1, X_2, \dots X_{m-2}$. In either case it is supposed that there must be at least two original independent data. If the equation (7) be alone employed, it would in the present instance furnish two equations, which may thus be written:

$$\frac{a_1(xX_1)}{X_1} + \frac{a_2(xX_2)}{X_2} \dots + \frac{a_m(xX_m)}{X_m} = x, \tag{1}$$

$$\frac{a_1(kX_1)}{X_1} + \frac{a_2(kX_2)}{X_2} \dots + \frac{a_m(kX_m)}{X_m} = k. \tag{2}$$

These equations are to be employed in the following manner:— Let $x_1, x_2, \dots x_n$ represent those events which consist in the formation of a correct opinion by the members of the court respectively. Let also w represent that event which consists in the guilt of the accused member. By the aid of these symbols we can logically express the functions $X_1, X_2, \dots X_{m-1}$, whose probabilities are given, as also the function X_m. Then from the function X_1 select those constituents which contain, as a factor, any particular symbol of the set $x_1, x_2, \dots x_n$, and also those constituents which contain as a factor w. In both results change $x_1, x_2, \dots x_n$ severally into x, and w into k. The above results will give (xX_1) and (kX_1). Effecting the same transformations throughout, the system (1), (2) will, upon the particular hypothesis involved, determine x and k.

14. We may collect from the above investigations the following facts and conclusions:

1st. That from the mere records of agreement and disagreement in the opinions of any body of men, no definite numerical conclusions can be drawn respecting either the probability of correct judgment in an individual member of the body, or the merit of the questions submitted to its consideration.

2nd. That such conclusions may be drawn upon various distinct hypotheses, as—1st, Upon the usual hypothesis of the absolute independence of individual judgments; 2ndly, upon certain definite modifications of that hypothesis warranted by the actual data; 3rdly, upon a distinct principle of solution suggested by the appearance of a common form in the solutions obtained by the modifications above adverted to.

Lastly. That whatever of doubt may attach to the final results, rests not upon the imperfection of the method, which adapts itself equally to all

命题四。

13. 给定从记录到的一致同意事例、或刑事法庭判决中指定的数值多数中得出的任何概率系统，要求在与上一命题相似的假说之下，求出法庭成员正确判断的平均概率，以及被告人有罪的一般概率。

这个问题的解与上一个问题的解只有细微差别，可以归到同一组一般公式(4)和(5)，或(7)。应当注意，由于有两个元素 c 和 k 需要确定，就必须保留两个函数 X ^{1} , X ^{2} , ...X m – ^{1}，让我们假定 X ^{1}和 X m – ^{1}作最终比较，在数据的表达中或者采用其余 m – 3 个函数，或者采用两个各自的集合 X ^{2} , X ^{3} , ...X m – ^{1}和 X ^{1} , X ^{2} , ...X m – ^{2}。无论哪种情形，都假定必须至少有两个原始独立数据。若仅采用方程(7)，则在当前情形下它将提供两个方程，可写成如下形式：

$$\frac{a_1(xX_1)}{X_1} + \frac{a_2(xX_2)}{X_2} \dots + \frac{a_m(xX_m)}{X_m} = x, \tag{1}$$

$$\frac{a_1(kX_1)}{X_1} + \frac{a_2(kX_2)}{X_2} \dots + \frac{a_m(kX_m)}{X_m} = k. \tag{2}$$

这些方程将按以下方式使用：- 设 x ^{1} , x ^{2} , ...x n 分别表示那些由法庭成员形成正确意见的事件。又设 w 表示由被控成员有罪所构成的那个事件。借助这些记号，我们可以在逻辑上表达函数 X ^{1} , X ^{2} , ...X m – ^{1}，其概率已给出，也可表达函数 X m。然后从函数 X ^{1}中选出那些以集合 x ^{1} , x ^{2} , ...x n 中任一特定记号作为因子的成分，也选出那些以 w 作为因子的成分。在两个结果中，将 x ^{1} , x ^{2} , ...x n 分别变为 x，并将 w 变为 k。上述结果将给出(xX ^{1})和(kX ^{1})。在整体上施行同样的变换，系统(1),(2)将根据所涉及的特定假说确定 x 和 k。

14. 我们可以从上述考察中汇集以下事实和结论：

第一，仅凭任何一群人意见中一致与分歧的记录，既不能就其中个别成员作出正确判断的概率得出任何确定的数值结论，也不能就该群体所审议的问题的价值得出任何确定的数值结论。

第二，这样的结论可以依据若干不同的假说得出，即：第一，依据各个判断绝对独立的通常假说；第二，依据由实际数据所保证的对该假说的某些确定修正；第三，依据一种独特的解法原则，该原则是由上述修正所得的解中出现共同形式而提示的。

最后，无论最终结果有何疑问，都不在于方法的不完善，因为该方法同样适用于所有情况

hypotheses, but upon the uncertainty of the hypotheses themselves.

It seems, however, probable that with even the widest limits of hypothesis, consistent with the taking into account of *all* the data of experience, the deviation of the results obtained would be but slight, and that their mean values might be determined with great confidence by the methods of Prop. III. Of those methods I should be disposed to give the preference to the first. Such a principle of mean solution having been agreed upon, other considerations seem to indicate that the values of c and k for tribunals and assemblies possessing a definite constitution, and governed in their deliberations by fixed rules, would remain nearly constant, subject, however, to a small secular variation, dependent upon the progress of knowledge and of justice among mankind. There exist at present few, if any, data proper for their determination.

假设，而是基于假设本身的不确定性。

然而，似乎很可能的是，即使在假说的最宽泛限度内，只要与考虑所有经验材料相一致，所得结果的偏差也只会是微小的，并且它们的平均值可以借助命题III的方法以极大的信心确定。在那些方法中，我倾向于优先选择第一种。一旦商定了这样一种平均解原则，其他考虑似乎表明，对于具有确定宪制、并在其审议中受固定规则支配的法庭和议会，c 和 k 的值将几乎保持恒定，然而会受制于一种小的长期变化，这种变化取决于人类知识和正义的进步。目前适合确定它们的材料即使有，也很少。

Chapter XXII

ON THE NATURE OF SCIENCE, AND THE CONSTITUTION OF THE INTELLECT.

1. What I mean by the constitution of a system is the aggregate of those causes and tendencies which produce its observed character, when operating, without interference, under those conditions to which the system is conceived to be adapted. Our judgment of such adaptation must be founded upon a study of the circumstances in which the system attains its freest action, produces its most harmonious results, or fulfils in some other way the apparent design of its construction. There are cases in which we know distinctly the causes upon which the operation of a system depends, as well as its conditions and its end. This is the most perfect kind of knowledge relatively to the subject under consideration. There are also cases in which we know only imperfectly or partially the causes which are at work, but are able, nevertheless, to determine to some extent the laws of their action, and, beyond this, to discover general tendencies, and to infer ulterior purpose. It has thus, I think rightly, been concluded that there is a moral faculty in our nature, not because we can understand the special instruments by which it works, as we connect the organ with the faculty of sight, nor upon the ground that men agree in the adoption of universal rules of conduct; but because while, in some form or other, the sentiment of moral approbation or disapprobation manifests itself in all, it tends, wherever human progress is observable, wherever society is not either stationary or hastening to decay, to attach itself to certain classes of actions, consentaneously, and after a manner indicative both of permanency and of law. Always and everywhere the manifestation of Order affords a presumption, not measurable indeed, but real (XX. 22), of the fulfilment of an end or purpose, and the existence of a ground of orderly causation.

2. The particular question of the constitution of the intellect has, it is almost needless to say, attracted the efforts of speculative ingenuity in every age. For it not only addresses itself to that desire of knowledge which the greatest masters of ancient thought believed to be innate in our species, but it adds to the ordinary strength of this motive the inducement of a human and personal interest. A genuine devotion to truth is, indeed, seldom partial in its aims, but while it prompts to expatiate over the fair fields of outward

第二十二章

论科学的性质与智力的构成

1. 我所理解的一个系统的 constitution，是指那些原因和倾向的总和，这些原因和倾向在系统被认为适应的那些条件下不受干扰地运作时，产生其被观察到的特性。我们对这种适应的判断必须建立在对以下情况的研究之上：系统在其中达到最自由的作用、产生最和谐的结果，或以某种其他方式实现其构造的明显设计。有些情况是我们清楚地知道一个系统的运作所依赖的原因，也知道它的条件和目的。这是相对于所考虑的主词而言最完美的一种知识。也有些情况是我们只能不完善地或部分地知道起作用的原因，但仍然能够在某种程度上确定其作用的规律，并且在此之外发现一般倾向并推断进一步的目的。因此，我认为正确地，人们得出结论说我们的本性中有一种道德能力，不是因为我们能够理解它运作的特殊工具，如同我们把器官与视觉能力联系起来那样，也不是因为人们在采纳普遍行为规则上一致；而是因为，虽然道德赞许或不赞许的情感以某种形式在所有人身上表现出来，但无论在哪里只要人类进步可被观察，无论在哪里社会不是停滞就是趋于衰败，它都倾向于同时附着于某些类别的行为，并且以一种既表明持久性又表明规律性的方式附着于它们。秩序的表现总是并且处处提供一种推定--确实不可度量，但却是真实的(XX.22)--即某个目的或意图得到实现，以及有序因果关系的基础存在。

2. 几乎无需说，关于理智 constitution 的这一特定问题，在每个时代都吸引了思辨才智的努力。因为它不仅诉诸于古代思想最伟大的大师们相信内在于我们物种中的那种知识欲，而且在这种动机的通常力量之外，又加上了人类的和个人的兴趣的诱因。对真理的真正献身，确实很少在其目标上是偏狭的，但当它促使人们漫游于外部

observation, forbids to neglect the study of our own faculties. Even in ages the most devoted to material interests, some portion of the current of thought has been reflected inwards, and the desire to comprehend that by which all else is comprehended has only been baffled in order to be renewed.

It is probable that this *pertinacity* of effort would not have been maintained among sincere inquirers after truth, had the conviction been general that such speculations are hopelessly barren. We may conceive that it has been felt that if something of error and uncertainty, always incidental to a state of partial information, must ever be attached to the results of such inquiries, a residue of positive knowledge may yet remain; that the contradictions which are met with are more often verbal than real; above all, that even *probable* conclusions derive here an interest and a value from their subject, which render them not unworthy to claim regard beside the more definite and more splendid results of physical science. Such considerations seem to be perfectly legitimate. Insoluble as many of the problems connected with the inquiry into the nature and constitution of the mind must be presumed to be, there are not wanting others upon which a limited but not doubtful knowledge, others upon which the conclusions of a highly probable analogy, are attainable. As the realms of day and night are not strictly conterminous, but are separated by a crepuscular zone, through which the light of the one fades gradually off into the darkness of the other, so it may be said that every region of positive knowledge lies surrounded by a debateable and speculative territory, over which it in some degree extends its influence and its light. Thus there may be questions relating to the constitution of the intellect which, though they do not admit, in the present state of knowledge, of an absolute decision, may receive so much of reflected information as to render their probable solution not difficult; and there may also be questions relating to the nature of science, and even to particular truths and doctrines of science, upon which they who accept the general principles of this work cannot but be led to entertain positive opinions, differing, it may be, from those which are usually received in the present day.[1] In what follows I shall recapitulate some of the more definite conclusions established in the former parts of this treatise, and shall then indicate one or two trains of thought, connected with the general objects above adverted to, which they seem to me calculated to suggest.

3. Among those conclusions, relating to the intellectual constitution, which may be considered as belonging to the realm of positive knowledge, we may reckon the scientific laws of thought and reasoning, which have formed the basis of the general methods of this treatise, together with the principles, Chap, v., by which their application has been determined. The resolution of the domain of thought into two spheres, distinct but coexistent (IV. XI.); the subjection of the

[1]The following illustration may suffice:–

It is maintained by some of the highest modern authorities in grammar that conjunctions connect propositions only. Now, without inquiring directly whether this opinion is sound or not, it is obvious that it cannot consistently be held by any who admit the scientific principles of this treatise; for to such it would seem to involve a denial, either, 1st, of the possibility of *performing*, or 2ndly, of the possibility of *expressing*, a mental operation, the laws of which, viewed in both these relations, have been investigated and applied in the present work—(Latham on the English Language; Sir John Stoddart's Universal Grammar, &c.)

3. 在那些关于理智构造、可被视为属于实证知识领域的结论中，我们可以计入已构成本论著一般方法之基础的思维与推理的科学规律，以及第五章中据以确定其应用的那些原则。将思想领域划分为两个不同但共存的领域(IV.XI.)；使

很可能，如果这样一种信念--即这类思辨毫无希望地贫瘠--是普遍的，那么在对真理的真诚探究者中间，这种努力的顽强性就不会维持下去。我们可以设想，人们已经感到：如果某种错误和不确定性--总是伴随部分信息状态而来--必定附着于这类探究的结果，那么仍可能留下一点实证知识；所遇到的反驳更多是言辞上的而非实质上的；最重要的是，甚至或然的结论也在这里从其主词中获得一种兴趣和价值，使它们并非不值得在物理科学更确定、更辉煌的结果旁边受到重视。这样的考虑似乎是完全正当的。尽管与探究心灵的本性和 constitution 有关的许多问题必须被假定为不可解，但也不乏另一些问题，对它们可以获得有限但并非可疑的知识，另一些问题，对它们可以获得高度或然类比的结论。正如白昼与黑夜的领域并非严格接壤，而是被一个朦胧地带分隔开来，通过这个地带，一方的光逐渐消隐于另一方的黑暗之中，同样可以说，每一个实证知识领域都被一个可争论的和思辨的领域所环绕，它在一定程度上将其影响和光照扩展到那个领域之上。因此，可能存在一些与理智 constitution 有关的问题，尽管在目前的知识状态下它们不允许绝对的决定，却可能接受如此多的反射信息，以致它们的或然解决并不困难；也可能存在一些与科学的性质、甚至与科学的特定真理和学说有关的问题，对于这些问题，接受本著作一般原则的人不可能不被引导去持有确定的意见，这些意见也许不同于当今通常接受的意见。[1]在下文中，我将重述本论著前几部分所建立的若干较为确定的结论，然后指出一两条与上述一般对象相关的思路，我认为这些结论似乎能够提示这些思路。

3. 在那些与理智的构造有关、可以被视为属于实证知识领域的结论之中，我们可以计入那些构成本论著一般方法之基础的思维与推理的科学规律，连同第五章中据以确定其应用的那些原则。将思维领域分解为两个不同但共存的领域(IV.XI.)；使

1 下面的例示或许足够：--

一些现代最权威的语法学家主张，连词只连接命题。现在，不必直接探究这一观点是否成立，显而易见的是，任何承认本论著科学原理的人都不能一贯地坚持它；因为对这些人来说，这似乎意味着要么否认执行的可能性，要么否认表达一种心智活动的可能性，而这种心智活动的规律，从这两种关系来看，已在本著作中得到了研究和应用--(Latham，《论英语语言》；John Stoddart 爵士，《普遍语法》，等等)

intellectual operations within those spheres to a common system of laws (XI.); the general mathematical character of those laws, and their actual expression (II. III.); the extent of their affinity with the laws of thought in the domain of number, and the point of their divergence therefrom; the dominant character of the two limiting conceptions of universe and eternity among all the subjects of thought with which Logic is concerned; the relation of those conceptions to the fundamental conception of unity in the science of number,— these, with many similar results, are not to be ranked as merely probable or analogical conclusions, but are entitled to be regarded as truths of science. Whether they be termed metaphysical or not, is a matter of indifference. The nature of the evidence upon which they rest, though in kind distinct, is not inferior in value to any which can be adduced in support of the general truths of physical science.

Again, it is agreed that there is a certain order observable in the progress of all the exacter forms of knowledge. The study of every department of physical science begins with observation, it advances by the collation of facts to a presumptive acquaintance with their connecting law, the validity of such presumption it tests by new experiments so devised as to augment, if the presumption be well founded, its probability indefinitely; and finally, the law of the phænomenon having been with sufficient confidence determined, the investigation of causes, conducted by the due mixture of hypothesis and deduction, crowns the inquiry. In this advancing order of knowledge, the particular faculties and laws whose nature has been considered in this work bear their part. It is evident, therefore, that if we would impartially investigate either the nature of science, or the intellectual constitution in its relation to science, no part of the two series above presented ought to be regarded as isolated. More especially ought those truths which stand in any kind of *supplemental* relation to each other to be considered in their mutual bearing and connexion.

4. Thus the necessity of an experimental basis for all positive knowledge, viewed in connexion with the existence and the peculiar character of that system of mental laws, and principles, and operations, to which attention has been directed, tends to throw light upon some important questions by which the world of speculative thought is still in a great measure divided. How, from the particular facts which experience presents, do we arrive at the general propositions of science? What is the nature of these propositions? Are they solely the collections of experience, or does the mind supply some connecting principle of its own? In a word, what is the nature of scientific truth, and what are the grounds of that confidence with which it claims to be received?

That to such questions as the above, no single and general answer can be given, must be evident. There are cases in which they do not even need discussion. Instances are familiar, in which general propositions merely express *per enumerationem simplicem*, a fact established by actual observation in all the cases to which the proposition applies. The astronomer asserts upon this ground, that all the known planets move from west to east round the sun. But there are also cases in which general propositions are assumed from observation of their truth in particular instances, and extension of that truth to instances

理智的操作在这些领域内被归于一个共同的规律体系(XI.)；这些规律的一般数学特征及其实际表达(II.III.)；它们与数领域中的思维规律的亲缘程度，以及它们与思维规律分歧之处；宇宙与永恒这两个极限概念在逻辑所关注的一切思维对象中所具有的主导特征；这些概念与数科学中统一性这一基本概念的关系，--这些以及许多类似的结果，不应被列为仅仅是或然的或类比的结论，而应被视为科学的真理。它们是否被称为形而上学的，无关紧要。它们所依据的证据之性质，虽然在种类上不同，但在价值上并不逊于任何可以用来支持物理科学一般真理的证据。

再者，人们一致认为，在所有较精确的知识形式之进展中，可以观察到某种秩序。对物理科学每一个部门的研究都始于观察，它通过事实的整理而推进到对它们联系规律的推定性的认识，这种推定性的有效性，它通过新的实验来检验，这些实验被设计得使这种推定如果是确有根据的，其概率就会无限增大；最后，当现象规律已以足够的信心被确定下来，对原因的研究，通过假说与演绎的适当混合来进行，便为探究加冕。在这种知识的前进秩序中，本著作所考察其性质的那些特殊能力与规律各自有其作用。因此，显而易见，如果我们想要不偏不倚地研究科学的性质，或研究理智构造在其与科学的关系中的性质，上述两个系列中的任何部分都不应被视为孤立的。尤其是那些彼此处于任何种类的补充关系中的真理，更应在它们相互的关联与联系中加以考察。

4.因此，一切实证知识都需要一个实验基础，这一必要性，与那套已被予以关注的心理规律、原则和操作体系的存在及其特殊性质联系起来看，倾向于阐明一些思辨思想界在很大程度上仍然为之分裂的重要问题。我们如何从经验所呈现的特殊事实达到科学的一般命题?这些命题的性质是什么?它们仅仅是经验的汇集，还是心灵提供了某种自身的连接原则?一言以蔽之，科学真理的性质是什么，它要求被接受的那种信心的根据又是什么?

对于上述这类问题，不可能给出单一的、一般的答案，这一点必定是显而易见的。有些情况下它们甚至不需要讨论。有些实例是熟悉的，其中一般命题仅仅表达*per enumerationem simplicem*，即一个由实际观察在该命题所适用的一切情况中确立的事实。天文学家据此断言，所有已知行星都自西向东绕太阳运行。但也有一些情况，其中一般命题是从对其在特定实例中的真理性的观察，以及将该真理性扩展到

unobserved. No principle of merely deductive reasoning can warrant such a procedure. When from a large number of observations on the planet Mars, Kepler inferred that it revolved in an ellipse, the conclusion was larger than his premises, or indeed than any premises which mere observation could give. What other element, then, is necessary to give even a prospective validity to such generalizations as this? It is the ability inherent in our nature to appreciate Order, and the concurrent presumption, however founded, that the phænomena of Nature are connected by a principle of Order. Without these, the general truths of physical science could never have been ascertained. Grant that the procedure thus established can only conduct us to probable or to approximate results; it only follows, that the larger number of the generalizations of physical science possess but a probable or approximate truth. The security of the tenure of knowledge consists in this, that wheresoever such conclusions do truly represent the constitution of Nature, our confidence in their truth receives indefinite confirmation, and soon becomes undistinguishable from certainty. The existence of that principle above represented as the basis of inductive reasoning enables us to solve the much disputed question as to the necessity of general propositions in reasoning. The logician affirms, that it is impossible to deduce any conclusion from particular premises. Modern writers of high repute have contended, that all reasoning is from particular to particular truths. They instance, that in concluding from the possession of a property by certain members of a class, its possession by some other member, it is not necessary to establish the intermediate *general* conclusion which affirms its possession by *all* the members of the class in common. Now whether it is so or not, that principle of order or analogy upon which the reasoning is conducted must either be stated or apprehended as a general truth, to give validity to the final conclusion. In this form, at least, the necessity of general propositions as the basis of inference is confirmed,—a necessity which, however, I conceive to be involved in the very existence, and still more in the peculiar *nature*, of those faculties whose laws have been investigated in this work. For if the process of reasoning be carefully analyzed, it will appear that abstraction is made of all peculiarities of the individual to which the conclusion refers, and the attention confined to those properties by which its membership of the class is defined.

5. But besides the general propositions which are derived by induction from the collated facts of experience, there exist others belonging to the domain of what is termed *necessary* truth. Such are the general propositions of Arithmetic, as well as the propositions expressing the laws of thought upon which the general methods of this treatise are founded; and these propositions are not only capable of being rigorously verified in particular instances, but are made manifest in all their generality from the study of particular instances. Again, there exist general propositions expressive of necessary truths, but incapable, from the imperfection of the senses, of being exactly verified. Some, if not all, of the propositions of Geometry are of this nature; but it is not in the region of Geometry alone that such propositions are found. The question concerning their nature and origin is a very ancient one, and as it is more intimately connected with the inquiry into the constitution of the intellect than any other to which allusion

未观察到的实例而被假定的。仅仅演绎推理的原则不能为这样的程序提供保证。当开普勒从对火星的大量观察中推断它沿椭圆运行，这个结论大于他的前提，或者实际上大于任何仅仅观察所能给予的前提。那么，还需要什么其他要素才能赋予这类概括以哪怕前瞻性的有效性呢?那就是我们本性中固有的领会秩序的能力，以及同时存在的推定--无论其根据如何--即自然现象是由一个秩序原则联系在一起的。没有这些，物理科学的一般真理就永远不可能被确定。就算承认这样建立起来的程序只能引导我们达到或然的或近似的结论；那也只是说明，物理科学的较大部分概括只具有或然的或近似的真理性。知识之占有权的安全性在于这一点：凡是这类结论确实真正代表了自然的构造，我们对它们真理性的信心就会得到无限的确认，并很快变得与确定性无法区分。上述被表述为归纳推理基础的那个原则的存在，使我们能够解决关于推理中一般命题之必要性的那个争论甚多的问题。逻辑学家断言，从特称前提不可能推出任何结论。现代享有盛誉的作家们主张，一切推理都是从特殊到特殊的真理。他们举例说，在从一个类的某些成员具有某种属性推出另一些成员也具有该属性时，没有必要确立那个中间一般结论，即断言该类的所有成员共同具有该属性。现在，无论是否如此，推理所依据的那个秩序或类比原则，必须被陈述或被领会为一个一般真理，才能赋予最终结论以有效性。至少在这种形式中，一般命题作为推论基础的必要性得到了确认，--然而，我认为这种必要性蕴含于那些能力的存在本身之中，更蕴含于本著作所考察其规律的那些能力的特殊性质之中。因为如果仔细分析推理过程，就会看出，它抽去了结论所指涉的个体的全部特殊性，而将注意力局限于那些界定其类属性的属性。

5. 但是，除了由经验事实归纳而来的一般命题之外，还存在属于所谓必然真理领域内的其他命题。算术的一般命题，以及表达本论著所依据的思维规律的一般命题，即属此类；这些命题不仅能够在特殊事例中得到严格的验证，而且通过对特殊事例的研究而在其全部普遍性中得以显明。再者，还存在表达必然真理但在感官的不完善之下无法被精确验证的一般命题。几何学的某些命题(如果不是全部的话)即属此种性质；但此类命题并非仅见于几何学领域。关于其性质与起源的问题是一个极为古老的问题，并且由于它比任何其他所提及的问题都更为紧密地与对理智构造的探究相关联，

has been made, it will not be irrelevant to consider it here. Among the opinions which have most widely prevailed upon the subject are the following. It has been maintained, that propositions of the class referred to exist in the mind independently of experience, and that those conceptions which are the subjects of them are the imprints of eternal archetypes. With such archetypes, conceived, however, to possess a reality of which all the objects of sense are but a faint shadow or dim suggestion, Plato furnished his ideal world. It has, on the other hand, been variously contended, that the subjects of such propositions are copies of individual objects of experience; that they are mere names; that they are individual objects of experience themselves; and that the propositions which relate to them are, on account of the imperfection of those objects, but partially true; lastly, that they are intellectual products formed by abstraction from the sensible perceptions of individual things, but so formed as to become, what the individual things never can be, subjects of science, i.e. subjects concerning which exact and general propositions may be affirmed. And there exist, perhaps, yet other views, in some of which the sensible, in others the intellectual or ideal, element predominates.

Now if the last of the views above adverted to be taken (for it is not proposed to consider either the purely ideal or the purely nominalist view) and if it be inquired what, in the sense above stated, are the proper objects of science, objects in relation to which its propositions are true without any mixture of error, it is conceived that but one answer can be given. It is, that neither do individual objects of experience, nor with all probability do the mental images which they suggest, possess any strict claim to this title. It seems to be certain, that neither in nature nor in art do we meet with anything absolutely agreeing with the geometrical definition of a straight line, or of a triangle, or of a circle, though the deviation therefrom may be inappreciable by sense; and it may be conceived as at least doubtful, whether we can form a perfect mental image, or conception, with which the agreement shall be more exact. But it is not doubtful that such conceptions, however imperfect, do point to something beyond themselves, in the gradual approach towards which all imperfection tends to disappear. Although the perfect triangle, or square, or circle, exists not in nature, eludes all our powers of *representative* conception, and is presented to us in thought only, as the limit of an indefinite process of abstraction, yet, by a wonderful faculty of the understanding, it may be made the subject of propositions which are *absolutely* true. The domain of reason is thus revealed to us as larger than that of imagination. Should any, indeed, think that we are able to picture to ourselves, with rigid accuracy, the scientific elements of form, direction, magnitude, &c., these things, as actually conceived, will, in the view of such persons, be the proper objects of science. But if, as seems to me the more just opinion, an incurable imperfection attaches to all our attempts to realize with precision these elements, then we can only affirm, that the more external objects do approach in reality, or the conceptions of fancy by abstraction, to certain *limiting* states, never, it may be, actually attained, the more do the general propositions of science concerning those things or conceptions approach to absolute truth, the actual deviation therefrom tending to disappear. To some

在此处加以考虑并非不相干。关于此一主题最为流行的观点如下。有人主张，所提及的那类命题独立于经验而存在于心灵之中，而作为其主词的那些概念则是永恒原型的印记。柏拉图以其理想世界赋予了此类原型--尽管这些原型被设想为具有一种实在性，而一切感官对象不过是其微弱的影子或模糊的暗示。另一方面，则有各种不同的主张：此类命题的主词是个别经验对象的摹本；它们仅仅是名称；它们本身就是个别的经验对象；而关于它们的命题由于那些对象的不完善而只是部分地为真；最后，它们是通过从对个别事物的感性知觉中抽象而形成的理智产物，但其形成方式使之成为个别事物永远不可能成为的东西，即科学的主词，亦即可以对其肯定精确而普遍命题的主词。或许还存在其他观点，其中有些以感性元素为主，另一些则以理智的或观念的元素为主。

现在，如果采取上述最后一种观点(因为此处不拟考虑纯粹观念的或纯粹唯名论的观点)，并且如果追问在上述意义上什么是科学的真正对象--即其命题在其中为真而丝毫不掺杂错误的那些对象--那么可以设想只能给出一个答案。那就是：个别经验对象，以及它们所暗示的心理影像(就其全部可能性而言)，都不严格具有这一称号的资格。看来可以确定的是，无论在自然中还是在技艺中，我们都遇不到任何与直线、三角形或圆的几何定义绝对符合的东西，尽管其偏差可能为感官所不可觉察；而且至少可以设想为可疑的是，我们能否形成一个完美的心理影像或概念，使之与上述定义的符合更为精确。但无可怀疑的是，此类概念尽管不完善，确实指向自身之外的某种东西，在向其逐渐接近的过程中一切不完善都趋于消失。尽管完美的三角形、正方形或圆不存在于自然之中，逃避我们一切表象概念的能力，并且仅在思想中作为无限抽象过程的极限而呈现于我们，然而凭借知性的一种奇妙能力，它可以成为绝对为真的命题的主词。理性的领域由此向我们显示为比想象的领域更为广阔。诚然，如果有人以为我们能够以严格的精确性为自己描绘出形式、方向、大小等等的科学元素，那么在这些人的观点中，这些事物正如实际被设想的那样，将是科学的真正对象。但如果--在我看来这是更为公正的意见--我们一切精确地实现这些元素的尝试都带有不可治愈的不完善性，那么我们只能断言：外部对象在现实中，或想象的概念通过抽象，越是接近某些极限状态--这些状态也许永远不能被实际达到--关于那些事物或概念的科学一般命题就越是接近绝对真理，而其实际偏差则趋于消失。在某种

extent, the same observations are applicable also to the physical sciences. What have been termed the "fundamental ideas" of those sciences as force, polarity, crystallization, &c.,[2]are neither, as I conceive, intellectual products independent of experience, nor mere copies of external things; but while, on the one hand, they have a necessary antecedent in experience, on the other hand they require for their formation the exercise of the power of abstraction, in obedience to some general faculty or disposition of our nature, which ever prompts us to the research, and qualifies us for the appreciation, of order.[3]Thus we study approximately the effects of gravitation on the motions of the heavenly bodies, by a reference to the *limiting* supposition, that the planets are perfect spheres or spheroids. We determine approximately the path of a ray of light through the atmosphere, by a process in which abstraction is made of all disturbing influences of temperature. And such is the order of procedure in all the higher walks of human knowledge. Now what is remarkable in connexion with these processes of the intellect is the disposition, and the corresponding ability, to ascend from the imperfect representations of sense and the diversities of individual experience, to the perception of general, and it may be of immutable truths. Whereever this disposition and this ability unite, each series of connected facts in nature may furnish the intimations of an order more exact than that which it directly manifests. For it may serve as ground and occasion for the exercise of those powers, whose office it is to apprehend the general truths which are indeed exemplified, but never with perfect fidelity, in a world of changeful phænomena.

6. The truth that the ultimate laws of thought are mathematical in their form, viewed in connexion with the fact of the possibility of error, establishes a ground for some remarkable conclusions. If we directed our attention to the scientific truth alone, we might be led to infer an almost exact parallelism between the intellectual operations and the movements of external nature. Suppose any one conversant with physical science, but unaccustomed to reflect upon the nature of his own faculties, to have been informed, that it had been proved, that the laws of those faculties were mathematical; it is probable that after the first feelings of incredulity had subsided, the impression would arise, that the order of thought must, *therefore*, be as necessary as that of the material universe. We know that in the realm of natural science, the absolute

[2]Whewell's Philosophy of the Inductive Sciences, pp. 71, 77, 213.

[3]Of the idea of order it has been profoundly said, that it carries within itself its own justification or its own control, the very trustworthiness of our faculties being judged by the conformity of their results to an order which satisfies the reason. "L'idée de l'ordre a cela de singulier et d'eminent, qu'elle porte en elle même sa justification ou son contrôle. Pour trouver si nos autres facultés nous trompent ou nous ne trompent pas, nous examinons si les notions qu'elles nous donnent s'enchaînent on ne s'enchaînent pas suivant un ordre qui satisfasse la raison."—*Cournot, Essai sur les fondements de nos Connaissances.* Admitting this principle as the guide of those powers of abstraction which we undoubtedly possess, it seems unphilosophical to assume that the fundamental ideas of the sciences are not derivable from experience. Doubtless the capacities which have been given to us for the comprehension of the actual world would avail us in a differently constituted scene, if in some form or other the dominion of order was still maintained. It is conceivable that in such a new theatre of speculation, the laws of the intellectual procedure remaining the same, the fundamental ideas of the sciences might be wholly different from those with which we are at present acquainted.

程度上，同样的观察也适用于物理科学。那些被称为各门科学的"基本观念"的东西，如力、极性、结晶等等，[2]在我看来，既不是独立于经验的理智产物，也不是外部事物的单纯摹本；而是，一方面它们在经验中有其必然的前件，另一方面它们的形成需要抽象能力的运用，这种运用服从于我们本性中某种普遍的才能或倾向，这种才能或倾向永远驱使我们从事研究，并使我们有资格鉴赏秩序。[3]因此，我们通过参照行星是完美球体或椭球体的极限假设，近似地研究引力对天体运动的结果。我们通过一个抽去一切温度干扰影响的过程，近似地确定光线穿过大气的路径。人类知识的一切较高领域中的程序次序莫不如此。与这些理智过程相关联而值得注意的，是那种从感官的不完善表象和个别经验的多样性上升到对普遍的、也许是永恒真理的知觉的倾向以及相应的能力。凡此种倾向与能力相结合之处，自然中每一系列相互关联的事实都可以提供一种比其直接显现的秩序更为精确的秩序的暗示。因为它可以作为那些能力之运用的根据和契机，而那些能力的职责在于把握普遍真理--这些真理在变化现象的世界中确实得到例证，但从来不是以完美的忠实得到例证的。

6. 思维规律的终极规律在其形式上是数学的这一真理，与错误之可能性这一事实联系起来看，为某些值得注意的结论确立了根据。如果我们仅仅注意科学真理，我们可能被引向推断：理智操作与外部自然的运动之间存在着几乎精确的平行关系。假设任何一位熟悉物理科学但不习惯于反思自身能力之性质的人被告知，那些能力的规律已被证明是数学的；很可能在最初的怀疑感消退之后，会产生这样的印象：思想的秩序因此必定像物质宇宙的秩序一样必然。我们知道，在自然科学的领域中，绝对的

2 休厄尔《归纳科学哲学》，第71、77、213页。

3 关于秩序的观念，有人曾深刻地说过，它自身之内就带有自己的辩护或自己的制约，我们各种官能的可靠性正是由其结果是否符合一种满足理性的秩序来评判的。"秩序观念的特殊与卓越之处在于，它自身之内就带有自己的辩护或自己的制约。要发现我们的其他官能是否欺骗我们或没有欺骗我们，我们考察它们给予我们的观念是否按照一种满足理性的秩序相互联系或不相联系。"--库尔诺，《论我们知识的基础》。既然承认这一原则为我们无疑具有的那些抽象能力的向导，那么假定各科学的基本观念不能从经验中导出，似乎是不合哲学的。毫无疑问，赋予我们用以理解现实世界的能力，在一个构造不同的场景中也会对我们有用，只要秩序的统治以某种形式仍然得到维持。可以设想，在这样一个新的思辨舞台上，理智程序的规律保持不变，而各科学的基本观念则可能与我们目前所知的那些完全不同。

connexion between the initial and final elements of a problem, exhibited in the mathematical form, fitly symbolizes that physical necessity which binds together effect and cause. The necessary sequence of states and conditions in the inorganic world, and the necessary connexion of premises and conclusion in the processes of exact demonstration thereto applied, seem to be co-ordinate. It may possibly be a question, to which of the two series the primary application of the term "necessary" is due; whether to the observed constancy of Nature, or to the indissoluble connexion of propositions in all valid reasoning upon her works. Historically we should perhaps give the preference to the former, philosophically to the latter view. But the fact of the connexion is indisputable, and the analogy to which it points is obvious.

Were, then, the laws of valid reasoning uniformly obeyed, a very close parallelism would exist between the operations of the intellect and those of external Nature. Subjection to laws mathematical in their form and expression, even the subjection of an absolute obedience, would stamp upon the two series one common character. The reign of necessity over the intellectual and the physical world would be alike complete and universal.

But while the observation of external Nature testifies with ever-strengthening evidence to the fact, that uniformity of operation and unvarying obedience to appointed laws prevail throughout her entire domain, the slightest attention to the processes of the intellectual world reveals to us another state of things. The mathematical laws of reasoning are, properly speaking, the laws of *right* reasoning only, and their actual transgression is a perpetually recurring phenomenon. Error, which has no place in the material system, occupies a large one here. We must accept this as one of those ultimate facts, the origin of which it lies beyond the province of science to determine. We must admit that there exist laws which even the rigour of their mathematical forms does not preserve from violation. We must ascribe to them an authority the essence of which does not consist in power, a supremacy which the analogy of the inviolable order of the natural world in no way assists us to comprehend.

As the distinction thus pointed out is *real*, it remains unaffected by any peculiarity in our views respecting other portions of the mental constitution. If we regard the intellect as free, and this is apparently the view most in accordance with the general spirit of these speculations, its freedom must be viewed as opposed to the dominion of necessity, not to the existence of a certain just supremacy of truth. The laws of correct inference may be violated, but they do not the less truly *exist* on this account. Equally do they remain unaffected in character and authority if the hypothesis of necessity in its extreme form be adopted. Let it be granted that the laws of valid reasoning, such as they are determined to be in this work, or, to speak more generally, such as they would finally appear in the conclusions of an exhaustive analysis, form but a *part* of the system of laws by which the actual processes of reasoning, whether right or wrong, are governed. Let it be granted that if that system were known to us in its completeness, we should perceive that the whole intellectual procedure was *necessary*, even as the movements of the inorganic world are necessary. And let it finally, as a consequence of this hypothesis, be granted that the phænomena of

一个问题的初始元素与最终元素之间的联系，以数学形式展示出来，恰当地象征着将结果与原因结合在一起的物理必然性。无机世界中状态与条件的必然序列，以及应用于其上的精确证明过程中前提与结论的必然联系，似乎是同等的。这可能是一个问题：术语"必然的"最初应用于这两个序列中的哪一个；是应用于观察到的自然的恒常性，还是应用于关于自然作品的 all 有效推理中命题之间不可分解的联系。从历史上看，我们或许应偏重前者，从哲学上看则偏重后者。但联系的事实是无可争辩的，它所指向的类比也是明显的。

那么，如果有效的推理规律被一致地遵守，那么在理智的运作与外部自然的运作之间就会存在一种非常紧密的平行关系。服从于在其形式和表达上都是数学的规律，甚至是服从于一种绝对的服从，就会在两个序列上打上一种共同的特征。必然性对理智世界和物理世界的统治同样会是完整而普遍的。

然而，尽管对外部自然的观察以日益增强的证据证明这一事实：运作的一致性和对既定规律的始终不渝的服从在其整个领域中盛行，但只要稍加注意理智世界的过程，就会向我们揭示另一种事态。推理的数学规律，确切地说，只是正确推理的规律，而它们的实际违反是一个反复出现的现象。错误在物质系统中没有位置，在这里却占据很大的位置。我们必须把这接受为那些终极事实之一，其起源的确定超出科学的范围。我们必须承认存在一些规律，即使其数学形式的严格性也不能使它们免于被违反。我们必须赋予它们一种权威，其本质不在于力量，一种至高地位，自然世界不可侵犯的秩序的类比绝不能帮助我们理解。

既然如此指出的区别是真实的，它就不受我们对心理构造其他部分的看法中任何特殊性的影响。如果我们把理智视为自由的，而这显然是与此类思辨的一般精神最相符的观点，那么它的自由必须被视为与必然性的统治相对立，而不是与真理的某种正当至高地位的存在相对立。正确推论的规律可能被违反，但尽管如此，它们并不因此就不存在。如果采取极端形式的必然性假说，它们在性质和权威上也同样不受影响。姑且承认，有效推理的规律--如它们在本著作中被确定的那样，或者更一般地说，如它们在详尽分析的结论中最终会呈现的那样--只是支配实际推理过程(无论正确还是错误)的规律体系的一个部分。姑且承认，如果我们完整地认识那个体系，我们就会觉察到整个理智程序是必然的，正如无机世界的运动是必然的一样。最后，作为这一假说的结果，姑且承认

incorrect reasoning or error, wheresoever presented, are due to the interference of other laws with those laws of which *right* reasoning is the product. Still it would remain that there exist among the intellectual laws a number marked out from the rest by this special character, viz., that every movement of the intellectual system which is accomplished solely under their direction is *right*, that every interference therewith by other laws is not interference only, but *violation*. It cannot but be felt that this circumstance would give to the laws in question a character of distinction and of predominance. They would but the more evidently seem to indicate a final purpose which is not always fulfilled, to possess an authority inherent and just, but not always commanding obedience.

Now a little consideration will show that there is nothing analogous to this in the government of the world by natural law. The realm of inorganic Nature admits neither of preference nor of distinctions. We cannot separate any portion of her laws from the rest, and pronounce them alone worthy of obedience,—alone charged with the fulfilment of her highest purpose. On the contrary, all her laws seem to stand co-ordinate, and the larger our acquaintance with them, the more necessary does their united action seem to the harmony and, so far as we can comprehend it, to the general *design* of the system. How often the most signal departures from apparent order in the inorganic world, such as the perturbations of the planetary system, the interruption of the process of crystallization by the intrusion of a foreign force, and others of a like nature, either merge into the conception of some more exalted scheme of order, or lose to a more attentive and instructed gaze their abnormal aspect, it is needless to remark. One explanation only of these facts can be given, viz., that the distinction between *true* and *false*, between *correct* and *incorrect*, exists in the processes of the intellect, but not in the region of a physical necessity. As we advance from the lower stages of organic being to the higher grade of conscious intelligence, this contrast gradually dawns upon us. Wherever the phænomena of life are manifested, the dominion of rigid law in some degree yields to that mysterious principle of activity. Thus, although the structure of the animal tribes is conformable to certain general types, yet are those types sometimes, perhaps, in relation to the highest standards of beauty and proportion, always, imperfectly realized. The two alternatives, between which Art in the present day fluctuates, are the exact imitation of individual forms, and the endeavour, by abstraction from all such, to arrive at the conception of an ideal grace and expression, never, it may be, perfectly manifested in forms of earthly mould. Again, those teleological adaptations by which, without the organic type being sacrificed, species become fitted to new conditions or abodes, are but slowly accomplished,—accomplished, however, not, apparently, by the fateful power of external circumstances, but by the calling forth of an energy from within. Life in all its forms may thus be contrasted with the passive fixity of inorganic nature. But inasmuch as the perfection of the types in which it is corporeally manifested is in some measure of an ideal character, inasmuch as we cannot precisely define the highest *suggested* excellency of form and of adaptation, the contrast is less marked here than that which exists between the intellectual processes and those of the purely material world. For the definite and technical character of the

不正确推理或错误的现象，无论出现在何处，都是由于其他规律与那些以正确推理为其产物的规律相干扰。即便如此，仍然会存在这样的情况：在理智规律中，有一些数目以这一特殊性质而从其余规律中区分出来，即：理智系统完全在它们的指导下所完成的每一次运动都是正确的，其他规律对它的每一次干扰都不只是干扰，而是违反。不能不感觉到，这种情况会赋予所讨论的规律一种卓越与优越的性质。它们只会更加明显地似乎指示一个并非总能实现的目的，拥有一种内在而正当的权威，但并非总能命令服从。

现在稍加思考就会表明，在自然规律对世界的治理中，没有任何与这种情况类似的东西。无机自然的领域既不允许偏好，也不允许区分。我们不能把她的任何一部分规律与其余部分分开，并宣称它们单独值得服从，--单独负责实现她的最高目的。相反，她的所有规律似乎都是并列的，我们对它们了解得越多，它们的联合作用对于体系的和谐、并且就我们所能理解的范围而言对于体系的一般设计，就显得越必然。无机世界中与表面秩序最显著的偏离，例如行星系的摄动、因外来力量的侵入而中断结晶过程，以及其他类似性质的情况，多么经常地要么融入某种更崇高秩序方案的观念之中，要么在更专注、更有教养的目光下失去其异常面貌，这无需赘述。对这些事实只能给出一种解释，即：真与假之间、正确与不正确之间的区分，存在于理智的过程中，而不存在于物理必然性的领域。当我们从有机存在的较低阶段前进到有意识理智的较高级别时，这种对照逐渐向我们显现。凡生命现象得以表现之处，严格规律的统治都在某种程度上让位于那种神秘的活动原则。因此，尽管动物族类的结构符合某些一般类型，然而这些类型有时也许、并且就美与比例的最高标准而言总是被不完美地实现。当今艺术在其间摇摆的两个选择是：对个体形式的精确模仿，以及通过从所有这类形式中抽象出来，努力达到一种理想的优雅与表现的观念--这种观念也许从未在地球泥土所塑的形式中被完美地表现出来。再者，那些目的论式的适应--通过它们，在不牺牲有机类型的情况下，物种变得适应新的条件或居所--只是缓慢地完成的；然而，显然不是通过外部环境的宿命力量完成的，而是通过从内部唤起一种能量完成的。因此，一切形式的生命都可以与无机自然的被动固定性相对照。但是，由于生命在其中以身体方式表现的类型之完美性在某种程度上具有理想性质，由于我们不能精确界定形式和适应之被提示的最高卓越性，所以这里的对照不如理智过程与纯粹物质世界过程之间的对照那样显著。因为那确定而专门的性质

mathematical laws by which both are governed, places in stronger light the fundamental difference between the kind of authority which, in their capacity of government, they respectively exercise.

7. There is yet another instance connected with the general objects of this chapter, in which the collation of truths or facts, drawn from different sources, suggests an instructive train of reflection. It consists in the comparison of the laws of thought, in their scientific expression, with the actual forms which physical speculation in early ages, and metaphysical speculation in all ages, have tended to assume. There are two illustrations of this remark, to which, in particular, I wish to direct attention here.

1st. It has been shown (III. 13) that there is a scientific connexion between the conceptions of unity in Number, and the universe in Logic. They occupy in their respective systems the same relative place, and are subject to the same formal laws. Now to the Greek mind, in that early stage of activity,—a stage not less marked, perhaps not less *necessary*, in the progression of the human intellect, than the era of Bacon or of Newton,—when the great problems of Nature began to unfold themselves, while the means of observation were as yet wanting, and its necessity not understood, the terms "Universe" and "The One" seem to have been regarded as almost identical. To assign the nature of that unity of which all existence was thought to be a manifestation, was the first aim of philosophy.[4] Thales sought for this fundamental unity in water. Anaximenes and Diogenes conceived it to be air. Hippasus of Metapontum, and Heraclitus the Ephesian, pronounced that it was fire. Less definite or less confident in his views, Parmenides simply declared that all existing things were One; Melissus that the Universe was infinite, unsusceptible of change or motion, One, like to itself, and that motion was not, but seemed to be.[5] In a spirit which, to the reflective mind of Aristotle, appeared sober when contrasted with the rashness of previous speculation, Anaxagoras of Clazomenæ, following, perhaps, the steps of his fellow-citizen, Hermotimus, sought in Intelligence the cause of the world and of its order.[6] The pantheistic tendency which pervaded many of these speculations is manifest in the language of Xenophanes, the founder of the Eleatic school, who, "surveying the expanse of heaven, declared that the One was God."[7] Perhaps there are few, if any, of the forms in which unity can be conceived, in the abstract as numerical or rational, in the concrete as a passive substance, or a central and living principle, of which we do not meet with applications in these ancient doctrines. The writings of Aristotle, to which I have chiefly referred, abound with allusions of this nature, though of the

[4]See various passages in Aristotle's Metaphysics, Book1.

[5]*'Εδόκει δὲ αὐτῷ τὸ πᾶν ἄπειρον εἶναι, καὶ ἀναλλοίωτον, καὶ ἀκίνητον, καὶ ἓν, ὅμοιον ἑαυτῷ καὶ πλῆρες. κινησίν τε μὴ εἶναι δοκεῖν δὲ εἶναι. —Diog. Laert.* IX. cap. 4.

[6]*Νοῦν δή τις εἰπὼν ἐνεῖναι, καθάπερ ἐν τοῖς ζῴοις, καὶ ἐν τῇ φύσει, τὸν αἴτιον τοῦ κόσμου καὶ τῆς τάξεως πάσης οἷον νήφων ἐφάνη παρ' εἰκῆ λέγοντας τοὺς πρότερον. Φανερῶς μὲν οὖν 'Αναξαγόραν ἴσμεν ἁψάμενον τούτων τῶν λόγων, αἰτίαν δ' ἔχει πρότερον 'Ερμότιμος ὁ Κλαζομένιος εἰπεῖν. —Arist. Met.* I. 3.

[7]*Ξενοφάνης δὲ ... εἰς τὸν ὅλον οὐρανὸν ἀποβλέψας, τὸ ἓν εἶναι φησι τὸν θεόν. —Ib.*

支配两者的数学规律，更加鲜明地揭示了它们各自以其政府身份所行使的那类权威之间的根本差异。

7. 还有另一个与本章一般对象相关的例子，其中来自不同来源的真理或事实的对照提示出一系列富有教益的反思。它在于把思维规律在其科学表达中的形式，与早期物理思辨以及一切时代的形而上学思辨所倾向于采取的实际形式加以比较。关于这一评论，有两个例证是我在此特别希望提请注意的。

1. 已经表明(III.13)，数值中 unity 的概念与逻辑中 universe 的概念之间存在科学的联系。它们在各自的体系中占据相同的相对位置，并服从相同的形式规律。然而，对于希腊心灵而言，在其活动的早期阶段--这一阶段在人类理智的进展中或许并不比培根或牛顿的时代更少标志性、或许并不更少必然性--当自然的重大问题开始展开，而观察的手段尚且缺乏、其必然性尚未被理解时，"Universe"和"The One"这两个项似乎被视为几乎同一。指定一切存在都被认为是其表现的那种 unity 的性质，是哲学的首要目标。[4]泰勒斯在水里寻求这种根本的 unity。阿那克西美尼和第欧根尼认为它是气。梅塔蓬图的希帕索斯和以弗所的赫拉克利特宣称它是火。巴门尼德在其观点中不那么确定或那么自信，只是宣布一切存在的事物都是 One；梅利苏斯说 Universe 是无限的、不受变化或运动影响的、One、与自身相似，并且运动不存在，只是似乎存在。[5]以一种在亚里士多德的反思性心灵看来与先前思辨的鲁莽相比显得清醒的精神，克拉佐门尼的阿那克萨戈拉也许追随其同邦人赫尔莫提穆斯的脚步，在 Intelligence 中寻求世界及其秩序的原因。[6]贯穿这些思辨中许多的泛神论倾向在色诺芬尼的语言中显而易见，他是埃利亚学派的创始人，他"眺望天空的广阔，宣布 One 就是 God。"[7]也许，unity 可以被设想的形式--抽象地作为数值的或理性的，具体地作为被动的实体，或作为中心的与活的原则--中，几乎没有哪一种我们不在这些古代学说中遇到其应用。我主要引用的亚里士多德的著作充满了这种性质的暗示，尽管

[4]See various passages in Aristotle's Metaphysics, Book1.

[5]*'Εδόκει δὲ αὐτῷ τὸ πᾶν ἄπειρον εἶναι, καὶ ἀναλλοίωτον, καὶ ἀκίνητον, καὶ ἓν, ὅμοιον ἑαυτῷ καὶ πλῆρες. κινησίν τε μὴ εἶναι δοκεῖν δὲ εἶναι. —Diog. Laert.* IX. cap. 4.

[6]*Νοῦν δή τις εἰπὼν ἐνεῖναι, καθάπερ ἐν τοῖς ζῴοις, καὶ ἐν τῇ φύσει, τὸν αἴτιον τοῦ κόσμου καὶ τῆς τάξεως πάσης οἷον νήφων ἐφάνη παρ' εἰκῆ λέγοντας τοὺς πρότερον. Φανερῶς μὲν οὖν 'Αναξαγόραν ἴσμεν ἁψάμενον τούτων τῶν λόγων, αἰτίαν δ' ἔχει πρότερον 'Ερμότιμος ὁ Κλαζομένιος εἰπεῖν. —Arist. Met.* I. 3.

[7]*Ξενοφάνης δὲ ... εἰς τὸν ὅλον οὐρανὸν ἀποβλέψας, τὸ ἓν εἶναι φησι τὸν θεόν. —Ib.*

larger number of those who once addicted themselves to such speculations, it is probable that the very names have perished. Strange, but suggestive truth, that while Nature in all but the aspect of the heavens must have appeared as little else than a scene of unexplained disorder, while the popular belief was distracted amid the multiplicity of its gods, —the conception of a primal unity, if only in a rude, material form, should have struck deepest root; surviving in many a thoughtful breast the chills of a lifelong disappointment, and an endless search![8]

2ndly. In equally intimate alliance with that law of thought which is expressed by an equation of the second degree, and which has been termed in this treatise the law of duality, stands the tendency of ancient thought to those forms of philosophical speculation which are known under the name of dualism. The theory of Empedocles,[9] which explained the apparent contradictions of nature by referring them to the two opposing principles of "strife" and "friendship;" and the theory of Leucippus,[10] which resolved all existence into the two elements of a *plenum* and a vacuum, are of this nature. The famous comparison of the universe to a lyre or a bow,[11] its "recurrent harmony" being the product of opposite states of tension, betrays the same origin. In the system of Pythagoras, which seems to have been a combination of dualism with other elements derived from the study of numbers, and of their relations, ten fundamental antitheses are recognised: finite and infinite, even and odd, unity and multitude, right and left, male and female, rest and motion, straight and curved, light and darkness, good and evil, the square and the oblong. In that of Alcmæon the same fundamental dualism is accepted, but without the definite and numerical limitation with which it is connected in the Pythagorean system. The grand development of this idea is, however, met with in that ancient Manichæan doctrine, which not only formed the basis of the religious system of Persia, but spread widely through other regions of the East, and became memorable in the history of the Christian Church. The origin of dualism as a speculative

[8]The following lines, preserved by Sextus Empiricus, and ascribed to Timon the Sillograph, are not devoid of pathos:—

> *ὡς καὶ ἐγὼν ὄφελον πυκινοῦ νόου ἀντιβολῆσαι*
> *ἀμφοτερόβλεπτος (δολίη δ' ὀϊῷ ἐξεπατήθην,*
> *πρεσβυγενὴς ἔτ ἐὼν) καὶ ἀναμφήριστος ἀπάσης*
> *σκεπτοσύνης· ὅππη γὰρ ἐμὸν νόον εἰρύσαιμι,*
> *εἰς ἓν τ' αὐτὸ τε πᾶν ἀνέλυετο.*

I quote them from Ritter, and venture to give the following version:—

> Be mine, to partial views no more confin'd
> Or sceptic doubts, the truth-illumin'd mind!
> For, long deceiv'd, yet still on Truth intent,
> Life's waning years in wand'rings wild are spent.
> Still restless thought the same high quest essays,
> And still the One, the All, eludes my gaze.

[9]Arist. Met. I. 4. 6.
[10]Arist. Met. I. 4, 9.
[11]παλίντροπος ἁρμονίη ὅκως περ τόξου καὶ λύρης.—*Heraclitus*, quoted in *Origenis Philosophumena*, IX. 9. Also *Plutarch, De Iside et Osiride.*

曾经沉溺于这类思辨的人中的较大多数而言，很可能连名字都已消亡。这是一个奇怪却富有提示性的真理：当自然除了天空的面貌之外，必定显得几乎只是一个未被解释的混乱场景，当流行信仰在其众神的多重性中分散不定时，--一种原初统一性的概念，哪怕只是以粗糙的物质形式出现，竟然扎下了最深的根；在许多深思的心灵中，它经受住了一生失望的寒意和无尽探索而存留下来![8]

2. 同样与那条由二次方程表达、并在本论著中被称为 duality 规律的思维规律密切相连的，是古代思想对那种以 dualism 之名著称的哲学思辨形式的倾向。恩培多克勒的学说，[9]通过把自然的表面矛盾归因于"strife"和"friendship"两个对立原则来解释它们；以及留基伯的学说，[10]把一切存在分解为*plenum*和真空两个元素，都属于这种性质。把 universe 比作里拉琴或弓的著名比喻，[11]其"回返的和谐"是对立张力状态的产物，透露出同样的起源。在毕达哥拉斯的体系中--它似乎是 dualism 与从数及其关系的研究中得出的其他元素的结合--承认十个基本对立：有限与无限、偶与奇、unity 与众多、右与左、男与女、静与动、直与曲、光与暗、善与恶、正方形与长方形。在阿尔克迈翁的体系中，同样的基本 dualism 被接受，但没有毕达哥拉斯体系中与之相连的那种确定的、数值的限制。然而，这一观念的重大展开见于那古老的摩尼教学说，它不仅构成波斯宗教体系的基础，而且广泛传播到东方其他地区，并在基督教会的历史上变得令人难忘。dualism 作为一种思辨的起源

8 以下诗句由塞克斯都·恩披里柯保存，并归于西洛格拉斐的提蒙，并非没有感人之处：--

> *ὡς καὶ ἐγὼν ὄφελον πυκινοῦ νόου ἀντιβολῆσαι*
> *ἀμφοτερόβλεπτος (δολίη δ' ὀϊῷ ἐξεπατήθην,*
> *πρεσβυγενὴς ἔτ ἐὼν) καὶ ἀναμφήριστος ἀπάσης*
> *σκεπτοσύνης· ὅππη γὰρ ἐμὸν νόον εἰρύσαιμι,*
> *εἰς ἓν τ' αὐτὸ τε πᾶν ἀνέλυετο.*

我引自里特尔，并冒昧给出以下译文：--

> 愿我所有，不再局限于偏狭之见，或怀疑论的疑虑，而是被
> 真理照亮的心灵!因为，长久受骗，却仍专注于真理，生命
> 的衰暮之年消耗在狂野的漫游中。不安的思想仍然尝试着同
> 样崇高的探求，而那太一、那全体，仍然躲避我的凝视。

[9]Arist. Met. I. 4. 6.
[10]Arist. Met. I. 4, 9.
[11]παλίντροπος ἁρμονίη ὅκως περ τόξου καὶ λύρης.—*Heraclitus*, quoted in *Origenis Philosophumena*, IX. 9. Also *Plutarch, De Iside et Osiride.*

opinion, not yet connected with the personification of the Evil Principle, but naturally succeeding those doctrines which had assumed the primal unity of Nature, is thus stated by Aristotle:—"Since there manifestly existed in Nature things opposite to the good, and not only order and beauty, but also disorder and deformity; and since the evil things did manifestly preponderate in number over the good, and the deformed over the beautiful, some one else at length introduced strife and friendship as the respective causes of these diverse phænornena."[12]And in Greece, indeed, it seems to have been chiefly as a philosophical opinion, or as an adjunct to philosophical speculation, that the dualistic theory obtained ground.[13]The moral application of the doctrine most in accordance with the Greek mind is preserved in the great Platonic antithesis of "being and non-being,"—the connexion of the former with whatsoever is good and true, with the eternal ideas, and the archetypal world: of the latter with evil, with error, with the perishable phænomena of the present scene. The two forms of speculation which we have considered were here blended together; nor was it during the youth and maturity of Greek philosophy alone that the tendencies of thought above described were manifested. Ages of imitation caught up and adopted as their own the same spirit. Especially wherever the genius of Plato exercised sway was this influence felt. The unity of all real being, its identity with truth and goodness considered as to their essence; the illusion, the profound unreality, of all merely phænomenal existence; such were the views,—such the dispositions of thought, which it chiefly tended to foster. Hence that strong tendency to mysticism which, when the days of renown, whether on the field of intellectual or on that of social enterprise, had ended in Greece, became prevalent in her schools of philosophy, and reached their culminating point among the Alexandrian Platonists. The supposititious treatises of Dionysius the Areopagite served to convey the same influence, much modified by its contact with Aristotelian doctrines, to the scholastic disputants of the middle ages. It can furnish no just ground of controversy to say, that the tone of thought thus encouraged was as little consistent with genuine devotion as with a sober philosophy. That kindly influence of human affections, that homely intercourse with the common things of life, which form so large a part of the true, because intended, discipline of our nature, would be ill replaced by the contemplation even of the highest object of thought, viewed by an excessive abstraction as something concerning which not a single intelligible proposition could either be affirmed or denied.[14]I would but slightly allude to those connected speculations on the Divine Nature which ascribed to it the perfect union of opposite qualities,[15]or to the remarkable treatises of Anselm,

[12]'*Επεὶ δὲ καὶ τἀναντία τοῖς ἀγαθοῖς ἐνόντα ἐφαίνετο ἐν τῇ φύσει, καὶ οὐ μόνον τάξις καὶ τὸ καλὸν ἀλλὰ καὶ ἀταξία καὶ τὸ αἰσχρόν, καὶ πλείω τὰ κακὰ τῶν ἀγαθῶν καὶ τὰ φαῦλα τῶν καλῶν, οὕτως ἄλλος τις φιλίαν εἰσήνεγκε καὶ νεῖκος, ἑκάτερον ἑκατέρων αἴτιον τούτων.—Arist. Metaphysica*, I. 4.

[13]Witness Aristotle's well-known derivation of the elements from the qualities "warm," and "dry," and their contraries. It is characteristic that Plato connects their generation with mathematical principles.–*Timæus*, cap. xi.

[14]Αυτος και ηψπερ τησιν εστι και απηαιρεσιν.

[15]See especially the lofty strain of Hildebert beginning "Alpha et Ω magne Deus." (Trench's

意见，尚未与恶的原则的人格化相联系，而是自然地继那些假定自然之原始统一的学说之后，亚里士多德如此表述："因为自然中显然存在着与善相反的事物，不仅有序与美，也有无序与畸形；又因为恶的事物在数量上明显超过善的事物，畸形的事物超过美的事物，终于有某人引入了争斗与友爱，作为这些纷繁现象各自的原因。"[12]而在希腊，二元论似乎主要是作为一种哲学意见，或作为哲学思辨的附属物，才获得了立足之地。[13]该学说最符合希腊心灵的道德应用，保存在柏拉图那伟大的"存在与非存在"的对立之中--前者与一切善与真、与永恒的理念、与原型世界相联系；后者与恶、与错误、与当前场景中可朽的现象相联系。我们所考察的两种思辨形式在此融合在一起；而且，上述思想倾向的显现，并非仅在希腊哲学的青春与成熟时期。模仿的时代拾取了同样的精神并据为己有。尤其在柏拉图的天才施展支配之处，这种影响更为显著。一切真实存在的统一，其与被视为就其本质而言的真理与善的同一；一切仅仅现象性的存在的幻象、深刻的不真实；这些便是它主要倾向于培育的观点--这样的思想倾向。因此产生了那种强烈的神秘主义倾向，当荣耀的日子--无论是在理智的领域还是在社会事业的领域--在希腊终结之后，这种倾向在她的哲学学派中盛行起来，并在亚历山大的柏拉图主义者中达到了顶峰。伪托的狄奥尼修斯(Dionysius the Areopagite)的论著，将同样的影响--因与亚里士多德学说的接触而大为改变--传递给了中世纪的经院论辩者。说如此受到鼓励的思想基调既与真正的虔诚、也与清醒的哲学同样不相容，这不能提供任何正当的争论理由。人类情感那种亲切的影响，与生活中寻常事物的那种朴素的交往，构成了我们本性之真正(因为是本来 intended)训练的一大部分，若以对甚至最高思想对象的沉思来取代它，那将是糟糕的替代--那种沉思以过度的抽象将其视为某种既不能肯定也不能否定任何可理解命题的东西。[14]我只想略微提及那些相关的关于神性本性的思辨，它们将对立性质的完美结合归于神性，[15]或提及安瑟伦(Anselm)那非凡的论著，

[12]'*Επεὶ δὲ καὶ τἀναντία τοῖς ἀγαθοῖς ἐνόντα ἐφαίνετο ἐν τῇ φύσει, καὶ οὐ μόνον τάξις καὶ τὸ καλὸν ἀλλὰ καὶ ἀταξία καὶ τὸ αἰσχρόν, καὶ πλείω τὰ κακὰ τῶν ἀγαθῶν καὶ τὰ φαῦλα τῶν καλῶν, οὕτως ἄλλος τις φιλίαν εἰσήνεγκε καὶ νεῖκος, ἑκάτερον ἑκατέρων αἴτιον τούτων.—Arist. Metaphysica*, I. 4.

13 见证亚里士多德从"热"和"干"这两种性质及其对立面推导出元素的著名做法。柏拉图将它们的生成与数学原理联系起来，这是其特点。--《蒂迈欧篇》，第11章。

14 他既是加也是减。

15 尤其参见希尔德伯特开始于"Alpha et Ωmagne Deus"的崇高诗篇。(特伦奇的

designed to establish a theory of the universe upon the analogies of thought and being.[16] The primal unity is there represented as having its abode in the one eternal Truth. The conformity of Nature to her laws, the obedience of moral agents to the dictates of rectitude, are the same Truth seen in action; the world itself being but an expression of the self-reflecting thought of its Author.[17]Still more marked was the revival of the older forms of speculation during the sixteenth and seventeenth centuries. The friends and associates of Lorenzo the Magnificent, the recluses known in England as the Cambridge Platonists, together with many meditative spirits scattered through Europe, devoted themselves anew, either to the task of solving the ancient problem, De Uno, Vero, Bono, or to that of proving that all such inquiries are futile and vain.[18]The logical elements which underlie all these speculations, and from which they appear to borrow at least their form, it would be easy to trace in the outlines of more modern systems,–more especially in that association of the doctrine of the absolute unity with the distinction of the *ego* and the *non-ego* as the type of Nature, which forms the basis of the philosophy of Hegel. The attempts of speculative minds to ascend to some high pinnacle of truth, from which they might survey the entire framework and connexion of things *in the order of deductive thought*, have differed less in the forms of theory which they have produced, than through the nature of the interpretations which have been assigned to those forms.[19]And herein lies the real question as to the influence of philosophical systems upon the disposition and the life. For though it is of slight moment that men should agree in tracing back all the forms and conditions of being to a primal unity, it is otherwise as concerns their conceptions of what that unity is, and what are the kinds of relation, beside that of mere causality,

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Sacred Latin Poetry.) The principle upon which all these speculations rest is thus stated in the treatise referred to in the last note. Ουδεν ουν ηατοπον, εξ αμψδρον εικονον επι το παντον αιτιον αναβαντας, ηψπερκοσμιοις οπητηαλμοις τηεοραεσαι παντα εν τπς παντον αιτιπς, και τα αλλαελοις εναντια μονοειδος και ηαεομενος .—*De Divinis Nominibus*, cap. v. And the kind of knowledge which it is thus sought to attain is described as a "darkness beyond light," *υπερφωτος γνοφος*. (*De Mystica Theologia*, cap. i.) Milton has a similar thought—

> "Dark with excessive bright Thy skirts appear."
> *Par. Lost*, Book iii.

Contrast with these the nobler simplicity of I John, i. 5.

[16]Monologium, Prosologium, and De Veritate.

[17]"Idcirco cum ipse summus spiritus dicit seipsum dicit omnia quæ facta sunt."—*Monolog.* cap. xxiii.

[18]See dissertations in Spinoza, Picus of Mirandula, H. More, &c. Modern discussions of this nature are chiefly in connexion with aesthetics, the ground of the application being contained in the formula of Augustine: "Omnis porro pulchritudinis forma, unitas est."

[19]For instance, the learned mysticism of Gioberti, widely as it differs in its spirit and its conclusions from the pantheism of Hegel (both being, perhaps, equally remote from truth), resembles it in applying both to *thought* and to *being* the principles of unity and duality. It is asked:—"Or non è egli chiaro che ogni discorso si riduce in fine in fine alle idee di Dio, del mondo, e della creazione, l'ultima delle quali è il legame delle due prime?" And this question being affirmatively answered in the formula, "l'Ente crea le esistenze," it is said of that formula,—"Essa abbraccia la realtà universale nella dualità del necessario e del contingente, esprime il vincolo di questi due ordini, e collocandolo nella creazion sostanziale, riduce la dualità reale a un principio unico, all unità primordiale dell'Ente non astratto, complessivo, e generico, ma concreto, individuato, assoluto, e creatore."—*Del Bello e del Buono*, pp. 30, 31.

旨在依据思想与存在的类比建立一种宇宙理论。[16]在那里，原始的统一被表述为居于那唯一的永恒真理之中。自然对其规律的符合，道德行为者对于正直之指令的服从，都是在行动中所见的同一真理；世界本身不过是其作者自我反思之思想的一种表达。[17]更为显著的是十六、十七世纪间较古老思辨形式的复兴。洛伦佐·德·美第奇(Lorenzo the Magnificent)的朋友与同伴，在英国被称为剑桥柏拉图主义者的隐修士们，以及散布于欧洲的许多沉思的精神，都重新献身于要么解决那古老的问题--De Uno, Vero, Bono(论太一、真、善)，要么证明一切此类探究皆属徒劳无益。[18]所有这些思辨之下的逻辑元素，以及它们似乎至少借取其形式所自的那些逻辑元素，很容易在更为现代的体系的轮廓中追溯出来--尤其是在那种将绝对统一的学说与自我和非我之区分相结合、作为自然之类型的做法中，这构成了黑格尔哲学的基础。思辨的心灵试图攀升到某个真理的高峰，从那里他们可以在演绎思想的秩序中纵览事物的整个框架与联系，这些尝试在它们所产生的理论形式上的差异，不如在所赋予这些形式的解释之性质上的差异为大。[19]而哲学体系对性情与生活之影响的真正问题正在于此。因为，尽管人们一致地将存在的一切形式与条件追溯到一个原始的统一，这并无多大意义，但就他们对那个统一是什么、以及除了单纯的因果性之外还有哪些关系种类的概念而言，情况则不然，

神圣拉丁诗歌。)所有这些思辨所依据的原则，在上一注释所提及的论著中如此表述：Ουδεν ουν ηατοπον, εξ αμψδρον εικονον επι το παντον αιπον αναβαντας, ηψπερκοσμιοις οπητηαλμοις τηεοραεσαι παντα εν τπς παντον αιπιπς, και τα αλλαελοις εναντια μονοειδος και ηαεομενος.- *De Divinis Nominibus*，第v章。而如此力求达到的那种知识，被描述为"超越光明的黑暗",υπερφωτος γνοφος。(*De Mystica Theologia*，第i章。)弥尔顿(Milton)有类似的想法--

> "因过亮而显得暗，你的裙裾显现。"《失乐园》，第三卷。

与此相比，看《约翰一书》第一章第五节那更为高尚的简洁。

16《独白篇》《论证篇》与《论真理》。

17 "因此，当那至高无上的灵言说自身时，他便言说了一切受造之物。"--《独白》，第二十三章。

18 参见斯宾诺莎、米尔兰杜拉的皮科、H.莫尔等人的论文。这类性质的现代讨论主要与美学相关，其应用的基础包含在奥古斯丁的公式中："Omnis porro pulchritudinis forma, unitas est."

19 例如，焦贝蒂(Gioberti)那博学的神秘主义，尽管其精神与结论与黑格尔的泛神论大相径庭(两者或许同样远离真理)，却在将统一与二重性的原则同时应用于思想和存在这一点上与之相似。有人问道："难道不清楚吗，一切论述最终都归结为上帝、世界和创造的理念，其中最后一个乃是前两个的纽带?"而这个问题在公式"存在者创造诸存在"中得到肯定回答之后，关于该公式又说--"它在其必然与偶然的二重性中囊括了普遍实在，表达了这两个秩序的纽带，并将其置于实体性的创造之中，从而将实在的二重性归结为一个唯一的原则，归结为存在者之原初的统一--那存在者不是抽象的、总括的、属类的，而是具体的、个体化的、绝对的、创造性的。"--*Del Bello e del Buono*，第30、31页。

which it sustains to themselves. Herein too may be felt the powerlessness of mere Logic, the insufficiency of the profoundest knowledge of the laws of the understanding, to resolve those problems which lie nearer to our hearts, as progressive years strip away from our life the illusions of its golden dawn.

8. If the extremely arbitrary character of human opinion be considered, it will not be expected, nor is it here maintained, that the above are the only forms in which speculative men have shaped their conjectural solutions of the problem of existence. Under particular influences other forms of doctrine have arisen, not unfrequently, however, masking those portrayed above.[20]

But the wide prevalence of the particular theories which we have considered, together with their manifest analogy with the expressed laws of thought, may justly be conceived to indicate a connexion between the two systems. As all other mental acts and procedures are beset by their peculiar fallacies, so the operation of that law of thought termed in this work the law of duality may have its own peculiar tendency to error, exalting mere want of agreement into contrariety, and thus form a world which we necessarily view as formed of parts supplemental to each other, framing the conception of a world fundamentally divided by opposing powers. Such, with some large but hasty inductions from phænomena, may have been the origin of dualism,—independently of the question whether dualism is in any form a true theory or not. Here, however, it is of more importance to consider in detail the bearing of these ancient forms of speculation, as revived in the present day, upon the progress of real knowledge; and upon this point I desire, in pursuance of what has been said in the previous section, to add the following remarks:

1st. All sound philosophy gives its verdict against such speculations, if regarded as a means of determining the actual constitution of things. It may be that the progress of natural knowledge tends towards the recognition of some central Unity in Nature. Of such unity as consists in the mutual relation of the parts of a system there can be little doubt, and able men have speculated, not without grounds, on a more intimate correlation of physical forces than the mere idea of a system would lead us to conjecture. Further, it may be that in the bosom of that supposed unity are involved some general principles of division and re-union, the sources, under the Supreme Will, of much of the *related* variety of Nature. The instances of sex and polarity have been adduced in support of such a view. As a supposition, I will venture to add, that it is not very improbable that, in some such way as this, the constitution of things without may correspond to that of the mind within. But such correspondence, if it shall ever be proved to exist, will appear as the last induction from human knowledge, not as the first principle of scientific inquiry. The natural order

[20]Evidence in support of this statement will be found in the remarkable treatise recently published under the title (the correctness of which seems doubtful) of *Origenis Philosophumena*. The early corruptions of Christianity of which it contains the record, though many of them, as is evident from their Ophite character, derived from the very dregs of paganism, manifest certain persistent forms of philosophical speculation. For the most part they either belong to the dualistic scheme, or recognise three principles, primary or derived, between two of which the dualistic relation may be traced—*Orig. Phil.*, pp. 135, 139, 150, 235, 253, 264.

它对自身所维持的关系。在此也可以感受到纯粹逻辑的无能为力，感受到对知性规律最深刻的知识不足以解决那些更贴近我们心灵的问题，随着渐进的岁月从我们的生命中剥去其金色黎明的幻象。

8.如果考虑到人类意见极其任意的性质，那么就不会期望，这里也不主张，上述就是思辨的人们为存在问题的猜测性解答所塑造的唯一形式。在特定影响下，其他形式的学说也出现了，然而常常掩盖着上述所描绘的那些形式。[20]

但是，我们所考察的那些特定理论的广泛流行，连同它们与所表述的思维规律的明显类比，可以正当地被认为指示了两个体系之间的联系。正如所有其他心灵活动与过程都受其特有谬误的困扰，那种在本著作中被称为二重性规律的思维规律的运作，也可能有其特有的致错倾向，将单纯的不一致抬高为对立，从而形成一个我们必然视为由相互补充的部分所构成的世界，构设出一个从根本上被对立力量所分裂的世界的概念。这，连同一些从现象得出的大胆而仓促的归纳，也许就是二元论的起源--这与二元论无论以何种形式是否为真理论的问题无关。然而，在此更重要的是详细考察这些古代思辨形式(在当今复兴)对真实知识之进展的关系；而关于这一点，我愿依照前一节所述，补充以下意见：

1. 一切健全的哲学，若被视作确定事物实际构造的手段，都会对这类思辨作出否定的裁决。自然知识的进步或许趋向于承认自然中某种核心的统一性。至于那种由系统各部分相互关联所构成的统一性，则几乎无可置疑；而有才能的人也曾不无根据地推测，物理力之间存在着比仅仅一个系统的观念所能引导我们设想的更为紧密的关联。此外，或许在那被设想的统一性怀抱中，蕴含着某些划分与重新结合的一般原则，这些原则在至高意志之下，是自然之相关多样性的诸多根源。性别与极性的事例已被援引来支持这样一种观点。作为一个假设，我冒昧补充一句：事物外在的构造以某种类似于此的方式与心灵内在的构造相对应，并非很不可能。但这样一种对应，若终有一日被证明存在，也将作为人类知识的最后归纳而出现，而非作为科学探究的第一原则。自然的秩序

20 支持这一陈述的证据，可见于最近以*Origenis Philosophumena*为题出版的非凡论著(其标题的正确性似乎可疑)。该书记录的基督教早期腐化，尽管其中许多显然从其俄斐特派特征来看源自异教的渣滓，却表现出某些持久的哲学思辨形式。它们大多要么属于二元论体系，要么承认三个本原--原初的或派生的--其中两个本原之间可追溯出二元论关系--*Orig.Phil.*，第135、139、150、235、253、264页。

of discovery is from the particular to the universal, and it may confidently be affirmed that we have not yet advanced sufficiently far on this track to enable us to determine what are the ultimate forms into which all the special differences of Nature shall merge, and from which they shall receive their explanation.

2ndly. Were this correspondence between the forms of thought and the actual constitution of Nature proved to exist, whatsoever connexion or relation it might be supposed to establish between the two systems, it would in no degree affect the question of their mutual independence. It would in no sense lead to the consequence that the one system is the mere *product* of the other. A too great addiction to metaphysical speculations seems, in some instances, to have produced a tendency toward this species of illusion. Thus, among the many attempts which have been made to explain the existence of evil, it has been sought to assign to the fact a merely *relative* character,—to found it upon a species of logical opposition to the equally relative element of good. It suffices to say, that the assumption is purely gratuitous. What evil may be in the eyes of Infinite wisdom and purity, we can at the best but dimly conjecture; but to us, in all its forms, whether of pain or defect, or moral transgression, or retributory wo, it can wear but one aspect,—that of a sad and stern reality, against which, upon somewhat more than the highest order of prudential considerations, the whole preventive force of our nature may be exerted. Now what has been said upon the particular question just considered, is equally applicable to many other of the debated points of philosophy; such, for instance, as the external reality of space and time. We have no warrant for resolving these into mere forms of the understanding, though they unquestionably determine the present sphere of our knowledge. And, to speak more generally, there is no warrant for the extremely *subjective* tendency of much modern speculation. Whenever, in the view of the intellect, different hypotheses are equally consistent with an observed fact, the instinctive testimony of consciousness as to their relative value must be allowed to possess *authority*.

3rdly. If the study of the laws of thought avails us neither to determine the actual constitution of things, nor to explain the facts involved in that constitution which have perplexed the wise and saddened the thoughtful in all ages,—still less does it enable us to rise above the present conditions of our being, or lend its sanction to the doctrine which affirms the possibility of an *intuitive* knowledge of the infinite, and the unconditioned,—whether such knowledge be sought for in the realm of Nature, or above that realm. We can never be said to *comprehend* that which is represented to thought as the limit of an indefinite process of abstraction. A progression *ad infinitum* is impossible to finite powers. But though we cannot comprehend the infinite, there may be even scientific grounds for believing that human nature is constituted in some relation to the infinite. We cannot perfectly express the laws of thought, or establish in the most general sense the methods of which they form the basis, without at least the implication of elements which ordinary language expresses by the terms “Universe” and “Eternity.” As in the pure abstractions of Geometry, so in the domain of Logic it is seen, that the empire of Truth is, in a certain sense, larger than that of Imagination. And as there are many special departments

发现是从特称到全称的，并且可以有信心地断言，我们在这条道路上尚未前进得足够远，以致能够确定一切自然的特殊差异将最终融入哪些终极形式，并从这些形式获得它们的解释。

2. 即使思维形式与自然实际构造之间的这种对应被证明存在，无论它被设想为在两个体系之间建立起何种联系或关系，都丝毫不会影响它们相互独立的问题。它在任何意义上都不会导致这样一个后果：一个体系仅仅是另一个体系的产物。对形而上学思辨的过度沉迷，在某些事例中似乎产生了趋向这类幻觉的倾向。因此，在为了解释恶的存在而作出的许多尝试中，有人试图赋予这一事实一种纯粹相对的性质--将其建立在与同样相对的善的元素的一种逻辑对立之上。只需说，这一假定纯属无端。恶在无限智慧与纯洁的眼中可能是什么，我们至多只能模糊地猜测；但对我们而言，在其一切形式中，无论是痛苦还是缺陷，无论是道德过失还是报应的灾祸，它只能呈现一个面貌--一个悲哀而严峻的实在，针对它，基于某种高于最高审慎考虑的东西，我们本性的全部预防力量都可以施展出来。现在，关于刚才所考察的这一特定问题所说的话，同样适用于哲学中许多其他有争议的问题；例如，空间与时间的外在实在性。我们没有任何理由将它们化为纯粹知性的形式，尽管它们无疑决定着我们现在知识的范围。而且，更一般地说，现代许多思辨中那种极端主观的倾向是毫无根据的。每当在理智看来，不同的假说与一个观察到的事实同样一致时，意识关于它们相对价值的本能证词就必须被允许具有权威。

3. 如果思维规律的研究既不能帮助我们确定事物的实际构造，也不能解释那构造中所包含的、曾使历代智者困惑、使深思者悲伤的事实--那么它更不能使我们超越我们存在的当前状况，或为那种断言对无限者与无条件者具有直觉知识之可能性的学说提供认可--无论这种知识是在自然领域之内还是之上寻求。我们永远不能说理解了那在思想中被表现为一个无限抽象过程之极限的东西。一个无限的进程对于有限的能力是不可能的。但尽管我们不能理解无限者，却甚至可能有科学根据相信人性是在某种与无限者的关系中构成的。我们若不至少隐含地涉及普通语言以“宇宙”和“永恒”这两个词所表达的元素，就不能完善地表达思维规律，或在其最一般的意义上确立以它们为基础的方法。正如在几何学的纯粹抽象中一样，在逻辑的领域中也可以看到，真理的帝国在某种意义上大于想象的帝国。而且，正如有许多特殊的知识部门

of knowledge which can only be completely surveyed from an external point, so the theory of the intellectual processes, as applied only to finite objects, seems to involve the recognition of a sphere of thought from which all limits are withdrawn. If then, on the one hand, we cannot discover in the laws of thought and their analogies a sufficient basis of proof for the conclusions of a too daring mysticism; on the other hand we should err in regarding them as wholly unsuggestive. As parts of our intellectual nature, it seems not improbable that they should manifest their presence otherwise than by merely prescribing the conditions of formal inference. Whatever grounds we have for connecting them with the peculiar tendencies of physical speculation among the Ionian and Italic philosophers, the same grounds exist for associating them with a disposition of thought at once more common and more legitimate. To no casual influences, at least, ought we to attribute that meditative spirit which then most delights to commune with the external magnificence of Nature, when most impressed with the consciousness of sempiternal verities,—which reads in the nocturnal heavens a bright manifestation of order; or feels in some wild scene among the hills, the intimations of more than that abstract eternity which had rolled away ere yet their dark foundations were laid.[21]

9. Refraining from the further prosecution of a train of thought which to some may appear to be of too speculative a character, let us briefly review the positive results to which we have been led. It has appeared that there exist in our nature faculties which enable us to ascend from the particular facts of experience to the general propositions which form the basis of Science; as well as faculties whose office it is to deduce from general propositions accepted as true the particular conclusions which they involve. It has been seen, that those faculties are subject in their operations to laws capable of precise scientific expression, but invested with an authority which, as contrasted with the authority of the laws of nature, is distinct, *sui generis*, and underived. Further, there has appeared to be a manifest fitness between the intellectual procedure thus made known to us, and the conditions of that system of things by which we are surrounded,—such conditions, I mean, as the existence of species connected by general resemblances, of facts associated under general laws; together with that union of permanency with order, which while it gives stability to acquired knowledge, lays a foundation for the hope of indefinite progression. Human nature, quite independently of its observed or manifested tendencies, is seen to be *constituted* in a certain relation to Truth; and this relation, considered as a subject of speculative knowledge, is as capable of being studied in its details, is, moreover, as worthy of being so studied, as are the several departments of physical science, considered in the same aspect. I would especially direct attention to that view of the constitution of the intellect which represents it as subject to laws determinate in their character, but not operating by the power of necessity; which exhibits it as redeemed from the dominion of fate, without being abandoned to the lawlessness of chance. We cannot embrace this view without accepting at least as *probable* the intimations which, upon

[21] Psalm xc. 2

只能从外部观点才能完整地考察，同样，仅应用于有限对象的理智过程理论，似乎也涉及对一个一切界限都被撤去的思想领域的承认。因此，如果一方面我们不能在思维规律及其类比中发现一种过于大胆的神秘主义结论的充分证明基础；另一方面，我们若将它们视为完全无所启示，那就错了。作为我们理智本性的一部分，它们以仅仅规定形式推论之条件以外的方式表现自身，似乎并非不可能。无论我们有什么理由将它们与爱奥尼亚和意大利哲学家的物理思辨的独特倾向联系起来，同样的理由也存在于将它们与一种既更为常见又更为正当的思想倾向联系起来。至少，我们不应当把那种沉思的精神归因于任何偶然的影响--这种精神在最为感受到永恒真理之意识时，最乐于与自然的外在壮丽交流--它在夜空中读到秩序的明亮显现；或在山间某处荒野景色中，感受到某种超出那抽象永恒之暗示的东西，而那抽象永恒早在它们黑暗的基础奠定之前就已流逝。[21]

9. 我们不再进一步追究这一思路--在某些人看来它或许过于思辨--让我们简要回顾一下我们已被引向的那些积极成果。已经表明，我们的本性中存在着一些能力，它们使我们能够从经验的特称事实上升到构成科学基础的一般命题；同样也存在着一些能力，其职责是从被承认为真的一般命题中推出它们所包含的特称结论。已经看到，这些能力在其运作中服从于能够以精确的科学方式加以表述的规律，但又被赋予一种权威，这种权威与自然规律的权威相比，是独特的、*sui generis*、而非派生的。此外，已经表明，如此向我们揭示的理智程序与我们周围这一事物体系的诸条件之间存在着明显的契合--我指的是这样一些条件：由一般相似性联系起来的各个物种的存在，在一般规律之下相互关联的各个事实的存在；连同那种恒久性与秩序的结合，这种结合在赋予已获得的知识以稳定性的同时，也为无限进步的希望奠定了基础。人性，完全独立于其被观察到的或表现出的倾向，被看到是被构成在与真理的某种关系之中的；而这种关系，作为思辨知识的对象，正如物理科学的各个部门在同一方面被考察时一样，能够在其细节上被研究，而且同样值得被如此研究。我愿特别提请注意那种关于理智构造的观点，它把理智表述为服从于其性质确定的规律，但这些规律并非凭借必然性的力量而运作；它表明理智从命运的统治下被赎回，而又未被抛弃于偶然的无法则状态。我们不能接受这一观点，除非至少把那些暗示接受为或然的，这些暗示依据

21 诗篇 xc.2

the principle of analogy, it seems to furnish respecting another and a higher aspect of our nature,—its subjection in the sphere of duty as well as in that of knowledge to fixed laws whose authority does not consist in power,—its constitution with reference to an ideal standard and a final purpose. It has been thought, indeed, that scientific pursuits foster a disposition either to overlook the specific differences between the moral and the material world, or to regard the former as in no proper sense a subject for exact knowledge. Doubtless all exclusive pursuits tend to produce partial views, and it may be, that a mind long and deeply immersed in the contemplation of scenes over which the dominion of a physical necessity is unquestioned and supreme, may admit with difficulty the possibility of another order of things. But it is because of the *exclusiveness* of this devotion to a particular sphere of knowledge, that the prejudice in question takes possession, if at all, of the mind. The application of scientific methods to the study of the intellectual phænomena, conducted in an impartial spirit of inquiry, and without overlooking those elements of error and disturbance which must be accepted as *facts*, though they cannot be regarded as *laws*, in the constitution of our nature, seems to furnish the materials of a juster analogy.

10. If it be asked to what *practical* end such inquiries as the above point, it may be replied, that there exist various objects, in relation to which the courses of men's actions are mainly determined by their speculative views of human nature. Education, considered in its largest sense, is one of those objects. The ultimate ground of all inquiry into its nature and its methods must be laid in some previous theory of what man is, what are the ends for which his several faculties were designed, what are the motives which have power to influence them to sustained action, and to elicit their most perfect and most stable results. It may be doubted, whether these questions have ever been considered fully, and at the same time impartially, in the relations here suggested. The highest cultivation of taste by the study of the pure models of antiquity, the largest acquaintance with the facts and theories of modern physical science, viewed from this larger aspect of our nature, can only appear as parts of a perfect intellectual discipline. Looking from the same point of view upon the means to be employed, we might be led to inquire, whether that all but exclusive appeal which is made in the present day to the spirit of emulation or cupidity, does not tend to weaken the influence of those more enduring motives which seem to have been implanted in our nature for the immediate end in view. Upon these, and upon many other questions, the just limits of authority, the reconciliation of freedom of thought with discipline of feelings, habits, manners, and upon the whole *moral* aspect of the question,—what unfixedness of opinion, what diversity of practice, do we meet with! Yet, in the sober view of reason, there is no object within the compass of human endeavours which is of more weight and moment than this, considered, as I have said, in its largest meaning. Now, whatsoever tends to make more exact and definite our view of human nature, in any of its real aspects, tends, in the same proportion, to reduce these questions into narrower compass, and restrict the limits of their possible solution. Thus may even speculative inquiries prove fruitful of the most important principles of action.

类比的原则，似乎就我们本性的另一个更高方面提供了说明--即它在义务领域以及在知识领域中一样服从于固定的规律，这些规律的权威并不在于力量--它参照一个理想标准和一种最终目的而被构成。的确，人们曾认为，科学追求会养成一种倾向，要么忽视道德世界与物质世界之间的具体差异，要么把前者视为在任何恰当意义上都不是精确知识的对象。毫无疑问，一切排他性的追求都倾向于产生片面的观点，而且情况可能是，一个长期而深深地沉浸于对那些物理必然性的统治无可置疑且至高无上的景象的沉思之中的心灵，可能难以承认另一种事物秩序的可能性。但正是由于这种对某一特定知识领域的献身的排他性，所论的那种偏见才得以占据心灵，如果它确实占据了的话。把科学方法应用于理智现象的研究，以公正的探究精神进行，并且不忽视那些错误与扰乱的元素--它们必须被接受为事实，尽管在我们本性的构成中它们不能被看作规律--似乎提供了一种更为公正的类比的材料。

10. 如果有人问，上述这类研究指向什么实践的目的，那么可以回答说，存在着各种各样的对象，就这些对象而言，人们行动的方针主要由他们对人性的思辨观点所决定。教育，就其最广泛的意义而言，就是这些对象之一。一切对其本性和方法的探究的最终基础，都必须奠立于某种先前的理论之上：人是什么，他的各种能力被设计来达到的目的是什么，有哪些动机具有力量去影响它们持续行动并引出其最完美、最稳定的结果。可以怀疑，这些问题是否曾在此处所提示的种种关系中，被充分地、同时又公正地加以考察。通过研究古代的纯粹典范而对趣味进行最高程度的培养，对现代物理科学的事实和理论的最广泛的熟悉，从我们本性的这一更广阔的方面来看，只能表现为一种完善的理智训练的各部分。从同一观点来看待所要采用的手段，我们可能被引向询问：当今几乎唯一地诉诸竞争精神或贪欲的做法，是否并不倾向于削弱那些更为持久的动机的影响，这些动机似乎已被植入我们的本性之中，以直接服务于所着眼的目的。关于这些问题，以及关于许多其他问题--权威的正当界限，思想自由与情感、习惯、举止的纪律之间的调和，以及问题的整个道德的方面--我们遇到的是怎样的意见不定、怎样的实践分歧啊!然而，在理性的清醒看法中，在人类努力的范围之内，没有任何对象比这更为重要、更为关键，如我所说，就其最广泛的意义加以考察。现在，任何倾向于使我们对人性在其任何真实方面的看法变得更加精确和明确的东西，都按同样的比例倾向于把这些问题的范围缩小，并限制其可能解决的界限。这样，甚至思辨的探究也能证明富有最重要的行动原则。

11. Perhaps the most obviously legitimate bearing of such speculations would be upon the question of the place of Mathematics in the system of human knowledge, and the nature and office of mathematical studies, as a means of intellectual discipline. No one who has attended to the course of recent discussions can think this question an unimportant one. Those who have maintained that the position of Mathematics is in both respects a fundamental one, have drawn one of their strongest arguments from the actual constitution of things. The material frame is subject in all its parts to the relations of number. All dynamical, chemical, electrical, thermal, actions, seem not only to be measurable in themselves, but to be connected with each other, even to the extent of mutual convertibility, by numerical relations of a perfectly definite kind. But the opinion in question seems to me to rest upon a deeper basis than this. The laws of thought, in all its processes of conception and of reasoning, in all those operations of which language is the expression or the instrument, are of the same kind as are the laws of the acknowledged processes of Mathematics. It is not contended that it is necessary for us to acquaint ourselves with those laws in order to think coherently, or, in the ordinary sense of the terms, to reason well. Men draw inferences without any consciousness of those elements upon which the entire procedure depends. Still less is it desired to exalt the reasoning faculty over the faculties of observation, of reflection, and of judgment. But upon the very ground that human thought, traced to its ultimate elements, reveals itself in mathematical forms, we have a presumption that the mathematical sciences occupy, by the constitution of our nature, a fundamental place in human knowledge, and that no system of mental culture can be complete or fundamental, which altogether neglects them.

But the very same class of considerations shows with equal force the error of those who regard the study of Mathematics, and of their applications, as a sufficient basis either of knowledge or of discipline. If the constitution of the material frame is mathematical, it is not merely so. If the mind, in its capacity of formal reasoning, obeys, whether consciously or unconsciously, mathematical laws, it claims through its other capacities of sentiment and action, through its perceptions of beauty and of moral fitness, through its deep springs of emotion and affection, to hold relation to a different order of things. There is, moreover, a breadth of intellectual vision, a power of sympathy with truth in all its forms and manifestations, which is not measured by the force and subtlety of the dialectic faculty. Even the revelation of the material universe in its boundless magnitude, and pervading order, and constancy of law, is not necessarily the most fully apprehended by him who has traced with minutest accuracy the steps of the great demonstration. And if we embrace in our survey the interests and duties of life, how little do any processes of mere ratiocination enable us to comprehend the weightier questions which they present! As truly, therefore, as the cultivation of the mathematical or deductive faculty is a part of intellectual discipline, so truly is it only a part. The prejudice which would either banish or make supreme any one department of knowledge or faculty of mind, betrays not only error of judgment, but a defect of that intellectual modesty which is inseparable from a pure devotion to truth. It assumes the office of criticising

11. 也许这类思辨最明显正当的关联，将是关于数学在人类知识体系中的地位问题，以及数学研究作为一种理智训练手段的性质和职责问题。任何关注过晚近讨论进程的人都不会认为这个问题不重要。那些主张数学在这两方面都具有根本性地位的人，从事物的实际构成中引出了他们最强有力的论据之一。物质框架在其一切部分都服从于数的关系。一切力学的、化学的、电的、热的作用，似乎不仅在自身中是可度量的，而且通过一种完全确定的数值关系相互联系，甚至达到相互转化的程度。但在我看来，所论的这种意见立足于比这更深的基础之上。思维规律，在其一切概念和推理的过程中，在语言作为其表达或工具的一切那些运作中，与公认的数学过程的规律属于同一种类。这并不是主张，我们必须熟悉那些规律才能连贯地思考，或者按这些词的通常意义才能很好地推理。人们作出推论时，对整个过程所依赖的那些元素毫无意识。更不是想要把推理能力抬高到观察、反思和判断的能力之上。但正是基于人类思想在其终极元素上被追溯时以数学形式显示自身这一根据，我们有一种推定：数学科学凭借我们本性的构成，在人类知识中占据一个根本性的地位，而任何完全忽视它们的心理培养体系都不可能是完备的或根本性的。

但正是同一类考虑以同等的力量表明了那些人的错误，他们把数学及其应用的研究视为知识或训练的充分基础。如果物质框架的构成是数学的，它并不仅仅是数学的。如果心灵在其形式推理的能力中，无论有意识还是无意识，都服从数学规律，那么它通过其情感和行动的其他能力，通过其对美和道德适宜性的知觉，通过其情感和爱慕的深层源泉，要求与另一种事物秩序发生关系。此外，还有一种理智视野的广度，一种对一切形式和表现中的真理的同情力量，它不是由辩证能力的力度和精细来衡量的。甚至物质宇宙在其无限广袤、普遍秩序和规律恒常性中的启示，也未必被那个以最细微的精确性追溯了伟大证明的步骤的人最充分地领会。而如果我们把生活的利益和义务纳入我们的考察范围，那么任何纯粹推理的过程能使我们理解它们所提出的那些更为重要的问题又是多么少啊!因此，正如数学的或演绎的能力的培养是理智训练的一部分一样，它也真正地只是一部分。那种想要要么放逐要么使知识的任何一门部门或心灵的任何一种能力成为至上的偏见，不仅暴露出判断的错误，而且暴露出那种与对真理的纯粹献身不可分离的理智谦逊的缺陷。它僭取了批评的职责

a constitution of things which no human appointment has established, or can annul. It sets aside the ancient and just conception of truth as one though manifold. Much of this error, as actually existent among us, seems due to the special and isolated character of scientific teaching—which character it, in its turn, tends to foster. The study of philosophy, notwithstanding a few marked instances of exception, has failed to keep pace with the advance of the several departments of knowledge, whose mutual relations it is its province to determine. It is impossible, however, not to contemplate the particular evil in question as part of a larger system, and connect it with the too prevalent view of knowledge as a merely secular thing, and with the undue predominance, already adverted to, of those motives, legitimate within their proper limits, which are founded upon a regard to its secular advantages. In the extreme case it is not difficult to see that the continued operation of such motives, uncontrolled by any higher principles of action, uncorrected by the personal influence of superior minds, must tend to lower the standard of thought in reference to the objects of knowledge, and to render void and ineffectual whatsoever elements of a noble faith may still survive. And ever in proportion as these conditions are realized must the same effects follow. Hence, perhaps, it is that we sometimes find juster conceptions of the unity, the vital connexion, and the subordination to a moral purpose, of the different parts of Truth, among those who acknowledge nothing higher than the changing aspect of collective humanity, than among those who profess an intellectual allegiance to the Father of Lights. But these are questions which cannot further be pursued here. To some they will appear foreign to the professed design of this work. But the consideration of them has arisen naturally, either out of the speculations which that design involved, or in the course of reading and reflection which seemed necessary to its accomplishment.

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